I. Mathematics of finance - interest rates and growth rates

A. Amounts of money at different points in time are different goods

Amounts of money at different points in time are different goods. If you have the choice between 10000 rouble today and 20000 rouble tomorrow you probably will choose the higher amount of money. “Money today” and “ money tomorrow” are nearly perfect substitutes. Their rate of substitution is nearly 1. If you have the same choice between “Money today” and “money in seven years” your choice is not clear at all. “Money today” and “money in seven years” are imperfect substitutes. The reasons for this imperfection are

1. uncertainties and
2. interest.

Uncertainties can lie in your person or in the in the person, who promises the money, and you can be uncertain about the rate of interest and about the rate of inflation.

< Also see first exercise >

Continuous interest:

$K_t = K_o (1 + i^*)^t$ Annual payment of interest

$K_t = K_o (1 + i^*/ m)^{mt}$ With m: number of interest payments per year

Example $i^* = 4$ % per year = 0.04 as decursive interest (paid at end of year), t = 1; $K_0 = 100$.

m = 1: $K_t = 100 (1 + 0.04) = 104$;

m = 4: $K_t = 100 (1 + 0.01)^4 = 100*1.0406$

m = 8: $K_t = 100 (1 + 0.005)^8 = 100*1.0407$

You can see: if m is getting bigger, the final value rises, but the increase becomes more and more smaller. At infinitely small interest compounding periods, the final value moves towards:

$K_t = K_0 e^{it}$ $i = \ln (1 + i^*)$,

with e = 2.71828183… = Euler‘s number.

With i* 0.05 the continuous rate of interest is 0.0488. 0.0488 is the continuous interest, which is conforming to the decursive interest that means the one, which gives the same final value as the decursive interest (decursive interest means: interest is paid at the end of the year).

The 70ties rule: 70 / i = number of years in which an amount doubles (for a small i)

Deduction: $K_t = K_0 (1+i)^t \Leftrightarrow K_t / K_0 = (1+i)^t \Leftrightarrow \ln (K_t / K_0) = t \ln (1+i)$.

$\Rightarrow$ because of doubling $\ln 2 = t \ln (1+i) \Leftrightarrow \ln 2 / \ln (1+i) = t = 0.6931 / \ln 1.04$

= 0.6931/ 0.03922 = 17.76 this is approximated by 70/ 4 = 17.50.

B. Price –dividend ratios of companies with a growing dividend

Deduction of the factor of the present value of an annuity

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + ... + a_n \text{ , with } k = 1....n,$$

and with geometric sequence:

$$a_2 = x\, a_1 \rightarrow a_n = a_1\, x^{n-1}$$

$$\Rightarrow \sum_{k=1}^{n} a_k = a_1 + ax + ax^2 + ... + ax^{n-1} = s$$

(1) $s = a + ax + ax^2 + ... + ax^{n-1}$ (1)
(2) $sx = ax + ax^2 + \ldots + ax^{n-1} + ax^n$ (2)
(2) -(1) $\Rightarrow sx - s = ax^n - a = a(x^n - 1)$
$\Rightarrow s = a$ ---------
$s(x-1) = ax^n - a = a(x^n-1)$
$s = a\dfrac{x^n - 1}{x - 1}$ and with a = 1, $s = \dfrac{x^n - 1}{x - 1}$.

With: a = PMT = permanent cash flow,
x = 1 + i, and
s = Value of annuity at **end of period**

$$s = PMT \frac{x^n - 1}{x - 1} = PMT \frac{(1+i)^n - 1}{1 + i - 1} = PMT \frac{(1+i)^n - 1}{i}.$$

By multiplication with $1/(1+i)^n$ results

(3) Present value of annuity $PV = PMT \dfrac{(1+i)^n - 1}{(1+i)^n i}$.

$PV \lim n \rightarrow \infty = PMT \dfrac{1}{i}$ = Present value of eternal annuity = PMT* capitalization factor = PMT * 1/capitalization rate.
Example: capitalization rate i = 0,05; capitalization factor = 1/0,05 = 20.
Price - Earning – Ratios are capitalization factors: PV/PMT = 1/i

Example: Value of a house, wich yields a rent. With i = 0,05 => 1 / i = 20 => You have to pay 20fold the rent to by a house.

But what if the annuity is not fixed but is growing in time?

Rate of growth θ => $a_2 = a_1 * \theta$

(4) $$PV = \frac{PMT}{1+i} + \frac{PMT(1+\theta)}{(1+i)^2} + \frac{PMT(1+\theta)^2}{(1+i)^3} + ... + \frac{PMT(1+\theta)^{n-1}}{(1+i)^n}$$ = Present Value in t = 0

$$PV = \frac{PMT}{1+i}\left(1 + \frac{1+\theta}{1+i} + \frac{(1+\theta)^2}{(1+i)^2} + ... \frac{(1+\theta)^{n-1}}{(1+i)^{n-1}}\right) = \frac{PMT}{1+i}\frac{((1+\theta)/(1+i))^n - 1}{(1+\theta)/(1+i) - 1}$$

$$PV = \frac{PMT}{1+i}\frac{((1+\theta)/(1+i))^n - 1}{\frac{(1+\theta)-(1+i)}{1+i}} = PMT\frac{((1+\theta)/(1+i))^n - 1}{\theta - i}$$

$$PV \xrightarrow[\lim n\to\infty]{} = \textit{für } \theta > i : \infty$$

$$= \textit{für } \theta < i : PMT \, 1/_{i-\theta} \, .$$

This is the present value of an annuity eternally growing with θ.[1]

C. Stoppage of the debt service at unenforceable claims

1. When did debt crisis happen?

The expression “unenforceable claims is coined by

Niehans, Jürgen “Internationale Kredite mit undurchsetzbaren Forderungen”. In: „Die internationale Schuldenkrise. Ursachen-Konsequenz-Historische Erfahrungen“. Schriften des Vereins für Socialpolitik. Gesellschaft für Wirtschaft- und Sozialwissenschaften. Neue Folge. Band 155. p. 151-179.

The problem consists in the fact that today doesn’t exist enforcement with violence towards states. That‘s why a state can suspend its payments, even if it could still repay its debts.

The first debt crisis of the last 30 years was Poland‘s stoppage of payment. The Chancellor of the Federal Republic of Germany had asked rhetorically: “Why do we need private banks, if they can’t even give a credit of 1 billion to Poland?” As a result a consortium of banks granted the credit: until the stoppage of payment, the so called “umbrella theory” mislead various lenders. The “ umbrella theory” said: The Soviet Union will not allow that a state of the Warsaw Pact in convertible currency becomes insolvent. This theory was falsified in 1980.

[1] See: Williams, John Burr: The Theory of investment value, 2nd ed. Amsterdam 1956 (1938).

In 1982 Mexico suspends its payments: because of sinking oil prices and sinking US-imports caused by crisis and because of high US-interests. After this stoppage of payment of Mexico several so called “high indebted counties” and other Latin American states followed this step.
In the beginning of 1995, there was a danger of another stoppage of payment of Mexico. The Pesos was devalued by 40%. 20 billion of index – linked bonds could not be serviced. The IMF granted credits. In 1997 South Korea, Malaysia, Thailand and Indonesia came into shortage of liquid funds. (So called Asiatic debt crisis). In 1998 Russia stopped payments (compare for example Iwkoeln.de. IWD Nr.40 of October 1st 1998).
Credits are taken to satisfy the need of consumption of the population (Poland), to invest, but also to finance capital flight (intended or unintended).

2. Credits to countries with convertible currencies

If countries temporarily or even totally stop their payments, it affects those, who hold government bonds in their portfolio like banks, insurance companies and private investors.
Beside the direct owners of government bonds, owners of bank deposits and of insurance claims are affected indirectly. It affects a wide circle of debtors, which are electors at the same time. Therefore governments cannot afford to default because in this case they are voted out.

3. Credits to countries with inconvertible currency (foreign exchange controls)

If countries temporarily or completely stop their payments for claims on foreign exchange, it affects those, who hold these government bonds in their portfolio. The population is not allowed to own foreign exchange though. Consequently it doesn’t have foreign currency claims towards its state. Only foreigners are affected by the stoppage of payments.
The risk of a stoppage of payment is in this case a lot higher for creditors, because the state will only be servicing, as long as the newly taken credits do exceed the debt service!
As a first question we discuss:
What conditions the credit worthiness of a state?
Or formulated differently: How many debts can a state make?

Therefore we assume the following facts:

For all outflows of payments have to exist inflows of payments, as taxes, loan taking, or as of other ways of raising money. The inflow of payments of a government equals the tax revenues. Consequently the taxes (T) at a **public sector share in the GNP (**a) of 50% equal the amount:

T = a Y; with a = 0.5: T = 0.5*Y.

As the government needs its funds also to finance other things than the payment of interests (for instance salaries for public servants, police, education, national defence, road building), we assume that only 10% of its inflows are available to finance interests. Than the **debt service quota** (b) is b = 0.1 T). Interest rate shall be i = 0,08. Y shall be 100 Epsos.

We assume that all debts are rescheduled into eternal debts, as for example British consols.

Example: Debt maximum(B): B = 0.1* 0.5 * 100 Epsos/0.08 = 62.5 Epsos. The debt maximum is 62.5% of the GDP at an interest of 8% p.a. This amount of debt can maximally be serviced, if there is no growth. Beyond this debt maximum, the government is not anymore able to repay its debts. It must stop payments. The interest quota is then 0.625*0.08 =0.05. The maximum interest quota is consequently 5% of the GDP. This border is of course not rigid. In the Euro currency area the expenditures for interest were 4.2 % of GDP in the year 1999 and in the year 2002 they were 3.7 % of GDP. In the Federal Republic of Germany, the expenditures for interests in percent of public households come to 3.7 % of GDP; in 2002 they were 3.3 % of the GDP.

Has a state **with growth** of GDP and a debt position of 70 Pesos exhausted its limit?

The credit limit in a stationary economy B = abY/i would be with a = 0.5, b = 0.1, i = 0.08 and Y = 100 Epsos, 62.5Epsos, as calculated above.

The National Product at the point of time (t) results at growth out of National Product at the point of time 0 multiplied with the growth factor:

$$Y(t) = Y_0 e^{\theta t}$$

According to the equation (4), for the capitalization factor of a growing economy results:

B = abY/ (i-θ): 0.1* 0.5 * 100 Epsos/(0.08-θ) as the credit limit, estimating the rate of growth (θ). For θ = 2% p.a. results a credit limit of 0.01*0.05*100/ (0.08 - 0.02) = 83.3 % of the GDP compared to 62.5 % of the GNP without growth.

Corresponding to the estimated rate of growth result the following possibilities:

- High rate of growth (θ): the denominator becomes smaller, the credit limit higher.
- Negative rate of growth (θ): the denominator becomes bigger, which limits credit possibilities.

Consequently, the credit limit of a state depends, apart from other things, of the rate of growth (θ). The outstanding debts cannot always be collected. Contrary to private subjects or enterprises, claims towards sovereign states are hard to enforce, as seizure of assets is impossible. Empirically it was found out that international trade decreases by 10 % in average[2]. This is the most important remaining, negative consequence of a stoppage of a debt service in states without a convertible currency. Besides credit worthiness is damaged. We discuss the role of decreasing credit worthiness for the decision to stop the payments, in the following paragraph.

Historical examples for unenforceable claims can be found already in the 18. Century, also the expropriation of the Suez Channel by Egypt in the year 1957. Also current claims towards atomic powers as the United States and Russia are as hard to enforce as those of EU towards Italy as a member of the EU. Another current example: In 2002 Germany was happy to receive 500 Million DM from Russia as the legal successor of the Soviet Union instead of 6 billion of inherited claims of Democratic Republic of Germany.

Why do states service their debts also at a considerable obligation to pay interests?

Besides the international rating that leads to low interests, there is a simple, rational reason for servicing debts: the debts are (only) serviced as long as the net cash inflows are positive (in the long run).

For debt service to be maintained we have (1) $C= dB -i\,B > 0$.

This means: the net cash inflows have to positive on the long run.

We denote

B: Bonds (=debt)

dB: inflows from new indebtedness

i B: outflows for debt service.

The debts, a state can make, result according to the premise on top relative to GDP, with a percentage of $a*b/i-\theta = 0.05/(i-\theta)$

As the credit limit $B(t) = a*b*Y$ is a linear transformation of Y, it is valid:

$Y(t) = Y_o * e^{\theta t} \Rightarrow B(t) = B_0 * e^{\theta t}$, that means if GDP is growing, the credit limit grows as well.

Has a state reached the credit limit, he can take new credits only if the credit limit increases.

2 Rose Andrew K. : One Reason Countries Pay Their Debts: Renegotiation and International Trade. Staff Report FRBNY:NO: 142. DEC. 2001 http://app.ny.frb.org/rps/results.cfm?SearchType=JC (29.4.02)

Putting in into (1): C (t) = dB/dt – i B $e^{\theta t}$

We have: s = e^{at} $\Rightarrow$ ds/dt = a e^{at} (Comment: Continuous interests are easer differentiable.)

(2) C (t) = θ * B $e^{\theta t}$ – i B $e^{\theta t}$ = (θ -i)* B $e^{\theta t}$

It is about inflows, if θ > i $\Rightarrow$ the government is servicing, if i > θ (lasting) it stops its payments.

Mexico's stoppage of payment in 1982 shows the typical reasons.

Mexico: strong inflation in the United States between 1975 and 1980, then stabilisation ?demand for commodities of the United States decreased $\Rightarrow$ Export of commodities by Mexico decreases; at the same time the interest in the United States were increasing, because of the stabilization policy of the United States: After 1980 the oil prices began to sink. This as well led to sinking expectations for growth of Mexican exports.

Data for Mexico in 1982:

GDP in 1981: 270 billion US-$ at current prices and exchange rates

GDP in 1982: 190 billion US-$ at current prices and exchange rates

Source: DataStream

Total indebtedness 380 billion US-$; on the base of the GDP in 1981, it was 140 % of GDP; on the base of the GDP in 1982 it was 200%.

If the sum of all discounted future inflows is less than 0, it comes to a debt crisis.

C (t) = $\int_T^{\infty} (\theta B e^{\theta t -} i B e_{\theta t}) e^{-\theta t} < 0$.

This means: the payment of interests is stopped, if the sum of all discounted in- and out flows is negative, i > θ . They are serviced, if the sum of the all discounted payments is positive i < θ .

Stoppage of payment is a rational decision. The criterion for this decision is the comparison of i with θ .

The centrepiece of debt crisis is **expectation of growth**.

D. Limits to Government Debt

We denote:

B	Bonds, = $B_0 + \dot{b}Y_0 \int_0^T e^{\theta t} dt$ = $B_0 + \frac{\dot{b}Y_0}{\theta}(e^{\theta T} - 1)$.
Sign regulation for B:	positive net financial assets have the sign + Negative net financial assets have the sign -
Y	National income (including interests) = Net social product
i	Interest rate
θ	Rate of growth of the disposable national income
$\dot{B}$	Budget balance; $\dot{B} < 0$: deficit, $\dot{B} > 0$: surplus
$P\dot{B}$	Primary budget balance; $P\dot{B} < 0$: deficit $P\dot{B} > 0$: surplus; $P\dot{B} = \dot{B}$ - i B
$p\dot{b}$	Primary deficit quota $P\dot{B}$/Y
$\dot{b}$	Deficit quota $\dot{B}$/Y;
$(\cdot)_r$	Real amounts
$(.)_n$	Nnominal amounts

1. Under which circumstances is the debt quota (B_t/Y_t) permanently increasing?

Answer: when $\Delta B / B > \Delta Y / Y$, that is, if B is growing at a higher rate than Y
If you extent the first fraction with 1/Y, you can write as well:
(1) $\dot{b}/(B_t/Y_t) > \theta$, oder $\dot{b}/(B_t/Y_t) - \theta > 0$.

Proposition 1: If the rate of growth of the debt quota is bigger than the rate of growth of the national income, the debt quota is rising beyond all bounds.

2. Are permanent budget deficits leading to a debt quota rising beyond all bounds?

In the case, $\dot{b} = \Delta B / Y$ is not depending on time, this reflection leads to the determining equation for the amount of the debt quota, which occurs after an infinite number of periods (long-term equilibrium), if the deficit quota ($\dot{b}$) and the rate of growth (θ) are given **(Domar, Evsey D.: The "Burden of the Debt" and the National Income. In: AER vol. 34 (1944), S. 798-827.)**

If you divide the debts $B_t = B_0 + \frac{\dot{b}Y_0}{\theta}(e^{\theta T} - 1)$

by the national income, Y_T

you get $\frac{B_T}{Y_T} = \frac{B_0}{Y_0 e^{\theta T}} + \frac{\dot{b}}{\theta}(1 - e^{-\theta T})$.

For $\lim_{t \longrightarrow \infty}$ results

(2)B/Y = $\dot{b}$ /θ.

Proposition 2: Is the rate of growth 0, even the smallest permanent deficit quota is leading to a debt quota rising beyond all bounds.
However, if the rate of growth is positive, every deficit quota is leading permanently to a stable value for the debt quota regardless, if the starting point of the debt quota is higher or lower than this value.

Proposition 3: At a constant deficit quota, the debt quota is sinking with an increasing rate of growth and moves for θ $\rightarrow \infty$ *towards zero.*

Application: A deficit quota of 1% leads with a growth rate of 1,67% p.a. , to a debt quota of 60%, a deficit quota of 3% leads at a growth rate of 5% to a debt quota of 60%.

3. Under which circumstances, the interests can be paid only by a new credit, without that the debt quota is rising beyond all measurements?

We are asking under which circumstances, the debt quota is rising beyond all measurements and draw then the argumentum e contrarium (from the contrary).

.

The interests are calculated as i B, and the interest quota is calculated as i B/Y = i $\dot{b}$ /θ.
Is the inequality (1), $\dot{b}$ /(B/Y) - θ > 0, multiplied with B/Y; you have to distinguish 2 cases B positive und B negative. We are only looking at the more important case: **B in B/Y is negative** (that means a government bond exist). We obtain:

$\dot{b}$ - θ B/Y < 0.

The multiplication of an inequality with a negative number is turning the inequality sign around. Further we add to 0 = i B/Y - i B/Y Then we can, under the consideration of

3) $\dot{B} = P\dot{B}$ + i B, or $\dot{b} = P\dot{B}$ /Y + i B/Y = $p\dot{b}$ + i B/Y

(Verbal: The budget deficit is equal with the primary budget deficit (budget deficit without the consideration of the interest expenditures) plus interest payment),
examine:

(4) $\dot{b}$ - i B/Y - θ B/Y + i B/Y < 0, or:

(B/Y)(i-θ) + $p\dot{b}$ < 0

Permanent primary deficit means that $p\dot{b}$ is negative. Is as well i - θ positive, so is the inequality (4) always fulfilled (since B is negative). That means, that the debt quota is permanently increasing, as the inequality (4) is the condition for an increasing debt quota.

Should the debt quota not increase beyond all measurements, the following has to be valid

(5) (B/Y)(i-θ) + $p\dot{b} \geq 0$.

The possibility to reach a permanent primary deficit, without that the debt quota moves towards infinite, is only given, if i < θ

Is B negative and i > θ and should the debt quota not rise beyond all measurements, the following should be valid:

(5) $(B/Y)(i-\theta) + p\dot{b} \geq 0$,

So $p\dot{b}$ has to be positive in the long-term equilibrium, a primary excess has to be obtained. Should the debt quota not increase, the primary quota of excess has to be equal or bigger as the debt quota multiplied with minus (i-θ).

(6) $p\dot{b} \geq -B/Y\ (i-\theta)$, for i > θ and.

Under consideration of (2), this is

$p\dot{b} \geq -\ (i-\theta)\ \dot{b}/\theta$, für i > θ.

A permanent primary deficit quota in the long-term equilibrium with a public deficit, can only exist, if θ > i, whereby has to be valid:

$p\dot{b} \geq (\theta-i)\ \dot{b}/\theta$.

The primary deficit quota should not be smaller (oder absolut nicht größer) as the deficit quota multiplied with the difference of (θ - i)/θ.

Proposition4: For every deficit quota is valid, that the interests can be covered by new borrowings, if the rate of interest is not bigger than the rate of growth. (Equation4). The taxes can then be used completely for the primary national expenditures (expenses without interests).

Proposition 5: For every deficit quota is valid, that the interest quota at a constant deficit quota with increasing θ is getting smaller and for $\theta \rightarrow \infty$ moves towards cero.

4. Which deficit quota can be maintained at a given primary excess quota $p\dot{b}$ and given rate of growth θ, if i>θ?

Inequality (5) turns at a long-term equilibrium into:

$(\dot{b}/\theta)(i-\theta) + p\dot{b} \geq 0$.

Solved by $\dot{b}$ results for i>θ:

$\dot{b} \geq -\ p\dot{b}\ \theta/(i-\theta)$.

Proposition 6: At a given primary excess quota, the permanently possible deficit quota moves towards the primary excess quota weight with θ/(i-θ)

Application: If you assume realistically, that the solvency of a government depends on the maximally realisable primary excess quota $p\dot{b}$, so is the permanently realisable deficit quota at θ=0,015 and i=0,04 equal 0, 6 $p\dot{b}$. Is the primary excess quota for example 0,05 (a very high amount, which would be considered insupportable) or 0,02 (a just about tolerable amount), the deficit quota takes on minimally the value -0,03 respectively. -0,012.

5. How high the primary deficit quota has to be respectively can be, if $\dot{b}$, i, and θ are given?

Inequality (5) turns at the long-term equilibrium into:

$(\dot{b}/\theta)\ (i-\theta) + p\dot{b} \geq 0$.

Solved by $p\dot{b}$, it results:

$p\dot{b} \geq -\dot{b}$ (i-θ)/θ.

Proposition 7: the primary excess quota should not be smaller than the deficit quota, multiplied with (i - θ)/θ. Is θ > i, the inequality is the condition for primary deficit quota that is possible maximally in the in long-term equilibrium

5. How have the propositions three, four and five to be modified, if i and θ are not independant from each other.

The independence of i und θ, could be assumed, if i and θ are interpreted as real variables. Are i_n and θ_n however dependant as such that

$i_n = i_r + \pi + i_r\pi$ and $\theta_n = \theta_r + \pi + \theta_r\pi$, the interest quota i B/Y at long-term equilibrium turns into:

(7) $iB/Y = \dot{b}\dfrac{i_r + \pi + i_r\pi}{\theta_r + \pi + \theta_r\pi}$.

for $\pi \to \infty$, $i\,B/Y = \dot{b}\dfrac{\frac{i_r}{\pi} + (1 + i_r)}{\frac{\theta_r}{\pi} + (1 + \theta_r)}$ moves towards $\dot{b}\dfrac{1+i_r}{1+\theta_r}$.

Is the difference between i_r und θ_r very small, the interest quota moves towards the deficit quota, if $\pi \to \infty$ is valid.

Proposition 8:For the deficit quota is valid, that permanently the interests can mainly paid by new borrowings, if $\pi \to \infty$, regardless if $i_r > \theta_r$ or $i_r < \theta_r$ assumed that $i_r \approx \theta_r$ .Taxes can be almost completely used for primary national expenditure is· $i < \theta$, a permanent deficit of the primary household can be realised which is becoming more and more small compared to Y, and is $i > \theta$ an excess of the primary household has to be realised, which is also becoming more and more small compared to Y, if $\theta \to \infty$ is valid.

Proposition 9: For every deficit quota is valid, that the interest quota is getting smaller at a constant deficit quota with an increasing θ and for $\theta \to \infty$ moves *towards* $\dot{b}\dfrac{1+i_r}{1+\theta_r}$.

Burden of debt at different rates of growth and interest

Symbol: B = (Bonds),[DM] Y = Gross domestic product (GNP) [DM/Period]

θ = Rate of growth of the GNP, [1/Period] θ_n, θ_r = nominal respectively real rate of growth

i = interest rate, [1/Period] i_n, i_r = nominal respectively real interest rate

I = Interests [DM/Period] $\dot{b}$ = deficit quota = ΔB/Y [ ]

Table 1: Limiting value of national debt (for $t \to \infty$) in percent of gross domestic product (GDP) (B/Y=$\dot{b}$/θ) at a growth rate of GDP θ and a deficit quota $\dot{b}$=ΔB/Y

$\dot{b}$		0,001	0,010	0,020	0,030	0,050	0,100	0,200	0,500	1,000
θ		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0,0001	A	10	100	200	300	500	1000	2000	5000	10000
0,0167	C	0,060	0,600	1,200	1,800	3,000	6,000	12,000	30,000	60,000
0,0333	D	0,030	0,300	0,600	0,900	1,500	3,000	6,000	15,000	30,000
0,0500	E	0,020	0,200	0,400	0,600	1,000	2,000	4,000	10,000	20,000
0,0833	F	0,012	0,120	0,240	0,360	0,600	1,200	2,400	6,000	12,000
0,1667	G	0,006	0,060	0,120	0,180	0,300	0,600	1,200	3,000	6,000
0,3333	H	0,003	0,030	0,060	0,090	0,150	0,300	0,600	1,500	3,000
0,8333	I	0,001	0,012	0,024	0,036	0,060	0,120	0,240	0,600	1,200
1,6667	K	0,001	0,006	0,012	0,018	0,030	0,060	0,120	0,300	0,600

The national debt converges for $t \to \infty$ towards the amounts in the table: Field E4 for instance indicates that, $\dot{b}$ = 3% and θ ≥ 5%, so that the rate of debt after an infinite number of periods is not higher than 60%. Here it is insignificant, if the actual rate of debt is bigger or smaller than 60%. Table 1 says also that the deficit quota can be the higher; the higher the rate of growth is, before a critical rate of debt is exceeded. Is the rate of growth of the real GDP for example 0,02 p.a., the difference θ - 0,02 indicates the rate of inflation, which is at least required, to keep the condition B/Y = 60%

Table 2: Interests on the government bond as percentage of the GDP (I/Y) at a rate of interest of 4% p.a.: I/Y = 0,04 $\dot{b}$/θ after an infinite number of periods

$\dot{b}$		0,0125	0,0250	0,0500	0,1000	0,2000	0,4000	0,8000	1,0000
θ		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0,0100	A	0,0500	0,1000	0,2000	0,4000	0,8000	1,6000	3,2000	4,0000
0,0200	B	0,0250	0,0500	0,1000	0,2000	0,4000	0,8000	1,6000	2,0000
0,0400	C	0,0125	0,0250	0,0500	0,1000	0,2000	0,4000	0,8000	1,0000
0,0800	D	0,0063	0,0125	0,0250	0,0500	0,1000	0,2000	0,4000	0,5000
0,1600	E	0,0031	0,0063	0,0125	0,0250	0,0500	0,1000	0,2000	0,2500
0,3200	F	0,0016	0,0031	0,0063	0,0125	0,0250	0,0500	0,1000	0,1250
0,4000	G	0,0013	0,0025	0,0050	0,0100	0,0200	0,0400	0,0800	0,1000

The table shows analogously to table 1 in the diagonal that the deficit quota can be all the higher the higher is the rate of growth, before the interest quota (I / GDP) has reached a certain amount, for example the amount 0.1 in the diagonal. The columns show that the interest quota at a constant rate of (0,04), is decreasing with an increasing rate of growth. Row C indicates that the interest quota moves towards the deficit quota, if the rate of interest and the rate of growth of GDP are equal (i = θ). The interests are then covered by new borrowings. Taxes can be used for primary government expenditures. The primary expenditure quota (= expenditure quota - interest quota) is then equal with the tax quota. Differently said: the primary deficit quota (= deficit quota – interest quota) then equals 0.

Burden of debt at different rates of growth and interest with inflation

Table 3: Payments of interest as percentage of GDP at a real growth rate of 2%p.a., a real interest rate of 4 % p.a., and a nominal interest rate $i_n = (1+i_r)*(1+\theta_n)/(1+\theta_r) - 1$ Rate of inflation $\pi = (1+\theta_n)/(1+\theta_r)$

$\dot{b}$		0,0125	0,0250	0,0500	0,1000	0,2000	0,4000	0,8000	1,0000	Interes i_n	Rate of inflation π
θ_n		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0,01	A	0,0373	0,0745	0,1490	0,2980	0,5961	1,1922	2,3843	2,9804	0,03	-0,01
0,02	B	0,0250	0,0500	0,1000	0,2000	0,4000	0,8000	1,6000	2,0000	0,04	0,00
0,04	C	0,0189	0,0377	0,0755	0,1510	0,3020	0,6039	1,2078	1,5098	0,06	0,02
0,08	E	0,0158	0,0316	0,0632	0,1265	0,2529	0,5059	1,0118	1,2647	0,10	0,06
0,16	F	0,0143	0,0286	0,0571	0,1142	0,2284	0,4569	0,9137	1,1422	0,18	0,14
0,32	G	0,0135	0,0270	0,0540	0,1081	0,2162	0,4324	0,8647	1,0809	0,35	0,29
0,40	H	0,0134	0,0267	0,0534	0,1069	0,2137	0,4275	0,8549	1,0686	0,43	0,37
1,00	I	0,0130	0,0260	0,0520	0,1039	0,2078	0,4157	0,8314	1,0392	1,04	0,96
1000,00	J	0,0127	0,0255	0,0510	0,1020	0,2039	0,4079	0,8157	1,0196	1019,63	980,37

The table shows in column 3, $\dot{b}$ = 0,05, that the interest quota is sinking even at an increasing rate of nominal GNP if the interests are calculated according to the Fischer equation, but a lot weaker than at a constant interest in table 2. Apart, the interest quotas in the diagonal are higher at inflation and lower at a deflation compared to table 2.

But further table 3 shows:

- that higher deficit quotas can be realised with an increasing rate of inflation at a constant interest quota, (compare cell A4 (0,298) ≅ cell C5 (0,302)),
- but that rates of inflation have to move towards infinite for the interest quota to sink to the level of the deficit quota (column 3, respective last row).

Remarks

Historical: Many countries got rid of their debts in the past by inflation. England had after the Napoleon Wars and after the Second World War a considerably higher debt quotas as in recent times (in 2003 it was 39% of the GDP). After the Napoleonic War population growth led to a diminishing debt quota. After Second World War inflation combined with sound budget policy had the same effect. The excess inflation towards Germany shows in depreciation of the British Pound. It sank from 20 DM/£ sank to 3 DM/£ (1.48€/£ *1.99583 DM/€) Also the United States had a higher debt quota at the times of Second World War. Germany repudiated domestic debt (= refuse to pay debts) and could pursue the stabilization policy.

After the realisation of the Economic and Monetary Union, inflation policy pursued by an individual state is not possible anymore.

Primary quantities:

Budget deficit - Payment of interest = Primary budget deficit

Government spending - Payment of interest = Primary government spending

Example: Interest = 0.05 p.a. ; B/Y = 60 %

0.60* 0.05 = 0.03 of GDP are interests

Interest quota in % of the GDP? In 2003 in the Federal Republic of Germany: Deficit = -3.9 % of the GDP[3]

Interest quota Germany 2003 = 66.2 billion €[4]/2.1929 billion €[5]= -3.1%;

It results a primary deficit quota of -3.9 % -(-3.1%) = - 0.8 %.

Only more growth of the **legal** economy (the shadow economy is growing!) can improve the budget position. Clinton tried to get support with the slogan" It's the economy stupid." The meaning was: care for growth.

-

Discussion of abovementioned formulas

$$\frac{\dot{B}/Y}{B/Y} > \frac{\Delta Y}{Y} \Leftrightarrow \frac{\dot{b}}{B_t / Y_t} - \theta > 0$$

[3] Compare monthly bulletin of the European Central Bank (ECB)

[4] Compare monthly bulletin of the Deutsche Bundes Bank, 56. Jg, Nr. 3. March 2004, p. 53*

[5] Compare monthly bulletin of the Deutsche Bundesbank, 56. Jg. Nr. 3. March 2004, p. 60*

Deduction

$$\frac{B_t}{Y_t} = \frac{B_0}{Y_0 e^{\theta t}} + \frac{\dot{b} Y_0}{Y_t \theta}(e^{\theta t} - 1) = \frac{B_0}{Y_0 e^{\theta t}} + \frac{\dot{b}}{\theta}(1 - e^{-\theta t})$$

(If s = e ^{at}, the ∂s/∂t = a e ^{at} =>if original function S = (1/a) e ^{at}, then ∂s/∂t = e ^{at} and reversed $\int e^{at} dt = \frac{1}{a} e^{at}$)

$$\frac{B_T}{Y_T} = \frac{B_0}{Y_0 e^{\theta T}} + \frac{\dot{b}}{\theta}(1 - e^{-\theta T})$$

$$\lim_{t \to \infty} \frac{B_t}{Y_t} = 0 + (\dot{b} / \theta)(1 - 0) = \dot{b} / \theta \text{, if } \theta > 0$$

Example:

The criteria of Maastricht planned a maximum deficit of 3% of the GDP and a debt quota of 60 %. Growth Θ = 2%.

60 = 3 / Θ <=> Θ = 1 / 20 = 0,05 this means that the necessary rate of growth has to be 5 for the both criteria to be compatible on the long-term.

If instead of real 5% only real 2 % are achieved, you need 3 % inflation for the criteria to be compatible on the long-term.

Domar, Evsey D.: **The "Burden of the Debt" and the National Income. In: AER vol. 34 (1944), S. 798-827.**

Amount of debt and deficit quota are not alone decisive you have to take the rate of growth into consideration.

Table 1:

In the moment a rate of growth of 1.67 % are rather typical for the big EU-States. It results from the table that a maximum deficit quota of 1 % is allowed for the long-term achievement 60 % indebtedness quota. In this sense the proposition of the former Federal Minister of Finances Waigel (fixation of the deficit quota to 1%)is explainable.

Table 2 indicates, at which rate of growth the interest can be covered by net credit intake.

Fischer Equation:

$(1 + i_n) = (1 + i_r) * (1 + \pi)$

Nominal rate of interest factor = Real rate of interest factor * Rate of price increase

(As the Fischer equation is often not valid in reality: see WS 23 ➔ Negative Real rate of interest)

$$\frac{B_t}{Y_t} * i = \frac{\dot{b}}{\theta} * i$$

Theorem 3: from inequality (1)

$$\dot{b} / \left(B_t / Y_t \right) > \theta$$

extended with B/Y, for B<0, and + i (B/Y) - i (B/Y)

(4) $\dot{b} - i\frac{B}{Y} - \theta\frac{B}{Y} + i\frac{B}{Y} < 0$

Under consideration of the equation for the primary deficit, it results:

$$\dot{B} = P\dot{B} + iB$$

Example: $P\dot{B} = 10, \quad B = 200 \quad i = 0{,}04$

$\dot{B}$ = - 10 + 0,04 * -200 = -10 - 8 = -18 Primary deficit + interest rate * debts = deficit

$$\dot{b} = p\dot{b} + i\frac{B}{Y}$$

(5) $B / Y(i - \theta) + p\dot{b} < 0$.

This is the condition for the debts to rise beyond all bounds.

Then (6) $B / Y(i - \theta) + p\dot{b} \geq 0$ is the condition for the debt position **not** to rise beyond all bounds.

If $p\dot{b}$ is cero, we don't have a primary balance (balanced primary budget), then (i - θ) $\leq$ 0 has to be valid, for the inequality (6) to be fulfilled. Is $p\dot{b}$ permanently negative, it has to be valid:

(i - θ) <0,and only if $\dot{pb}$ is permanently positive i > θ can be valid, but the lesser, the more as $\dot{pb}$ moves towards cero and the greater B/Y is.

How great can the primary deficit be at realistic debts B/Y = 0.6 with i = 0.04 and θ = 0 0.05?

B / Y * (0,04 - 0,05) $\geq -\dot{pb}$ <=> -0,6 * (- 0,01) $\geq -\dot{pb}$ ′ <=> 0,006 $\geq -\dot{pb}$

-0,006 $\leq \dot{pb}$!

Consequently, the primary deficit has to absolutely less than $|0.006|$ of the BIP, if B/Y = - 0,6 and if the rate of growth lies one percent above the rate of interest.

II. <u>What is money?</u>

A. How does an object turn into money? In which forms did it appear in the past?

An object is money, if you can buy at many people (everywhere, ubiquitous) many or even all goods. This seems to be trivial. All goods, which can be bought for money, are called merchandises. The quality to be able to buy a bigger or smaller circle of goods is described by Gerloff[6] as as “purchase capacity”, not identical with the English expression purchasing power meaning a quantity of goods which you can buy for a certain amount of money. Dollar is ubiquitous money. Some kinds of money can only be used only in bigger and smaller payment circles. The Iraqi Denar was probably a pure domestic currency in 2000. Also inflating currencies are hardly accepted by foreign countries.

Money facilitates the exchange of goods, by decreasing the transaction costs considerably in an economy based division of labour, contrary to a situation, where everybody has to perform a chain of transactions to change his goods or his specialised service for the goods he desires.

It is easily understandable that an economy based on division of labour is not possible without money. Even Socialist countries could not renounce to the use of money.

This doesn’t mean that mankind has purposely invented money or introduced it as a means of organisation. According to all experiences in other sectors, we have to assume that money developed in an evolutionary process together with the division of labour. In any case its

[6] Gerloff, Wilhelm: Geld und Gesellschaft. Frankfurt 1952. p. 190-213

development lays in the darkness of the past. Already the earliest discovered documents – field stones-are interpreted as such that they show the sale of fields for a certain amount of silver. Laum, Bernard: Heiliges Geld. Tübingen 1924 tries to support the thesis that money has developed out of sacrificial offerings demanded by gods. But for the priests to accept it, they had to be of ubiquitous utility for them. A nice booklet is the book "Geld" in the edition Deutsche Bank, 1982. Money must have developed between the 10th and 5th Century before Christ, a period, which is known as the Neolithic Revolution. At shepherds, for example the Greeks, livestock was used as a medium of exchange.

The Latin terminus Pecunia = money comes from Latin Pecus = cattle. The English term Fee is linguistically related to the German word Vieh = Cattle. The Gothic term "scatta" for cattle turned into a Schatz (g) = treasure. In China and Egypt, and also with farming people, cereals served as a medium of exchange. In China also "tool money" developed. Spates and picks were needed everywhere and served as a medium of exchange. Later symbols of tools served as money.

With hunters, spearheads served as money, with fishers, fishing hooks. Also pressed tea in bars and salt were money Almost every useful product has taken on money function at a certain period in time for a while. In post-war Germany, American cigarettes were medium of exchange. In India cowry shells were used as money already 3000 before Christ. Later they spread all over Asia. Shell money was stringed in necklaces.

Precious metals seem to have served already very early as money, because so big amounts of values could be transported relatively easy. For the Lydia Empire, today Asia Minor, under the king Alyattes (615-560 before Christ) the first coins are proved already for the 7th Century before Christ. By Minting a certain weight and percentage of purity are guaranteed. Alyattes let a lions head print onto his coins, which consisted of electron, a gold-and silver alloy, which existed naturally. But some Greek discoveries indicate an earlier date. Also this development lays in the darkness of the past. But approximately since this period of time, coins started to conquer the world. A round piece of metal, a planchet is put between the upper-and lower die out of hardened metal. With hammer blows, the minting is carried out, and the planchet consisting of soft metal takes on the form of the coin. Athens minted a silver coin with the picture of an owl, purchased goods with it and put money at the disposal of the whole Antic World. Therefore the saying " It's not worth to carry owls to Athens."

It seems that credits existed as long as money. Already to the times of Hammurabi, approximately 1700 before Christ, a "depositum irregulare", abstract obligations, and an Anum

pisa, who often appears as a lender and possibly was a banker, existed.[7]A developed banking system is proved in Mesopotamia for 600-500 before Christ. Documents were burned into clay, and as clay had no value, and was durable, the clay boards are still preserved today. Documents show the existence of at least two bankers, Igibi and Marusu.[8] Also temples served as banks.[9] Besides other things clay boards with amounts of money were found, which served as bearer documents. At a bearer document, the rights out of the papers respectively out of the clay boards follow the rights on the paper respectively on the clay board. (Rights are anonymously transferable). The contrary is an order paper, where money is only paid to a particular person. These clay boards are analogous to modern bank notes. Also relatively complicated legal constructions, as for example reservation of ownership, are proved. Reservation of ownership means that a good delivered on credit remains property of the seller until it is finally paid or, formulated differently: Ownership should not pass before payment of purchase price. These legal figures again indicate an organised legal system.
Beside the money, which consists of materials, whose production requires a considerable amount of resources. (commodity money for example gold, silver, livestock, cereals), credit money existed long before Christ.

If a legal subject promised the delivery of commodity money, it is debtor of a claim of a creditor. Is the debtor known to repay its debts promptly in commodity money, this means without delay, the creditor can trade in such a claim himself. It is shiftable.

<See: WS 4>

The claim itself turns into money. Commodity money cannot be reproduced, as you like. Claims arising from contracts can be reproduced infinitely .Hereby two problems can arise:

- **The debtor can become insolvent.**
- **The money volume can increase so much that higher increases in prices are happening. It comes to inflation.**

[7] Edzar, Dietz Otto: Altbabylonissche Rechts- und Wirtschaftsurkunden aus Tell-ed Dur im Iraq Musuem, Bagdad . Munic 1970. Kohler J; Peiser, F. E; Ungnad, A: Hammurabis Gesetze BD: I-V. Leipzig 1904-1911. Driver, G,.k; Miles John C:: The Babylonian Laws. Vol. II. Oxford 1955 and Koschaker, Paul: Babylonisch-Assyrisches Bürgschaftrecht.

[8] Kohler, J; Ungnad, A.: Assyrische Rechtsurkunden. Leipzig 1913. Koschaker, Paul: Ein Altassyrisches Rechtsbuch. Berlin 1922 Ahmed, Sami Said: Southern Mesopotamia in the Time of Ashurbanipal. The Haag. Paris 1968. Kohler, J; Peiser, F. E.: Aus dem Babylonischen Rechtsleben. Vol. I-IV. Leipzig 1890/1901/04/08. San Nicolo, Mariano; Petschow, Herbert: Babylonische Rechtsurkunden aus dem 6. Jh. vor Christus.

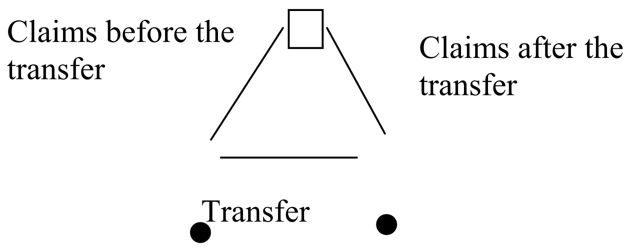
[9] Johns, C.H.W.: Babylonian and Assyrian Laws, Contracts and Letters. New York 1904

The last occurs, if credit money is used over a long period, so that people have widely moved away from commodity money, and if then the Sovereign doesn't allow the bankruptcy of the debtor of the claim used as money, and still guarantees the acceptance of its money.
George Friedrich Knapp has especially emphasized this aspect[10], by defining money as the object, whose acceptance is ordered by the government. This definition is surely not valid without reservations. As a reaction to Knapp, Gerloff has pointed out that a consent of society is required

B. The money of tomorrow

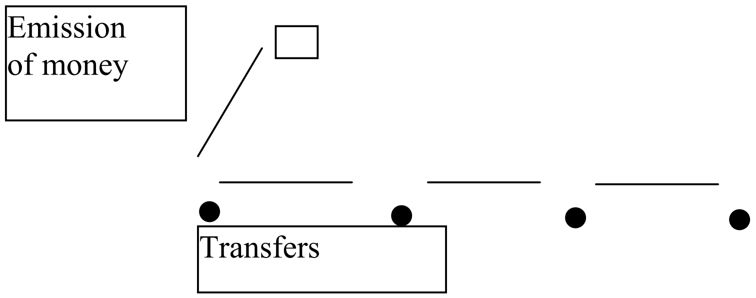
Monetary transaction use more and more electronic media. The canteen cart contains amounts of money, which were loaded on it by cash inputs into a machine, which can be debited again. You can pay in some shops with your check card and your pin number, what is replacing the checks more and more. In France people pay mainly with credit cards. You transfer money electronically and the stock market transactions are done from home with the computer. Direct Banking has revolutionized the banking activity. At an electronic transaction or at a payment by credit card, the person making the payment and the receiver are in contact with the bank.

□ Issuer
● User of Money
△ Public Key
Ô Secret Key



[10] Knapp, Georg Friedrich: Staatliche Theorie des Geldes. 3. Aufl. München und Leipzig 1921 (1905)

The issuer of banknotes does'nt know who presently owns the cash. With transfers, however, some bank will know who owns the claim..



Cash money serves, beside other things, the anonymity at payments. The question is, if anonymous money with the same functions can be created in an electronic form. A big supporter of e-cash is David Chaum with the company digi-cash. But its company seems to have become insolvent. At www. Google, you can find a lot about Chaum and about e-cash. Within this, also the work of Michael Froomkin: Flood Control in the Information Ocean: Living with Anonymity, Digital Cash, and Distributed Database. We will see that the creation of electronic cash is not a problem, but its abuse. The owner of electronic cash can copy the money, which exists as a file and spend it several times. According to Froomkin, this problem is not solved yet: Blinded Coins - Preventing Double Spending.
We will see at first how to create electronic money.

Electronic Money

Beutelspacher, Albrecht: Geheimsprachen (cryptography). Geschichte und Techniken. C.H. Beck - Wissen Nr. 2071. C.H. Beck: München 1997.

1. At first – Natural numbers

We look only at particular, natural numbers underneath a fixed quantity n, therefore

0, 1, 2, 3, ..., n-1

We use a calculation process, where the following is valid:
If we do calculations with these numbers, always a number smaller than n results.
Problem: n = 11, two arbitrary numbers 5 and 7 should be added up.

Solution: We replace the result by the rest, which results at the division with 11

5+7 = 1 (mod 11) or 5+7 mod 11 = 1.

2. How can two people determine a secret number in public?e

The partners agree on a prime number p.

The partners publicly agree on a number s, 1<s<p.

Alice determines her secret number a, Bob determines his secret number b.

Alice calculates $\alpha = s^a$ mod p and conveys α to Bob in public.

Bob calculates $\beta = s^b$ mod p and conveys β to Alice in public.

Alice calculates $k = \beta^a$ mod p,

Bob calculates $k' = \alpha^b$ mod p.

So both have calculated s^{ab} mod p. Consequently k = k´. That is their secret number.

An adversary, who knows p, s, α und β, cannot infer a and b from α and β, as the modulo - function, we are dealing with, is a trap - door – function. The function $a \to s^a$ mod p (The „discrete exponential function“) is easily carried out. Its inverse function, the “discrete, logarithmic function” can practically not be carried out according to today’s knowledge.

Example with numbers: p = 11; s = 5; a =3; b = 4;

Alice: $\alpha = 5^3$ mod 11 = 125 mod 11 = 4

Bob: $\beta = 5^4$ mod 11 = 625 mod 11 = 9

Alice: $k = \beta^\alpha = 9^3$ mod 11 = 729 mod 11 = (66) 3

Bob: $k' = \alpha^\beta = 4^4$ mod 11 = 256 mod 11 = (23) 3

Secret number k = k´ = 3

3. Secondly - Natural numbers

We look at natural numbers n, which result from the product of two different prime numbers, so

n = q * p with two different prime numbers p, q

Theorem:

The *Euler`s theorem* (Leonhard Euler, 1707 - 1783) is only valid for numbers of the type n = q * p.

For every natural number m with m ≤ n and every natural number s, it is valid

$$(1)\quad m^{s(p-1)(q-1)+1} \bmod n = m$$

Theorem:

For every natural number n, which is aliquant (=cannot be divided without remainder) to (p-1)(q-1), you can easily find a natural number d, so that it is valid:

- e * d = s(p-1)(q-1)+1

whereby s is a natural number, which results automatically from the calculation of d. The method, with which d is calculated, is called the Euclid's algorithm. **You can only calculate d, if the factors p and q are known.**

4. The RSA-Algorithm

Solution of the problem of the public exchange of keys by the discovery of the RSA-Algorithm (first published by Ronald Rivest, Adi Shamir, Leonard Adleman, 1977).

You take two great prime numbers and create the product. N = p * q

You determine two natural numbers e und d, so that it is valid:

e * d = s(p-1)(q-1)+1 gilt.

Secret key: d

Public key: e und n

You get the secret text c by raising the message m to the power of the public key e and by reducing modulo n:

$c = m^e \bmod n$

The receiver deciphers by raising the message c to the power of the secret key and by reducing modulo n:

$m' = c^d \bmod n$

$m' = c^d \bmod n = (m^e)^d \bmod n = m^{ed} \bmod n$

According to Euler´s theorem (1), it results $m^{ed} \bmod n = m$
therefore m’ = m

5. Digital signatures

Requirements of a digital signature:

- It has to be verifiable for everybody, that A has signed,
- Nobody can falsify the document
- Only A can put the signature.

A uses his private key to sign A; he calculates:

$$s = m^d \bmod n$$

You call s the digital or electronic signature of the message m. the signed document consists of m und his s.
The document can be verified by applying the public key of A on the signature s an

$m' = s^e \bmod n$ man überprüft ob m’ = m ist.

6. Hash functions

Attributes: Compression, one-way quality, freedom of collisions
With the help of a hash function, you add the digits (total of digits) of message M, to get h (M)) of the message m. Now you sign the “ in –between message” m = h (M) instead of the message M:

$$s = m^d \bmod n$$

The signed document consists out of the message M and the signature s. To verify, you calculate h (M) and checks then if this corresponds with.

$$m' = s^e \bmod n$$

7. Anonymous electronic money with the help of the blind signature

An arbitrary message, described by the natural number m < n is send to the bank by.

Alice. For this Alice chooses a random number z, for which a number z' exists for which it is valid:

(3) $z * z' \bmod n = 1$

z is put to the power of the public key (of the bank):

(4) $r = z^e \bmod n$.

n multiplied with **m** you calculate c:

(5) $c = (m * r) \bmod n$

and this is send to the bank.

The bank signs the received message.

(6) $s = c^d \bmod n$

and sends the signed message back to Alice.

Alice can now „unwrap" her money with

7. $s = c^d \bmod n = (m * r)^d \bmod n$
8. $s = m^d * r^d \bmod n$. (4) in (8), results in
9. $m^d * z^{ed} \bmod n$. (1) (Euler`s Theorem) in (9) results in
10. $s = m^d \bmod n$ by extending with „z´ mod n" we create
11. $s * z' \bmod n = (c^d * z') \bmod n$ and because of 10
12. $= (m^d z * z') \bmod n$
13. $= (m^d * 1) \bmod n =$ **m^d mod n (is the electronic money).**

8. Example with numbers

1. The banc (Bob) determines two prime numbers: p=3 and q=5, so that n=p*q=3*5=15.

The bank publishes n, the base of the modulo-calculation.

The bank determines two keys, the public key (e) and the secret key (d). e and d are determined as such that they fulfil the Euler`s theorem:

$$m^{s(p-1)(q-1)+1} \bmod n = m.$$

Consequently:

$e * d = s(p-1)(q-1)+1$.

We calculate:

$e*d = s*(p-1)(q-1)+1 = s*(3-1)(5-1)+1 = s*2*4+1 = s*8+1$.

The bank determines by iteration a multiple of 8, which increased by 1 results in a number, which can be factorised into two numbers. For s = 1, s is not aliquant to 15.For s = 2 results 17, a prime number. For s = 3, s is not aliquant to 15. For s = 4 results 33, which the bank factorises into 3*11. It determines e= 11 as the public key and d = 3 as the secret key. The secret key signifies “100 Rbl”, if the bank signs a message with this key. **The bank publishes 11.**

3 Alice determines z und z´ as such that it is valid:

(4) z*z´modn = 1

This is given for example, if z = 2 and z´ = 23 is.

(4) 2*23mod 15 = 1.

Alice determines:

$r=z^e$ mod $n=2^{11}$mod 15 = 2048 mod 15 = 8 (136 remainder 8).

She multiplies her message **m = 8**. for example “I want to create as much money as possible“ or „ I love you“ (transformed into a binary number) or „ These are 100 Rbl“ or also simply “8” with r and determines mod n:

(5) c = m*r mod n = 8*8 mod 15 = 4 (4 rest 4).

Alice sends c = 4 to the bank for signature and asks the bank to create 100 Rbl in electronic money.

.4. The bank signs the message c with its secret key d =3, which signifies “100 Rbl “

s = c^d mod n = 4^3 mod 15 = 64 mod 15 = **4**

and sends s back to Alice.

5 Alice unwraps s with

s*z´ mod n = m^d *z´*****z mod n = m^d*1 mod n = **m^d mod n**, the electronic money.

to put in: 4*23 mod n = 92 mod 15 = **2,**

or 8^3 mod n = 512 mod 15 = **2** **is the electronic money.**

If Alice passes the money to a third person, the receiver can apply the public key 11 to 2. Alice gives her message m to the receiver: „These are 100 Rbl" or „I love you“. The receiver applies the public key for 100 Rbl to the electronic money, to the message m. He calculates: 2^{11}mod 15 = 2048 mod 15 = 8.

The third person can read the message of Alice with the „100 Rbl“-key of the bank and knows that he received 100 Rbl.

It is important that the bank doesn’t see anymore whom it has sent the money to when it is cashed. The bank has received the message the message c of Alice. The receiver and later the bank verifies the authenticity of the money with the message m, which Alice communicates to it and which the receiver must not forget, because only with this

message the receiver can later be sure that the money is authentic. Therefore the bank doesn’t know that it has issued the money to Alice. The money is anonymous.
The name of Alice, who receives the money from the bank, can be encoded as well to avoid that the money is issued twice. At a second use of the money, the bank can read the name. This solution would not offer anonymity. According to Chaum and Beuchtelspacher, a realistic solution of the authenticity problem exists. The basic idea of this process is that the one, who pays, answers a couple of yes/no- questions to the receiver of the money, which only he can answer, if he has an authentic coin. At yes/ no-answers, the probability of one right answer is 0.5, but the probability of 10 answers is only $0.5^{10} = 1/1024$. The receiver could therefore reduce the probability, to receive a false coin, as much as he likes. We don’t want to discuss this method any further though. What can you do if someone spends money 1000 times very quickly and disappears then? Therefore it still seems to be a considerable problem at the moment. The cash card is possible though, which can be used then everywhere like a canteen card. The leading company for processing bank notes in Germany is Giesecke & Devrient. They reckon that banknotes still exist in 50 years. But they created a second base for themselves. They created the daughter company CpayS, which is occupied with electronic forms of money. from payment cards, which are still in the experimental stage over digital signatures to the newest cash mouse system, a payment card which can be used for electronic transfers. But also for this project, it is true: the way to the payment medium of the future will be plastered with the bleached bones of failed projects.

C. Functions of money
< See working sheet 2>

D. Quantitative definitions of money concepts
< See working sheet 5>

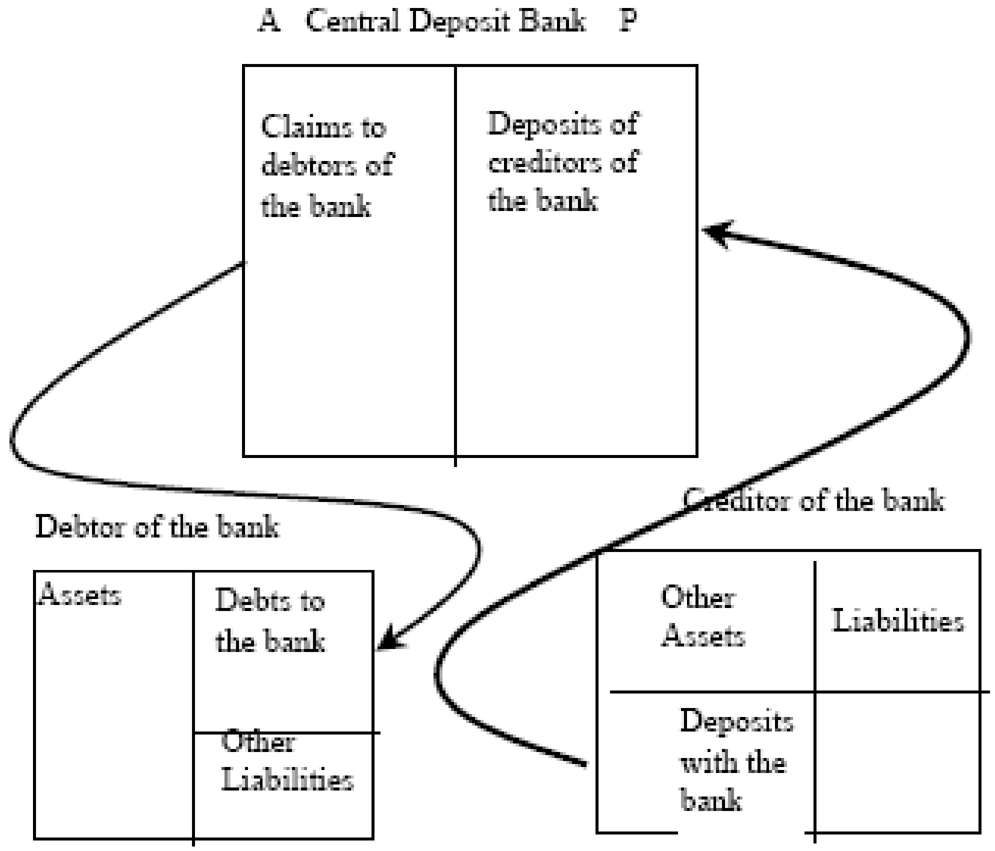
III. The production of money

A. Quantitative aspects

1. One tiered banking system– The credit mechanisms of Lautenbach[11]

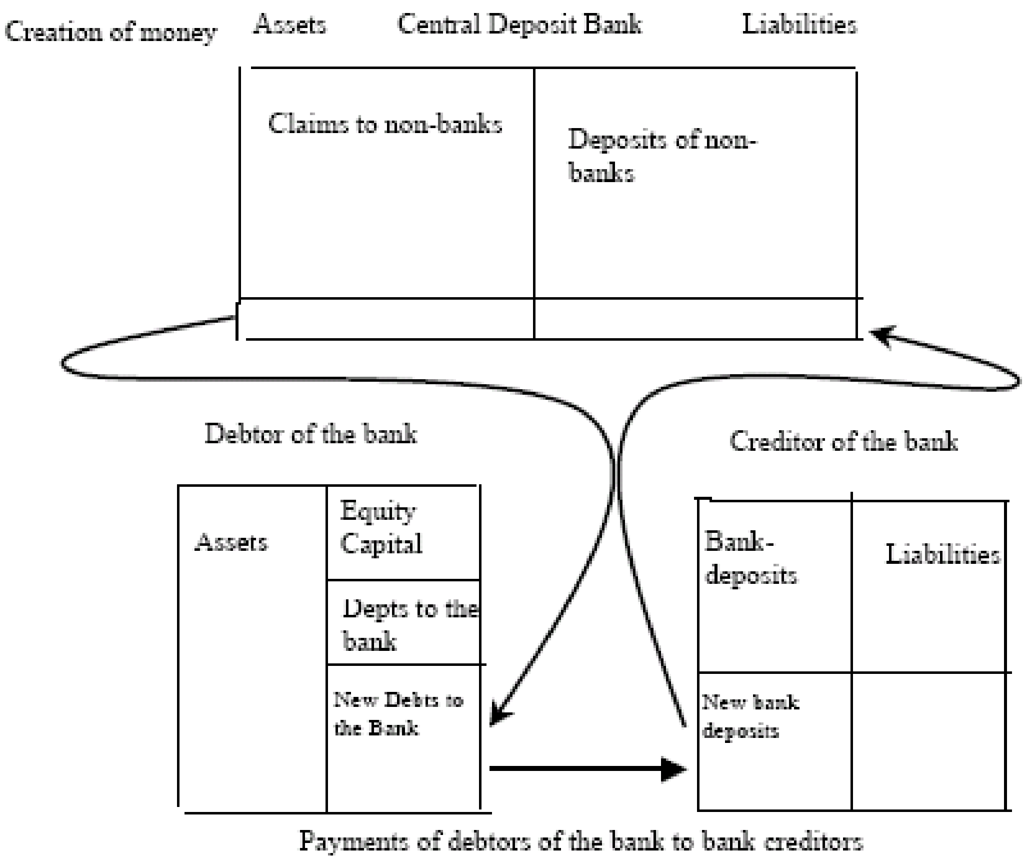
To make it easy, we assume that there is only one Central Deposit Bank, which handles all the payments. In this economy cash money doesn't exist. Therefore to all claims of the Central Bank in the economy correspond economic subjects, which owe money to the bank, therefore debtors of the bank. The bank has no equity. Ther are only liabilities of the Central Bank, which correspond to debts of the Central Bank to economic subjects, which have deposits with the bank, therefore creditors of the bank

[11] Lautenbach, Wilhelm: Zins, Kredit und Produktion. Tübingen 1952



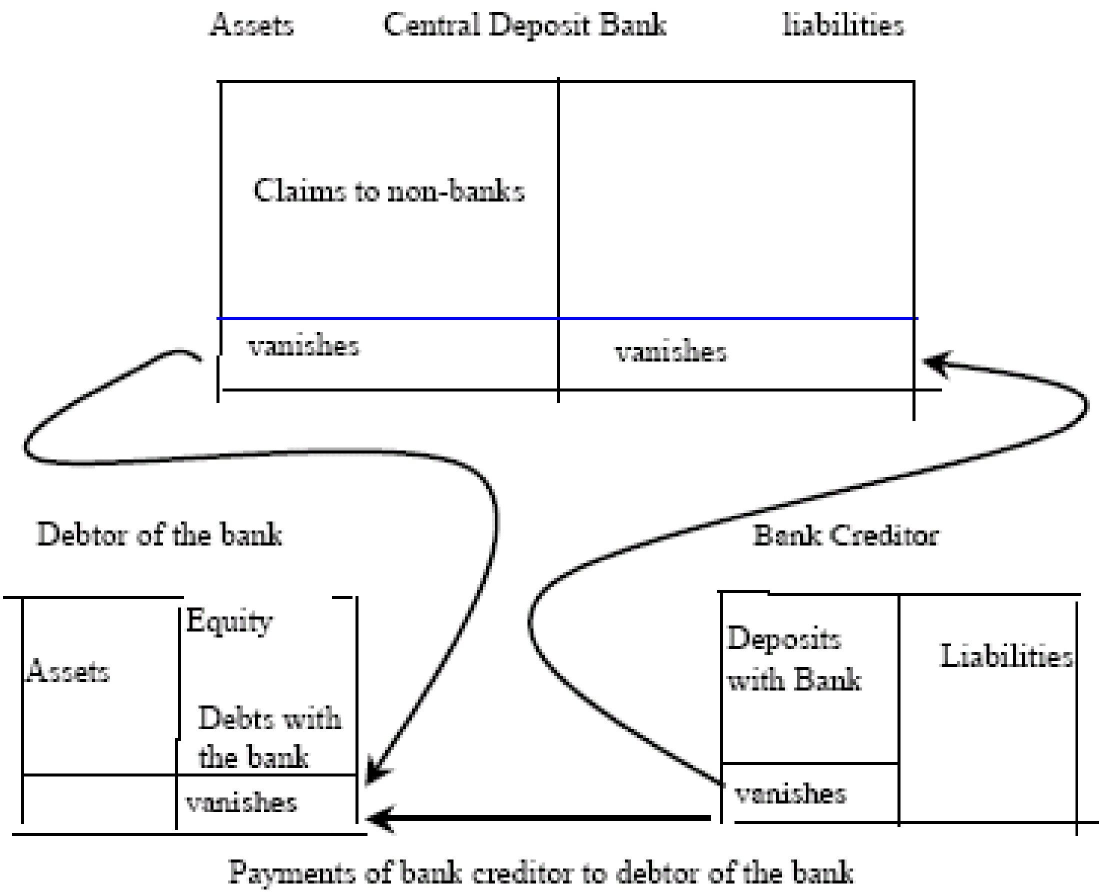
The bank balance remains the same, if a debtor of the bank is paying to another debtor of the bank.

Also if a bank creditor is paying to another bank creditor, the bank balance remains the same. The bank balance extends, if a bank creditor is paying to a bank debtor. It shortens, if a bank creditor is paying to a debtor of the bank. Creation of money means that the bank balance extends. This happens at payments of debtors of the bank to creditors of the bank.



With payments bank debtors to bank debtors, and with payments of bank creditors to bank creditors the volume of money doesn't change

Destruction of money:



Now it becomes clear: The money volume increases, if the debtor of the bank is paying to a creditor of the bank. The money volume only increases, if with an increase of economic activity also the indebtedness of the present debtors of the bank is increasing. If the debtors of the bank can free of debts because of rising profits at a good economic activity, the money volume decreases.

If indebtedness is increasing, because sales volume stagnates, but wages still have to be paid, even at a bad economic activity, money volume will be increasing.

These connections are important for monetary policies. They show, that an increasing money volume is not always a sign for a splendid economic activity, and that a splendid economic activity must not always be accompanied by an increasing volume of money.

2. Two tiered banking system - the multiplier

The ratio between the total volume of money created by the state (money of highest liquidity, monetary base, high powered money) and the money volume M1, the volume which is finally available for non-banks, is of special importance. The latter is different from the money of highest liquidity by not containing on one hand the deposits of the central bank with Monetary Financial Institutions (MFIs), but therefore containing on the other hand sight deposits of non-banks also with the MFIs. On balance, the money volume M1 is empirically higher than money of highest liquidity. But this is not necessarily so. The ratio between money volume M1 and the total volume of money of highest liquidity is called the multiplier, because it indicates to how much the money volume can be increased by the activities of IMFs compared to the money supply created by government. As the following table 4 shows, the multiplier fluctuated between 1.3 and 1.8 between 1965 and 1981in the Federal Republic of Germany.

The comparison of the rates of growth of central bank money and the one of the money volume M1 shows, that the multiplier was stable for a longer period in time in the GFR, even though fluctuations existed in certain years (Table 4). The ratio between a variant of the money volume (M)[12] and the total amount of money of highest liquidity (S) can be written as follows according to the definition of the money volume (1) M=S+D and
the money of highest liquidity (2) S´=S+R,
whereby S = money of highest liquidity in the hands of non-banks
R = reserves of banks of money of highest liquidity
D = deposits with banks

$$(3)\ M/S' = (S+D) / (S+R)$$

The quantities M and S are always interrelated: If M is interpreted as M1 then D comprises only sight-deposits of national non-banks, if M is interpreted as M2, then D comprises fix-term deposits with a maturity up to 4 years as well.

With the extension D /(R*S), you can write as well

$$(4)\ M = S * \frac{D/R(1+D/S)}{D/R+D/S} \quad .$$

[12] Depending on using the total amount of money of highest liquidity as a denominator or decreasing it by the stock of national public households and using high-powered money as a base, differences in the amount of the multiplier result , which are insignificant for the selected period of time though.

<u>Table 4:</u> *High-powered money, Money volume M1 and the multiplier in the Federal Republic of Germany between 1965-1981*

Time	Money volume M1[1] Billion DM	High-powered-money[2] Billion DM)	Multiplier[3]
	(1)	(2)	(3) = (1) / (2)
1965	79	49	1,6
1966	80	53	1,5
1967	88	51	1,7
1968	93	54	1,7
1969	99	56	1,8
1970	108	68	1,6
1971	122	79	1,5
1972	139	99	1,4
1973	143	107	1,3
1974	158	106	1,5
1975	180	108	1,7
1976	187	120	1,6
1977	208	129	1,6
1978	238	140	1,7
1979	248	156	1,6
1980	257	154	1,7
1981	255	153	1,7

Source: Monthly Bulletins of the Deutsche Bank

1.) Money volume M1 = money in circulation outside the banking system (without cash balances of MFIs)
+ sight deposits of national non-banks with MFIs

2.) High-powered money = circulation of bank notes
+ deposits of national MFIs with Deutsche Bundesbank
+ deposits of national enterprises and private persons with Deutsche Bundesbank
+ deposits of foreign investors with Deutsche Bundesbank
+ volumes of circulating token coins

3.) Multiplier = money volume M1/ high-powered money

Table 5: High-powered money, money volume M1 and the multiplier in European Monetary Union

	Central Bank Money (CBM) in Billion €	M1 Billion €	M1/CBM	M2 Billion €	M2/CBM	M3 Billion €	M3/CBM
(1)	(2)	(3)	(4)=(3)/(2)	(5)	(6)=(5)/(2)	(7)	(8)=(7)/(2)
January1st 1999	429	1,793	4.2	3,922	9.1	4,502	10.5
December 1999	492	1,964	4.0	4,133	8.4	4,791	9.8
December 2000	496	2,076	4.2	4,287	8.6	5,079	10.2
December 2001	414	2,153	5.2	4,629	11.2	5,413	13.1
December 2002	470	2,452	5.2	4,979	10.6	5,427	11.2
December 2003	552	2,673	4.8	5,225	9.5	6,136	11.1

Equation 4 shows clearly the dependence of the size of the multiplier (the fraction in 4) of the minimum reserves (1/D/R) and of the size of the deposit-cash- ratio of the public (D/S). The minimum reserves indicate the percentage of deposits of non-bank with banks. Minimum Reserve ratio time deposits of non-MFIs with MFIs are the Minimum Reserve Obligation. That is the amount of central bank money MFIs are obliged to hold. The deposit-cash- ratio reflects the payment habits of the public and the trust of the public into the solvency of deposit banks. If a lot is paid in cash and little by transfers, the deposit-cash-ratio is low. The same is true at mistrust into the solvency of banks as it could be observed mainly in the United States in the years 1931 to 1933.[13] Are D/R and /or (D/S) increasing, the multiplier is increasing as well as in the numerator they era multiplied with each other while they are added only in the denominator.

[13] See for instance Friedman, Schwarz (1971). A monetary history of the United States. p. 333

B. *Qualitative aspects: Liquidation of claims and risk transformation*

< See working sheets 4, 26 and 27

IV. Theory of interest

A. *Reasons for interests*[14]

Conditions for profit are:

Work and capital are necessary for production, complementary production factors, for example: Y = f (C, L, t) or Y = f (C, L). Such a function can be for example the Cobb-Douglas Production function: $Y = A * C^{\alpha} * L^{1-\alpha}$.

- The use of assets is assigned to people or groups.
- Assets are scare.

Conditions for accumulation of capital are:

- Higher incomes can be used to increase assets.

Conditions for interest are:

- Assets can be lent against payment.
- The expected marginal revenue (marginal utility) is greater for the debtor than for the creditor.

If a person doesn't support one theory, but considers several theories as correct, he is usually, pejoratively called an eclectic. An elegant way of eclecticism is to look for an comprehensive theory, which comprises the individual theories as special cases.

The upper theorem comprises a set of sufficient and necessary conditions (also the necessary condition, math.) for the existence of interests should represent such a comprehensive theory. Looking at the history of theories on interest, you will find out, that individual theories especially emphasize single conditions for the existence of interests. An ethic justification was connected to the explanation of interests. Scientific concepts, which are governed by interests are called ideologies. The question, under which circumstances unearned incomes develop, difers from the question, if such incomes are desired or if they are not desired.

The productivity theory of the interest: The basic idea is that interests exist because capital yields profit. Condition one is emphasised and interests are considered a production technical

[14] This paragraph is based on Stützel, Wolfgang: Kapital und Zins. In Evangelisches Soziallexikon. Kreuz Verlag. Stuttgart 1954. Column 539-548

necessity. In the struggle between the different social orders, you can see an ideology for interest income in this one-sided emphasize of the production process. One supporter of this theory is for example J.B. Say, Traite D`Economie Politique, 7. Volume Guillaumine & Cie.: Paris 1861 (1803).[15]

Böhm- Bawerk emphasizes the fact, that production of capital requires time. He gives the additional productivity of **Circuitous routes of production**, which are necessary, when capital is produced as a reason for interests. Here the factor (t) from the production function is emphasised, which normally is neglected. The **Abstinence theory of interests** by Wilhelm Nassau Senior emphasises as well the fact that production requires time. The temporary abstinence of the creditor is the justification for interests according to Senior. The " waiting-theories " of interest point in the same direction. But these theories cannot justify interests heirs obtain. Adam Smith, Thompson and Karl Marx attribute the profits to the work. **The labour theories of value** emphasise the factor (L) in the production function. Smith with the main work of economics "The wealth of nations" (1776) considers work as the source of all wealth. Thompson turns this in his main work "An inquiry into the principles of wealth" (1824)[16] into a theory of the exploitation of workers, which Karl Marx takes up again later. According to Karl Marx, capital that brings in interests is a social relation between people mediated by things. Here condition two is emphasised, therefore the fact that the use of wealth is normally attributed to special people or special groups of people. The same direction takes Franz Oppenheimer (teacher of Ludwig Erhard) with his theory of **Class monopoly rent.** The class monopolies develop from fighting conflicts, in which the winner oppresses and exploits the conquered people. Every individual capital is part of the class monopoly and every interest is a useable part of the class monopoly.

Condition four, that higher incomes can be used to increase wealth, is emphasised by Karl Marx. **. Accumulation theory** is his contribution to economics. His remaining contribution to economic theory is the idea of a growing economy.

Can condition two, that assets are assigned to particular people or groups, be removed? According to results of environmental research, **resources are overused, when they can be used by everybody, instead of being used efficiently.** Garret Hardin has summed this into a table in his essay " The tragedy of the Commons". [17]

[15] Quotation according to Böhm-Bawerk, Eugen von: Kapital und Kapitalzins. Geschichte und Kritik der Kapitalzinstheorien. Volume 4. Gustave Fischer: Jena 1921 (1884), p. 104. Besides work and capital, Say's production function already contains the natural resources, agents naturels.

[16] Translation of the title according to Gide, Charles; Rist Charles: Geschichte der volkswirtschaftlichen Lehrmeinungen. Third volume. Publisher Franz Oppenheim. Verlag Gustav Fischer: Jena 1923.

[17] Science 162 (December 13th 1986), p.1244. Alan C. Stockman: Introduction to Economics. Dryden Press: Fort Worth 1996 has drawn up the table.

Table 1: The tragedy of the commons

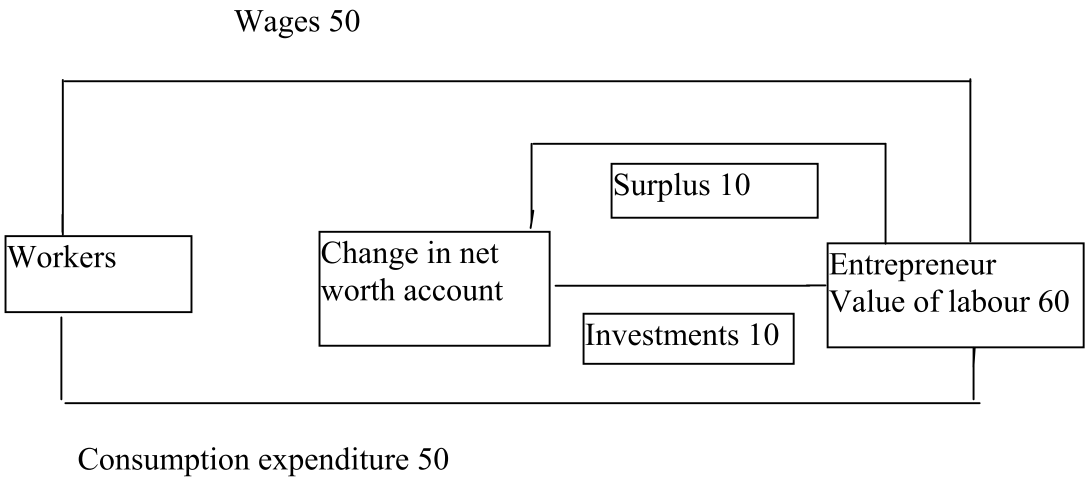
Data: Alternative yield to labour/fisher and day: Rbl 200; price 2 Rbl/fish

Fishing in Loch Ness					
Number of fishers	Number of caught fish / person and day	Number of caught fish in total/day	Private marginal revenue per person and day	Total social revenue per day	Marginal social revenue per day
(1)	(2)	(3) = (1) *(2)	(4) = (2)*2 Rbl/fish	(5) = (3)* 2 Rbl/fish	(6) = Δ (5)
1	400	400	800	800	800
2	350	700	700	1400	600
3	300	900	600	1800	400
4	250	1000	500	2000	**200**
5	200	1000	400	2000	0
6	150	900	300	1800	-200
7	100	700	**200**	1400	-400
8	50	400	100	800	-600

Full employment is a condition, what expresses in the fact that everybody has an alternative yield of 200 Rbl/day. The table shows that seven fishers are fishing in the sea at common property. Only the eighth fisher prefers to claim the alternative return of 200 Rbl. The sixth fisher has with 300 Rbl/day still more than the alternative return. It would efficient however, if the owner of the sea would only employ or allow four fishers. Then the number of fish caught would be at its maximum as well as profit with 2000 Rbl. The marginal social revenue of the fifth fisher would be cero. That's why he will not be employed. This means: at the population density today, common property almost always leads to sub-optimal results. Where still common property exists, problems arise: pollution of the air and water, deforestation of the rain forest, over-fishing of the seas, extermination of wildlife and rare species, pollution of the space with rubbish.

Clark, a scientist of the last century pointed out the consumption importance of credits. Credits are a medium to avoid any waiting and to already consume products, when you only started to produce.[18] The sixth and last **marginal condition** originates from the marginalist school therefore Menger, Wieser, but also Walras and Pareto.

Graphics 4.1: Circuit scheme of Marx with change in net worth account



If monopolies exist for especially new processes and methods of organisations, which are still not wide spread and which only **pioneers** have at their disposal, a rent yields from this knowledge. Schumpeter refers to this rent in his "Theory of the economic development" as **Quasi rent from innovation" or "Pioneer rent".**

Keynes's theory of interest is based in the knowledge that in modern money economies the capacity to pay at any time, the disposal over payment means, is wealth in itself.

Condition five that it has to be possible to borrow assets against interests, gets some attention by **the Islamic prohibition of interest** of the Old Testament and the temporarily existing Christian prohibition of interest. The Christian prohibition of interest can be interpreted as such, that the Roman tax administration wanted to monopolise the credit business as the church itself paid and received interests. In the range of the Islamic interest prohibition, investments as dormant equity holding with profit sharing is accepted. It can be valued positively that a creditor has to take care of investments. On the other hand, businessmen don't really like someone interfering in their affairs, so that only credit granting remains.

The English expression "interest" is the result of the Christian prohibition of interests: it is prohibited to demand interests. Is the money not paid back in time, the creditor can claim damages, the positive interest. Creditor and debtor conclude today a backdated contract, by

[18] Quotation according to Böhm - Bawerk, Eugen von: Kleinere Abhandlungen Über Kapital und Zins. Hölder /Pichler/ Tempsky: Wien and Leibzig 1926, p. 491

which the credit would have to be repaid today. The payment is arranged as “interest”, the compensation, because the debtor supposedly didn’t repay the money in time.
Proudhon, who, among other things, was in favour of definitive paper money (around 1840), published in1840 his work “ What is property?”- “Qu `est-ce que la propriété? (original title). He was then 31 years old.
Already at the first page, he gave the answer: Property is theft – “ La propriété, c`est le vol”. He considered interests as the **seigniorial right.** Of course condition two is emphasised by this: the use of assets is reserved to particular people or groups.
For one of these conditions to be declared as a reason, you have to be able to imagine, that this condition doesn’t exist or that it is historically variable. The socialists believed in any case, that the social order could be changed and partly believed in the utopia of the commons,[19]which in 1516 certainly was more realistic than in our overpopulated times. But also in real existing socialism use of assets was assigned to groups (cooperatives) or to the state or his subsidiaries.

B. *Height of the interest including the theory of term structure of interest rates*

1. Interest and rate of growth

Until now we have discovered that the sustained interest has to be greater than the sustained rate of growth of the national income, for the economy to be dynamically efficient. You can assume that this is normally the case. Would this not be the case, so many debtors would want to finance themselves by credit granting, that the interests would increase so much that they would be higher than the rate of growth. The interests can always be covered by new credit taking.

2. The marginal productivity theory

According to production theory, interest is equal to the marginal productivity of capital.
The Cobb Douglas’s production function has the following form:

$$(1) Y = F(C, L) = AC^{\alpha}L^{1-\alpha}.$$

[19] Thomas Morus: De optima statu rei puplicae deque nova insula Utopia. Löwen 1516.

Hereby signify: Y = National Product, (Yield)

A = Technology, A > 0

C = Capital Service

L = Labour Service

$0 < \alpha < 1.$ = Positive parameters

(2) MPC = $\partial Y/\partial C = \alpha A C^{\alpha-1} L^{1-\alpha} = \alpha Y/C$,

(3) MPL = $\partial Y/\partial L = (1-\alpha) A C^{\alpha} L^{-\alpha} = (1-\alpha)Y/L$.

Hereby signify:	MPL: Marginal product of labour
	MPC: Marginal product of capital
From (2) results:	MPC multiplied C = αY
and from (3) results	MPL multiplied L = $(1-\alpha)Y$.

Cobb-Douglas's production function is constructed on these properties. These mean:

- The partial marginal yields at a successive increased use of one production factor are decreasing (because of $\alpha < 1$)
- The production factors receive a certain share of the total product determined by the constant term (α) (right side of the equation), when they are paid with their marginal product (left side of the equation)
- The total product is just exhausted by the payment of the production factors. ($\alpha + (1-\alpha) = 1$).
- At a proportional increase of the amounts of both factors, the product increases by the same amount ("constant returns to scale")

$$F(zC,zL) = A(zC)^{\alpha}(zL)^{1-\alpha} = Az^{\alpha}C^{\alpha}z^{1-\alpha}L^{1-\alpha} = z^{\alpha}z^{1-\alpha}AC^{\alpha}L^{1-\alpha} = zY.$$

The marginal rate of substitution is for the Cobb-Douglas function:

$$-dL/dC = (\partial Y/\partial C)/(\partial Y/\partial L) = \alpha L/(1-\alpha)C.$$

The function $Y = AC^{\alpha}L^{\beta}$, with $0 < \alpha,\beta < 1$

is called the general Cobb - Douglas production function. Is $\alpha+\beta > 1$, the return is increasing disproportionately large at a proportional increase of all production factors (increasing returns to scale). Is $\alpha+\beta < 1$ the output increases disproportionately low compared to the increase of the use of the production factors.

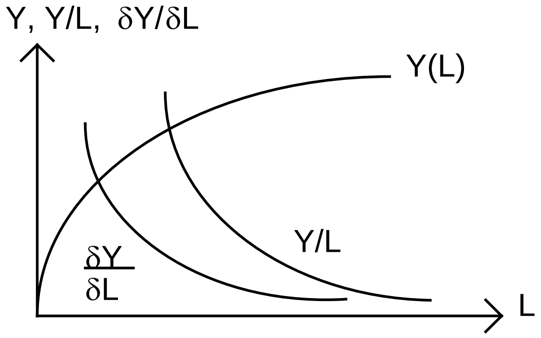


Abb. 4.20: Yield, Average Yield and Marginal Yield of the Cobb-Douglas Production Function

The yield in dependence of the labour is increasing at a decreasing rate. Average yield and marginal yield are decreasing. The marginal yield lies underneath the average yield.

Example with numbers (taken from Cezanne, Allgemeine Volkswirtschaftslehre):

$$Y = 1\,K^{0,4}L^{0,6}$$

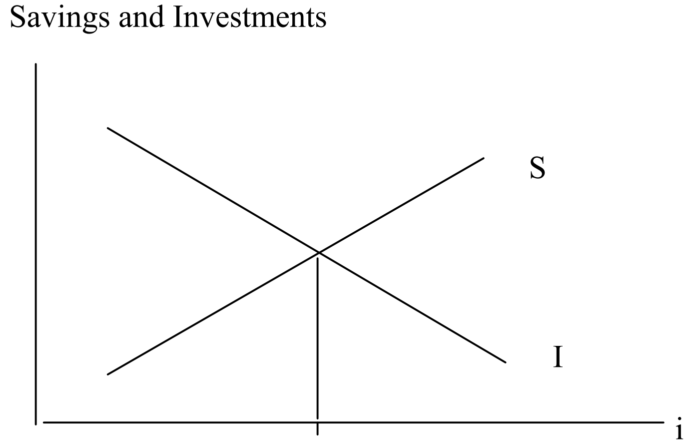
Development of yield at an variation of the production factors in the Cobb- Douglas production function

K										
8	0	2,30	**3,48**	4,44	5,28	6,03	6,73	7,38	8,00	
7	0	2,18	3,30	4,21	5,00	5,72	6,38	7,00	7,58	
6	0	2,05	3,10	3,96	4,70	5,38	6,00	6,58	7,13	
5	0	1,90	2,89	3,68	4,37	5,00	5,58	6,12	6,63	
4	0	1,74	2,64	**3,37**	4,00	4,57	5,10	5,60	6,06	
3	0	1,55	2,35	3,00	3,57	4,08	4,55	4,99	5,40	
2	0	1,32	2,00	2,55	3,03	**3,47**	3,87	4,24	4,59	
1	0	1,00	1,52	1,93	2,30	2,63	2,93	3,21	**3,48**	
0	0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	8	L

You can see the constant returns to scale in the main diagonal. Moving to the right in any row or to the top in any column, you can notice the decreasing increases in returns. The connection of the numbers printed in bold shows approximately the location of an iso revenue line. (yield isoquant).

3. A theory, called “classic theory” by Keynes

The classic theory consists in the fact, that the interest expresses savings and investments.[20]

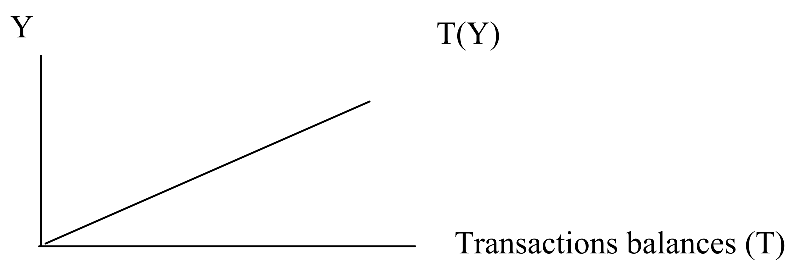


Equilibrium interest rate

Is the inclination to invest increasing (Moving the investment curve to the right), interest rate is increasing. Is the inclination to save increasing (Moving the savings curve to the left), the interest rate id decreasing. Keynes refers to Alfred Marshalls: Principles of Economics.

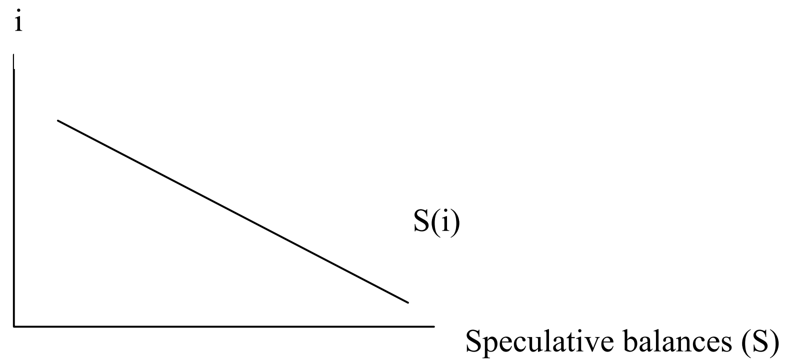
4. The Keynes `s Theory of Interest (Liquidity Preference Theory of Interest)

According to Keynes three motives of cash holdings exist: The transaction motive, the precautionary motive and the speculative motive. The cash. holding is apart from the transaction motive as well dependent of the national income



[20] The diagram can be found in : Keynes, John Maynard: The General Theory of Employment, Interest and Money. Macmillan &Co: London 1960 (1936), p.180.

The speculative balance is dependant of the interest. This means a dependence of expectations of changes in the interest rate, if the long-term interest is considered constant.
The precautionary motive of the cash holdings expresses the desire of people to keep a certain amount of their assets as cash, as always-unexpected events and desires exist. The speculative balance as the transactions balance is dependant of Y.



According to Tobin and Baumol , the transaction s balance is not only dependant of national income, but also inversely of the interest.

Symbols:

b Transactions costs of cash withdrawals

c/2 Average cash balance

c Amount of withdrawal

Y Income/Period

N =Y/c Number of cash withdrawals/Period

i Interest rate

The total cost/period are

K = i*c/2 + b*Y/c

To get a minimum of the function we differentiate:

$$\frac{\partial K}{\partial c} = \frac{i}{2} - b * Y * c^{-2} = 0$$

$$\frac{i}{2} = b * Y * c^{-2}$$

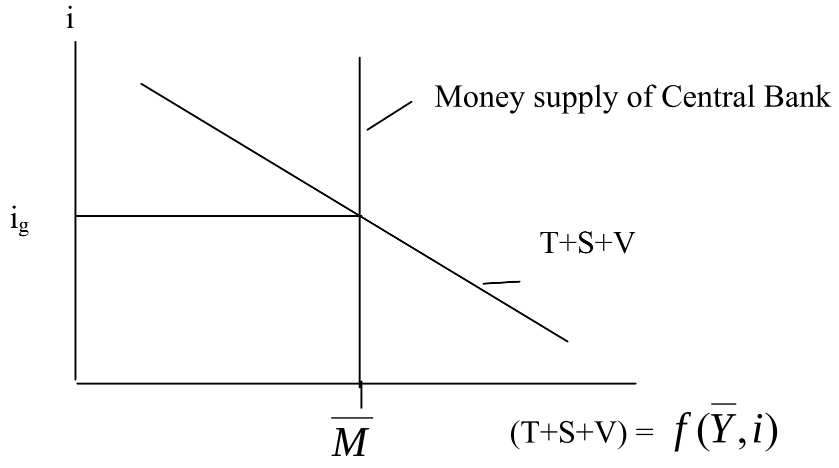
$$c^2 = \frac{2b * Y}{i}$$

$$c = \sqrt{\frac{2b * Y}{i}}.$$

This means: The transactions balances are increasing with the root of Y and with the root of transaction costs, and it is decreasing with the root of i.

For a given national income, the demand for cash is decreasing in dependence of interest.

Money supply and money demand according to Keynes



5. Theory of term structure of interest rates

We denote: R = (1+i), with i: interest rate;

${}_tR_k$ Factor of the effective rate of interest observed in t for the remaining maturity k:

${}_{t,t+j}R_k$ Factor of the effective rate of interest in t, traded for the point of time t+j for the remaining maturity k;

t = Month, Years depending on the context.

$${}_6R_{24} = ({}_6R_{12} * {}_{6,\,6+12}R_{12})^{1/2}$$

$${}_6R_{12} = ({}_6R_1^{\,1/12} * {}_{6,6+1}R_1^{\,1/12} * \ldots * {}_{6,6+11}R_1^{\,1/12})$$

The following connections are based on arbitrage between spot- and futures markets. Before futures markets existed, and also in our days, as for many time limits futures markets don't exist, such thoughts were called expectations theory. It says that market participants have certain expectations for future interest rates, and that because of these expectations the described connections exist. This is questionable, if no futures markets exist.

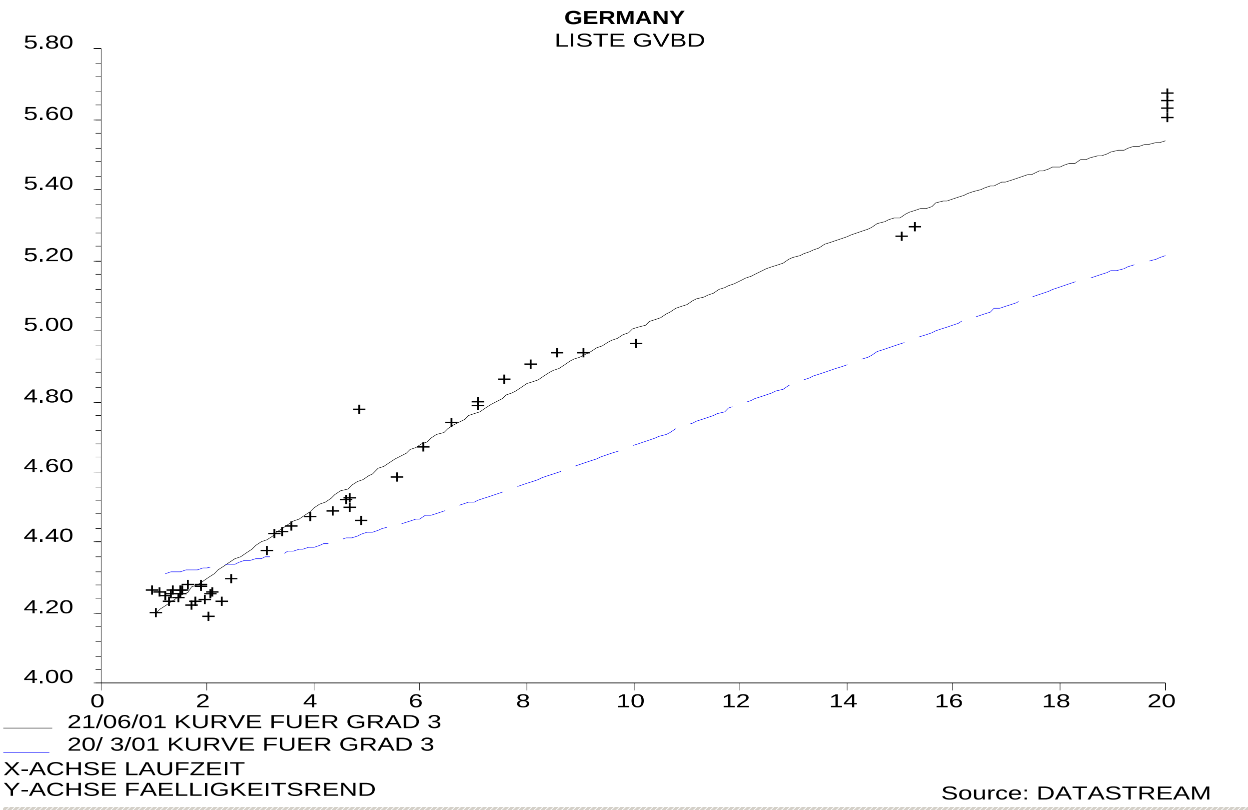
The contrary thesis is the market segmentation approach. It says that demand and supply on every individual market at any point of time determine the interest individually, and that there is no relation between the rates of interest of individual markets.

An increasing yield curve signifies that the short term interests expected for the future are lying above the long-term interests of the yield curve. Higher short-term interests for the future signify higher inflation expectations.

As a result of the Central bank reducing interests, often occurs a decrease of the interest on the short end and a rise of interests on the long end. You can explain this as such, as the Central Bank is reducing interests it increases inflation expectations and therefore future long-term interests are rising.

The broken line is the yield curve before the lowering of interests. The solid line with the points is the yield curve after the lowering of interests. The result of the lowering of interests at the short end is a steeper yield curve and higher long term interests. But higher long-term interests don't necessarily have to originate from higher inflation expectations in the Euro – area. The long-term interest rates in the Euro- area are still taking place parallel to the ones of the United States. The rates of interest were also rising in the United States between March and June. The relation to the interest rates of the United States is based on the uncovered interest parity, if for example it is considered the most probable that the future exchange rates will be the same the one today.

Yield curve in January and June after the lowering of the rates of interest of May 10th 2001: Short- term interests have decreased and long-term interests have increased.



Example with data of the London International Financial Futures Exchange (LIFFE)

Liffe 27.6.97: ${}_0R_1 = 3{,}27\%$ p.a. ${}_0R_2 = 3{,}55\%$ p.a.

June	Sept.	Dec.	March	June	Sept.	Dec.	March	June	Sept
0	3	6	9	12	15	18	21	24	27 Month
3,06	96,85	96,75	96,66	96,51	96,29	96,02	95,78	95,57	

$$\sqrt[2]{\underbrace{1{,}0306^{1/4}*1{,}0315^{1/4}*1{,}0325^{1/4}*1{,}0334^{1/4}}_{1{,}03199}*\underbrace{1{,}0349^{1/4}*1{,}0371^{1/4}*1{,}0398^{1/4}*1{,}0422^{1/4}}_{1{,}038496}}$$

1,031999 1,038496

One-year interest ${}_1R_1$ ${}_{0,2}R_1$

$(1{,}031999*1{,}038496)^{1/2} = 1{,}071728^{1/2} = 1{,}035243$ ➔ 3,52 % p.a. as two years interest ${}_0R_2$

V. Quantity theorie

< see working sheets 15-20 and Currency Developments>

A. *Non-accelerating wage rate unemployment (NAWRU) and Non accelerating inflation rate of unemployment (NAIRU)*

<Currency Developments: Phillips curve>

B. *Policy of the European Central Bank (ECB)*

1. Monetary instruments of the ECB

The single monetary policy in stage tree[21]

Eurosystem open market operations and standing facilities

Monetary policy operations	Types of transactions[22]		Maturity	Frequency
	Liquidity providing	Liquidity absorbing		
Open Market Transactions				
Main refinancing operations	Reverse transactions	-	- Two weeks	- weekly
Longer-term refinancing operations	- Reverse transactions	-	- Three month	- Monthly
Fine tuning operations	- Reverse transactions -Foreign exchange swap -Outright purchases	-Foreign exchange swaps -Collection of fixed-term deposits -Reverse transactions -Outright sales	- Non- standardised	- Non- regular
Structural operations	-Reverse transactions -Outright purchases	-Issue of debt certificates Outright sales	Standartised/non-standartised -	-Regular and non-regular - Non-regular
Standing facilities				
Marginal lending facilities	- Reverse transactions	-	-Overnight	- Access at the discretion of counterparties
Deposit facilities	-	- Deposits	-Overnight	

[21] Source European Central Bank: The Monetary Policy of the ECB. European Central Bank: Frankfurt am Main

[22] See Box 4.2 for a description of the types of open market transactions

"Reverse transactions" are security purchases of the ECB with repurchase agreements by the MFIs. These transactions are also called repurchase operations. The Central Bank purchases securities of banks for a certain time. The interests are covered by the difference between purchase– and sale price of the securities. Securities suited for security transactions are listed in a two-stage index of the ECB. In the first stage securities are listed, which are selected by criteria valid in the total Euro-area. In the second stage criteria for the areas of the individual Central Banks are defined to do justice to national peculiarities.[23]

The issuer of securities can be private and public unit based within the Euro-area, the ESCB or international organisations. The securities have to be deposited with a National Central Bank (NCB) or with a central securities depositary (CSD).

A further instrument is the minimum reserve policy. Minimum reserves are deposits of "monetary financial institutions" (MFIs) with the European Central Bank. It is at the moment two percent of deposits of non-MFIs with maturity or a period of notice up to and including two years plus money market papers in the hands of non-IMFs. Interbank liabilities are not subject to minimum reserve requirements.

The minimum reserve requirements for this month are calculated at the end and of deposits of the previous month. The minimum reserve is calculated from the daily, average deposit between the 24th of a month and the 23rd of the following month. Example: The minimum reserve requirements determined at the end of January will be valid for the period starting at February 24th until March 23rd.

2. Use of individual instruments

MFIs can raise money in a short time by repurchase operations at a high rate of interest, the marginal lending rate, if they have an urgent need of liquidity.[24] The banks can deposit money with the Central Bank at a deposit rate, which is very low. In between lays the main refinancing rate, around which the Euro over-night indexed, average (EONIA) for money market transactions of the banks oscillates. Money dealers of banks are trading on the phone with German banks, over night money at the money market. With this money you have to tell your partner until 10.30 in the morning, when you are going to reclaim it. In European money market

---

[23] European Monetary Institute: The single Monetary Policy in Stage three. Specification of the operational framework. European Monetary Institute: Frankfurt, January 1997, p. 21

[24] Liquidity is a characteristic of an economic subject (ES) in the sense of solvency. This is normally given at intact financial situations of the ES (solvency), and is a yes/no attribute. Liquidity is countable, if it refers to the liquid means of an ES`s, here namely Central Bank money. Different from this, is the total amount of CB-money in the hand of MFIs , the so-called liquidity of the banking system. This is controlled by the Central Bank and has to be always as high as the minimum reserve requirements, if the IMFs should fulfil the Minimum reserve requirements.

dealings, over-night money is common. It is repaid, if it is not agreed on a prolongation. The Trans-European Automated Real Time Gross Settlement Express Transfer - (TARGET) system was created to carry out money market dealings. If the Money dealer of the Saar-LB (Public bank at Saarbrucken) does a money market dealing with a European partner, he sees the money with the next actualisation, which is made every half hour, on his screen (his account). The 1-month money market rate, Euro Inter Bank Offered Rate (EURIBOR), the rate, to which banks offer euro-interbank deposits to other banks, lays above the main refinancing rate, because banks lend money on the long-term at higher interests than they themselves have to pay to the ECB. Marginal landing rate and deposit rate represent in normal times upper- and lower limit for money market rates. Only if the European Central Bank doesn't lend money at marginal lending rates, the money market rates can be higher respectively, if the ECB at the deposit rate doesn't accept money, the money market rates can be lower.

The Central Bank puts money to the disposal of the bank by so-called structural operations for three month by a monthly rhythm, each time a third of the total amount In average these allotments amount to 50 billion €[25]. Through main refinancing operations, 70 billion € are put at the banks disposals weekly. As you can see in table 1.1 of the monthly bulletins of the ECB the € deposits of the bank amount to approximately 110 billion €. This means that the totality of banks have to lend from the Central Bank weekly approximately 60% and every 14 days 100% of their central bank money. Therefore it is clear that the Central Bank can safely control the short-term interest rates. Its influence on the long-term interest rates is considerably weaker though because of several factors (inflation rates, US- interests).

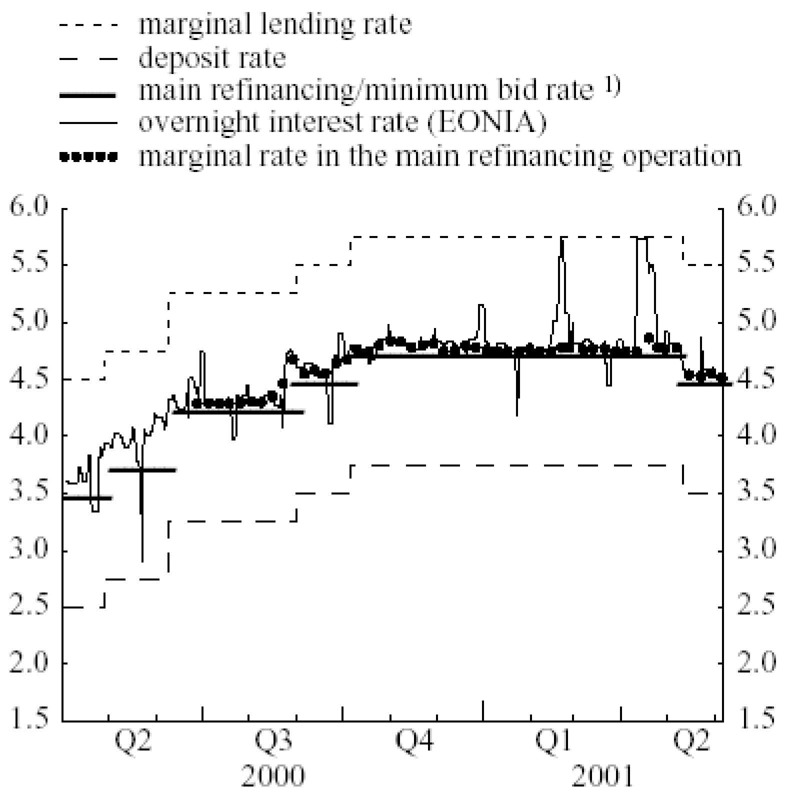
For the Central Bank money, which can be obtained by MFIs, the Central bank uses different forms of auctions:

- Fixed rate tender: the Central Bank fixes the interest rate and the amount to be auctioned. MFIs bid the amount of money they want. It comes to stagging, this means in anticipation of a part-allotment, the banks subscribe for more than they actually need or want. The lowest allotment rate was 0.8 %. You can see banks behave like small shareholders with companies going public during a bull market.
- Dutch interest tender (Dutch auction): ECB determines an amount to be auctioned. The highest offers are taken into consideration. A single allotment interest rate is selected and determined by the marginal bidder.
- US-style-variable-rate-tender: ECB determines an amount to be auctioned. MFIs with the highest offers obtain the money at different interest rates, namely the rates they bit.

[25] This number and the following are taken from the European Central Bank: Annual Report 1999, p. 48-50

ECB interest rates and money market rates

(percentages per annum; daily data)



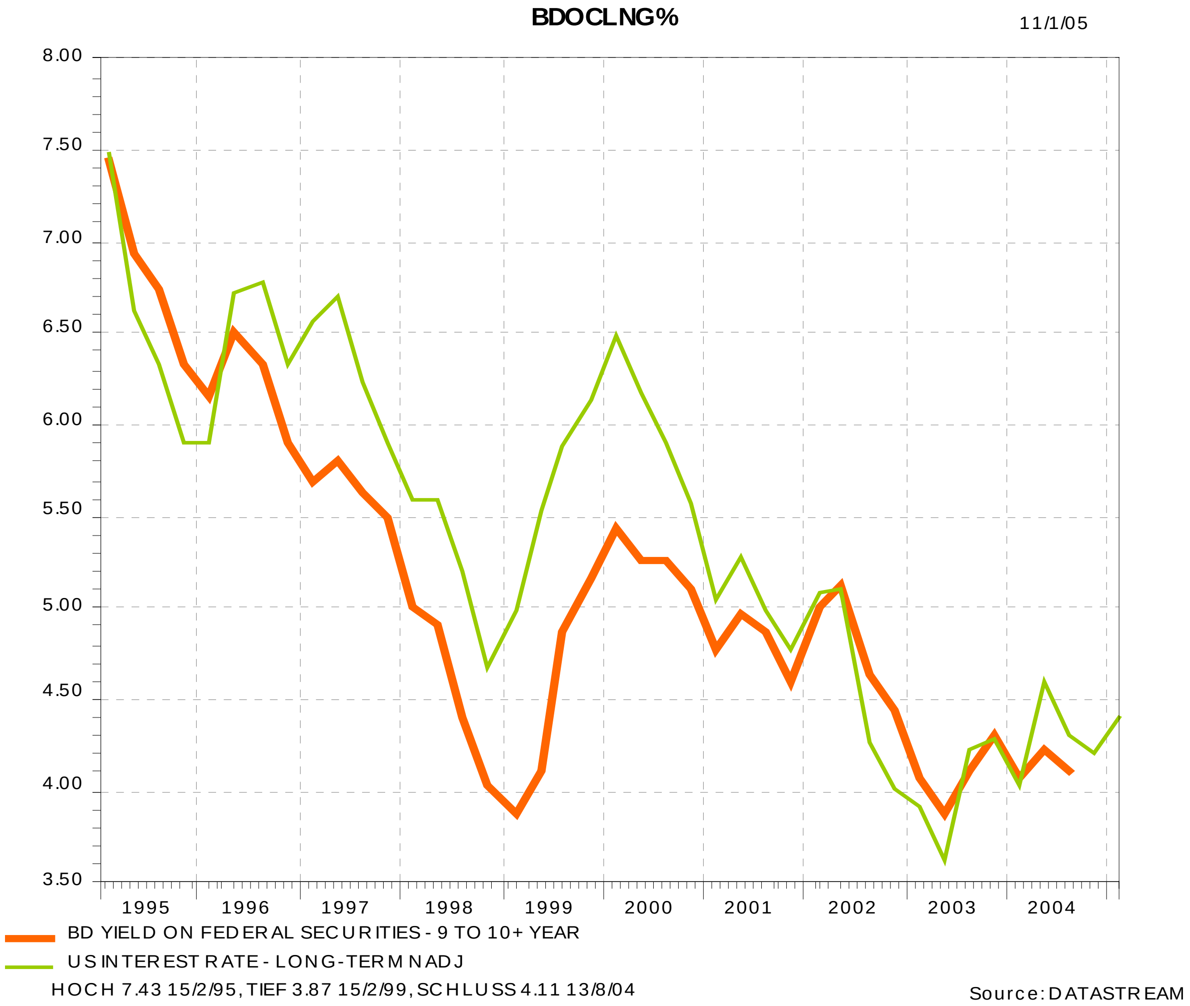
Sources: ECB and Reuters.

1) Starting from the operation settled on 28 June 2000, the main refinancing rate refers to the minimum bid rate applied to variable rate tenders.

ECB • Monthly Bulletin • June 2001

Graphics V.1: Comparison of ECB-interest-and money market rates

The minimum reserve policy is a powerful instrument. As powerful that it can hardly be used. Let's assume that the Central Bank leaves the liquidity of the banks constant and increases the minimum reserve rates. Then reserve-carrying liabilities have to sink. So the banks have two possibilities: Either they decrease the credit granting to non-banks (balance sheet of the consolidated balance sheet sinks) or they place bank bonds at high interest rates, to en courage non-MFIs to invest short-term bank deposits on the long-term. Both lead most probably to a very strong increase of interest rates, to a crash at the stock market and to recession. That's why minimum reserve rates hardly change in the United States.



Graphics V.2: Parallels between German and US ten-year–rates of interest as an example for the limits of the power of the ECB over long-term interest

3. Strategies of the ECB according to its publications

The ECB comments its monetary policy in three documents: The EMI publishes a bulletin over the single monetary policy of the ECB[26] and the monthly bulletin 1999 contains comments to the monetary policy of the ECB.[27] The third possible source is the complete lecture of the monthly bulletins of the ECB. In 2001, the ECB has published a paper: The Monetary Policy of the ECB. If for example the stock prices are constantly commented on in the monthly bulletins, and if they are even dedicated a whole box in Mai 2000, you can see even without the explicit confirmation, that the ECB observes the stock prices.[28]

According to article 105 of the EU-Treaty, the ECB is obliged to guarantee price stability. The ECB- Council has decided: Price stability is defined as the rise of the harmonised indexes of the

[26] European Monetary Institute: The Single Monetary Policy in Stage Three: Specification of the operational framework. Frankfurt January 1997.

[27] Die stabilitätsorientierte geldpolitische Strategie des Eurosystems. In Monatsbericht der EZB. Januar 1999. p.43-56

[28] Jüngste Tendenzen bei der Volatilität von Aktienindizes. In: Monatsbericht der ECB. May 2000, p. 20-22

consumer's price (HICP) for the EURO-currency area of under 2% p.a. compared to the previous year.[29] The price stability has to be maintained at medium-term. The advantages of price stability are explained by the ECB as follows.

- Price stability ameliorates the transparency of the relative pricing mechanisms and enables therefore the better allocation of resources.
- Price stability lowers the inflation premium in the nominal rate of interests.
- At inflation, resources are wasted to cover against inflation.
- At inflation uncontrolled re-distribution of incomes and resources happens.

The points mentioned are supported by literature. The wide literature on the Philips curve, which contains the contradiction between growth and price stability is not mentioned here. Also a non-inflationary, steady growth as a task of the Union is mentioned among the goals of the ECB in article 2 of the EU-treaty.[30] The opinion of the ECB is, that it serves the growth the best by guaranteeing price stability.
On the other hand, the ECB wants a rate of inflation not far below two percent. Some of the reasons are:

- It is not possible to take into account adequately technical progress in price indices.
- In a recession the central bank wants to lower real interest rates. Bit interest rates cannot sink below zero (Japanese rates were near zero in the past). If the rate of inflation is two percent the real rate of interest can become negative.
- There are uncertainties in the calculation of price indexes amounting to one percent p.a.

The ECB describes its strategy as two-pillar strategy[31]:

- Announcement of a monetary growth target for the money volume M3 of 4.3% for 1999 (actual rate of growth 5.9 %), 4.5 % for 2000 and 2001[32] the monetary growth target is determined as follows. An estimation of less than 2% inflation + GDP growth estimated as 2-2.5% + (-0.5 to -1 %) decrease of velocity of circulation $\Rightarrow$ 4.5% monetary growth
- "A widely backed-up assessment on the prospects for the future price development and on the risks for the price stability in the Euro–area in total."[33]

[29] Die stabilitätsorientierte geldpolitische Strategie des Eurosystems. In Monatsbericht der EZB. Januar 1999. p.43-56, p. 43
[30] Die stabilitätsorientierte geldpolitische Strategie des Eurosystems. In Monatsbericht der EZB. Januar 1999. p.43-56, p. 43
[31] You could also say that the strategy of the ECB stands on two legs.
[32] For example: Monthly bulletin of the EZB: März 2001, p. 7
[33] Die stabilitätsorientierte geldpolitische Strategie des Eurosystems. In Monatsbericht der EZB. Januar 1999. p.43-56, p. 43

The latter signifies that for example a rising US-$-price can serve as a justification of an increase of the interest rate, as rising import prices represent a danger for price stability. The expenditures of the Euro-area without transfers in 1999 were 1.159 billions € according to table 8.2 of the monthly bulletin of the ECB. The GDP was 6.108 billions according to table 5 and the import quota of goods and services 19%. (Data taken from the English edition of the monthly bulletin April 2000)

The ECB reserves itself the right neither to react automatically neither to an exceeding of the monetary target as an intermediate target nor to the exceeding of the final target of a rate of price in crease of 2 % p.a. It rather strives for a realisation of the targets on the medium-term. All this hardly enables an external observer to make a prediction on the monetary policy of the ECB. It rather opens discretionary powers to the ECB. The use of wide discretionary powers is also called discretionary behaviour. The contrary would be a behaviour bound to the rules, as Henry C Simons, father of the Chicago school, claimed it.[34]

4. The interpretation of the monetary politics of the Federal Reserve System (FED) and of the Deutsche Bundesbank by third parties – rules for a monetary policy

As an external observer, Taylor has tried to explain the monetary theory of the United States by simple rules. These examinations were then applied on the monetary politic of the Federal Republic of Germany (GFR) The empirically determined regularities can as well normatively be interpreted as rules for the transactions of the Central Bank, which, of course, the Central Bank denies.

Let's take the interest rate as an operative variable. Then it should be valid for the money market interest (R_t) that a deviation from a neutral value (R_t^*) signifies that also a value, which is used as an intermediate variable (x_t) deviates from its standard value (X_t^*). Then R_t and X_t are standing in a linear functional relation. Targets or intermediate targets can be for example: money aggregates, exchange rates or the nominal GNP.

$$R_t - R_t^* = a\,(X_t - X_t^*)$$ [35]

A different approach is to determinate the deviations of the operative variable from their standard value directly from the deviations of the final target variable.

[34] Simons, Henry C.: Rules versus authorities in monetary policy. In: The Journal of Political Economy. Vol.44 (1936). p. 1-30

[35] Orphanides, Anastasios: The quest for prosperity without inflation. In: European Central Bank. Working Paper Series. Working Paper No. 15, p 5

Taylor explains the monetary policy of the FED with the help of the following equation:[36]

$$R_t - R_t^* = y\,(\pi_t - \pi^*) + \delta\,\frac{(y_t - y^*)}{y^*}.$$

Here R_t = Federal Funds Rate for t = short-term money rate in t

R_t^* = real interest (2% p.a.) plus π, the neutral interest rate in t

π^* = 2 % p.a. = targeted rate of inflation

γ_t = GDP in t

γ^* = potential GDP.

γ and δ = Proportional factors, at Taylor each is ½,

as price stability, growth and full employment are equal targets in the United States. The difference between the current rate of inflation and targeted rate of inflation is called inflationary gap. Is the inflationary gap positive, Taylor predicts a higher R_t, that is a higher short-term money rate. Is the real GDP lower than the production potential, the difference in percent to the production potential is called out put gap, and Taylor predicts a lower short-term money market interest.

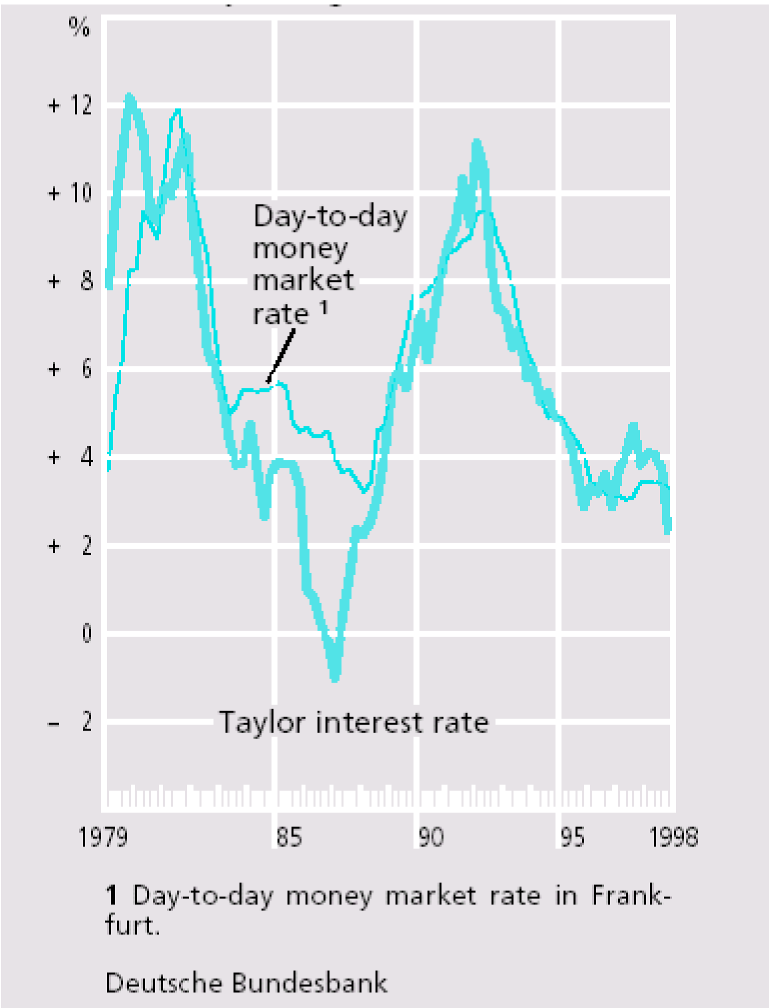
The Deutsche Bundesbank has calculated the Taylor-interest for Germany.[37] The curve seems to bring the German monetary policy near to some extent. The Taylor interest expresses stronger fluctuations of the money rates as thy can actually be observed in reality.

Day-to-day money market rate and Taylor interest rate for Germany

Quarterly averages

[36] Taylor, John B.: Discretion versus policy rules in practice. Carnegie- Rochster-Conference Series on public policy 39. December. p. 195-214. Quoted according to: Orphanides, Anastasios: The quest for prosperity without inflation. In: European Central Bank. Working Paper Series. Working Paper No. 15

[37] Deutsche Bundesbank: Taylor- zins und Monetary Conditions Index. Monatsbericht der Deutschen Bundesbank. 5. Jg. No.4. April 1999. p. 47-63



Graphics V.3: Overnight rates and Taylor interest for Germany

Source: Deutsche Bundesbank: Taylor-Interest Rate and monetary condition index. In: Monthly Report 51. Jg. Nr.4 April 1999, p. 47 –63, p.51

The problem of a monetary policy bound to rules consists in

- false rules
- time lags
- false data

To false rules: In the 30ties existed a rule, which has proved over thousands of years. The rule that every bank, also the Central Bank should change its bank notes into gold (or silver), and that this exchange should be defended even with higher interests. Friedman/Schwartz called the policy of the FED afterwards as unsuitable, as it extended the central bank money, but didn't completely compensate through the expansion of central bank money the decrease of M1 caused by the break down of banks. The FED is criticised that the old rule was not thrown overboard in favour of discretionary behaviour.

To time lags:

The point of time, when the monetary measures are necessary, differs from the point in time, when actions are taken:

- Time lag in the bank of issue
- time lag in the collection of information (statistic-lag)
- time lag until recognition (recognition –lag)
- time lag until the decision is taken - time-lag by the administration (decision- or administrative lag)

The point of time, at which a measures is taken differs from the point of time, at which the measures have an effect on the target (s).

- time lag at the banks; intermediate lag
- time lag at the non-banks; outside lag[38]

A pro-cyclical effect occurs, when a restrictive measure taken in a time of up swing only starts to show effect during the down swing and then only reinforces the latter:

To false data:

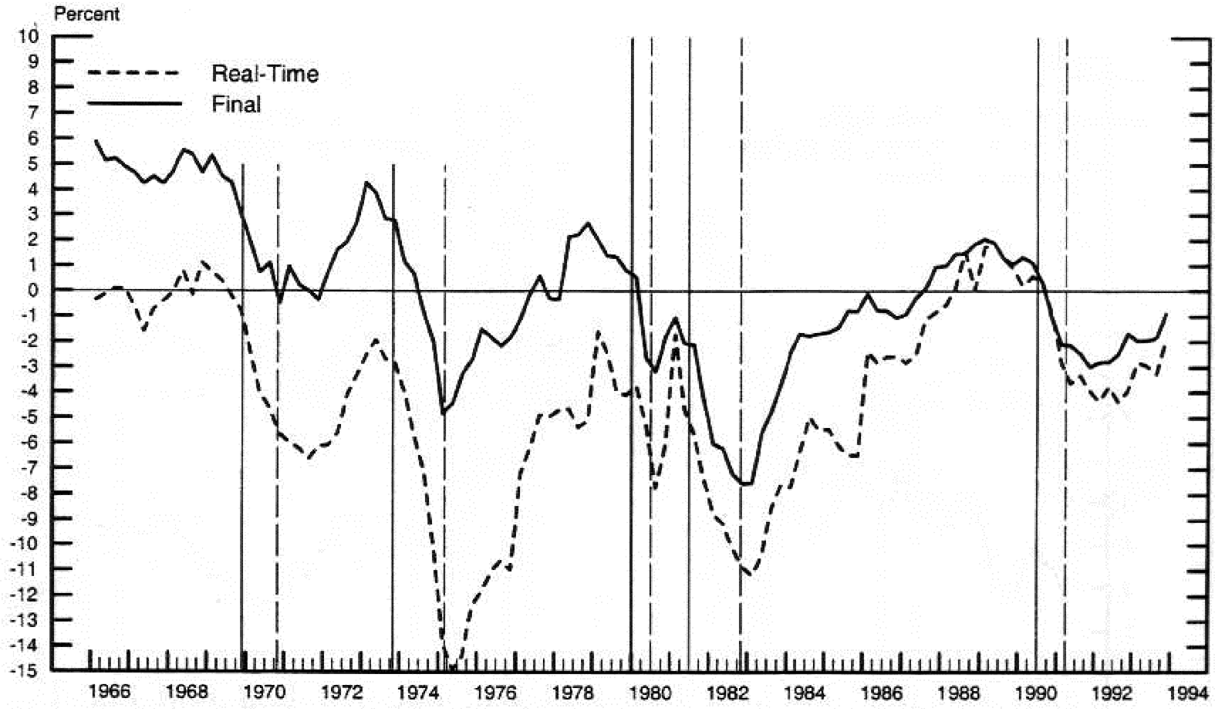
Orphanides[39] puts the US-inflation from 1975 until 1980 down to the fact that the policy of the FED oriented itself too much on the policy that corresponds to the Taylor’s rule. The data for the out put gap for the years between 1975 and 80 were much too high, - compared to the ones which existed at the end of 1994.

[38] Compare Issing, Otmar: Einführung in die Geldpolitik. 5. Auflage 1993 (1980), p. 162/3

[39] Orhanides, Anastasios: The quest for prosperity without inflation. In European Central Bank. Working Paper series. Working Paper No. 15, p. 4 and the graphics on page 47.

Figure 4

The Output Gap in Real-Time and Final Data



Graphics V.4: The out put gap in real time and final data

VI. Insolvencies of banks

A. Insolvencies of MFIs

The causes of bank insolvencies are

The most important reasons for insolvency of banks are:

- criminal activity by employees or owners, fraud and theft;
- credits to employees or owners, including to public entities;
- lack of diversification - large scale credits;
- maturity transformation pursued by banks on large scale;
- a too large business volume compared to equity, called overbanking;
- absorption of the risk of price changes by pursuing businesses unfamiliar to

banks.

Criminal activity: It is often the case that the cashier disappears with the cash.

It also has to be considered fraudulent, if credits without sufficient coverage are granted to employees, owners or their friends. The biggest crisis of this kind was the Federal and Loan (S&L) crisis in the United States in the 80ties of the last centurie. About 1500 institutes were failing. The reason was the deregulation of banks in the era of Reagan on the one hand and the existence of bank deposit insurances in the United States on the other hand. This created a moral hazard, which many managers gave in to. The American taxpayer paid for the S%L debacle about 135 billion US-$. Many of the credits, which failed, were given to members of the managing board or their friends.

Another reason for the S&L crisis can be found in the large amount of **maturity transformation**: Savers had short-term deposits and the S&L gave mortgage credits at fixed interests.[40] At an increase of the interest rate from 3 to 4%, the value of a one-year annuity sinks from 100 to 99. The value of an eternal rent sinks according to the equation Present Value = Annuity/Capitalisation Rate from 100 to 75. The equity of the S&L became negative by the increase of the interest rates.

Large credits played a role in the insolvency of Jürgen Schneider in 1994. He had credits with the Centralboden Bank, a daughter company of the Deutsche Bank, which were on the scale of the equity of the bank[41]. They are not considered large credits, if they are within the premium reserve stock of a mortgage bank. A credit is allowed to be within the premium reserve stock, if the bank has lent no more than 60% of the value of the real estate. The bank suspected that the value estimation were pure fantasy of Scheider, but didn't dare to insist on an examination, as itself would have been insolvent, if it would have taken the credits out of the premium reserve stock. It was absorbed by the Deutsche Bank after the Schneider`s insolvency. If a bank lends large scale credits, which are as large or even larger than their equity it becomes dependent on the creditor. (Schneider`s insolvency is a good example for the fact that private parties the same as states serve their debts only as long as the new credit taking exceeds the debt service. (Monetary and credit theory, debt crises.))

---

[40] Litan, Robert L.: Saving and Loan crisis. In: The New Palgrave Dictionary of Money &Finance. Ed.: Newman, Peter; Milgate, Murry; Eatwell, John. Macmillan, The Stockton Press: London and New York 1992. Vol. 3. p. 389-392.

[41] Schneider, Jürgen: Bekenntnisse eines Baulöwen. (Confessions of a building speculator). Ullstein: München 2001, p.258. „Only in this way (by putting my borrowings within the premium reserve stock) it was possible that the Centralboden Bank, a bank with the equity of 750 000 DM, could give me a credit of one and a half billion". My translation.

Large credits were the reason for the insolvency of the Leipziger Bank in 1900. In 1931 the Darmstädter- und National Bank (DANAT Bank) broke down as a result of a large-scale credit to the "Nordwolle". The break down f the DANAT-Bank was the trigger of the German bank crisis and of the introduction of foreign exchange controls in Germany. The large-scale borrower "Nordwolle" not even delivered balance sheets. Hermann Joseph Abs, after the Second World War, a member of the managing board and later of the supervisory board of the Deutsche Bank, then was an employee of the DANAT bank. He was in charge of the credits to "Nordwolle". He claimed a balance sheet of "Nordwolle". Nordwolle in turn claimed the dismissal of Abs.

This directs the attention to the problem of the **competition in credit worthiness**, which can be pernicious for banks. Desired is a competition in interest rates not in credit worthiness. Competition in credit worthiness plays also a role in the case of Jürgen Schneider, as most of the banks renounced to a for small costumers normal test of his declarations. The Herstattbank was an example for **overbanking**. It failed 1974, one year after flexible exchange rates came into existence. Its business volume consisted out of dealings in foreign exchange futures, which were not closed positions. Contrary to the principles of proper accounting the losses of open positions didn't appear in the profit and loss account. The equity ratio considering the dealings in futures was not in percent, but only in per mille.

What does the bank have to do to remain solvent? It has to keep liquid assets (compare working shet 4), which are shiftable to other banks. At every credit granting, the bank has to ask itself, if another bank would grant credit, so that the credit could be passed on to another bank in an emergency case. Not its own assessment of the credit worthiness is decisive for the credit granting, but the bank must assess its opinion about the fact: How is the assessment of other banks of the credit worthiness of the borrower. Not the amount of cash holdings is decisive for the liquidity of banks, but the solvency of the bank, as it can shift liquid assets to other banks or even to the Central Bank. In Germany exist the " Liquiditätskonsortial Bank", which keeps ready to purchase solvent assets of banks in case of a liquidity crisis. Deutsche Bundesbank is a partner of the Liquiditätskonsortial Bank, which means that it can provide large sums of money.

B. Insolvencies of Central Banks

When a bank is insolvent, the creditors obtain a settlement quota or a percentage of recovery. How can banks of issue protect from insolvency? An insolvency of banks of issue is only likely to happen, if central banks are obliged to change their bank notes into coins of precious metal or into foreign exchange. This was the case in the 19th century or at fixed exchange rates. The banks of issue have to have assets at their disposal, which at a run can be shifted to the international markets or can be exchanged at other banks of issue. A bank of issue, which depreciates its currency, can be compared with a bank of issue, which pays a settlement quota to its creditors. Flexible exchange rates are the comparable with a bank of issue, for which the state announces a debt deferral (moratorium). A moratorium signifies a stoppage of payment without that the creditor has the possibility to enforce the sale of the debtor's assets. The flexible exchange rate is then the price for a claim towards an insolvent bank. If the price of the Euro sinks, it is comparable with the sinking of Argentine bonds, which are dealt far below par.

Naturally currency devaluations are not talked about in ugly categories as percentage of recovery or even fraud, as with mister Schneider. But the Argentines, whose accounts were blocked in December 2001, and which in February 2002 obtained for one Peso only 71 US-Cent instead of one Dollar before or even less at flexible exchange rates, indicate by their protests, that they feel well deceived or like a bankrupt's creditors.

Where are the reasons for depreciation? It results mostly from the fact that the credit policy of the Central Bank was too expansive. This means: credits are granted at rates of interests, which are too low or creditworthiness is not given in the opinion of international banks. International credits are not shiftable, as the rates of interest are too low, and often as well because they lack credit worthiness. Lacking credit worthiness of a bank again has its reason in the lacking credit worthiness of the credits granted by it. What should be reached by depreciation? The Dollar revenues of exporters yield more currency units if the exporter reckons in domestic currency. Therefore it t is exported more at constant wages and constant other costs. At the same time import goods with given US-$ costs become more expensive within the domestic area. Besides, the speculations on devaluation vanish. By this vanish the high interests, which are necessary to deter the speculators. Were high interests necessary for a longer period of time, then the lowering of the rates of interest promote investments.

This is the immediate result. Business activity is flourishing and unemployment sinks. But the labour union will claim more money, as holidays abroad have become more expensive. Increase in wages can be obtained more easily, as unemployment decreases (Philips curve). But after a while inflation has eaten up all the competition advantages. The situation is ready for a new devaluation. After the devaluation starts the process of adjustment. The more often devaluations take place, the quicker processes of adjustment of the domestic economy happen. In Germany between 1922 an 1923, the prices for the domestic market formed, by the shops fixing the prices only after the opening of the foreign exchange market by multiplying prices of the previous day with the price-increasing factor of the exchange rate. This means: the adjustment of domestic, allocative decisions happens immediately. Then devaluation doesn't have a stimulating effect.

You can say as well: At a rational expectation of all market participants, the effects of devaluation are clearly seen, and it has no more effect on the allocation of resources.

Fixed, but adjustable parities should built up trust, so that stable expectations are created. But the state can occasionally use devaluations as an instrument by disappointing the expectations of the market participants. A policy based on time-inconsistency doesn't meet the moral requirements and fails on the long-term, because economic subjects are able to learn. The IMF-agreement with fixed exchange rates had a stabilizing effect until about 1965. Until 1971 more and more parity changes happened, so that they finally passed to flexible exchange rates. Only the foundation of the European Monetary Union (EMU)brought the turning away from policy based on time-inconsistency, as it showed to be inappropriate. Italy for example couldn't sell long-term government bonds at all. That's why the Italian public debt is short-term. The French interests increased so much that the state didn't see an advantage in the devaluation policy. Only with the converge process, the long-term interests of France have adjusted to the lower German ones.

VII. Exchange rate theory

For the purchasing power parity see

<Working sheets 21, 22, and 31>

Purchasing Power Parity –PPP says:

(1)$e = P/P^*$

whereby (e) denotes the exchange rate and P respectively P* the domestic and foreign price vectors. Differences from the purchasing power parity are justified with tariffs, transport costs and search costs.[42]

A narrower formulation of the PPP consists as such, as you assume that exchange rates are determined by tradables:

(2)$E = P_T/P_T^*$

P can be taken apart into the prices of tradables (Subscript T) and non-tradables (Subscript N)

(3)P = α P_T + (1-α) $P_{N,}$with

(4)ß =P_N/P_T oder P_N = ß PT.

We determine the relation of prices of non-tradables to tradables.

When putting (2) into (1), it results in:

(5) P = α P_T+ (1- α) ß $P_{T.}$

= P_T (α +(1- α) ß)

= P_T γ .

Corresponding equations can be introduced for the foreign country. The exchange rate in the newer form of the PPP turns into:

$$(6)\, e = \frac{P_T}{P_T^*} = \frac{P\gamma^*}{P^*\gamma}.$$

γ is a factor, which contains the ratio of tradables to non-tradables, which therefore is non-monetry, not monetary. Non-monetry factors can signify preferences, production technology and factor endowment

We describe the influence of productivity changes on the exchange rate in the following. The nominal wages (w) for tradables and non tradables are the same as the marginal product of labour (q) multiplied with the product price (P):

(7.1) w = q_T P_T and

(7.2) w = q_N P_N

With the labour markets homogeneous, „w" is in both equations the same. Wages are the same at bilateral polypoly (competition by many small agents on both sides of the market) (7.1) and (7.2) can be turned into:

(8.1) P_t = w/q_T or w = $P_T q_T$

(8.2) P_N = w/q_N = P_T q_T/ q_N

Respective equations are valid for the foreign country.

Puting(8.2) in (3) [P = α P_T + (1-α) P_N] we get

(1) (9) P = α P_T + (1−α) P_T (qt / q_N) or

$$(10)\quad P_T = P\frac{1}{\alpha + (1-\alpha)\, q_T / q_n}$$

Corresponding equations are valid for the foreign country. If we put equation (6.3) into the equation (1), it results in: whereby (a*) is the term for foreign countries.

[42] Streissler, Erich W. : Exchange rates and financial markets. Routledge: London and New York 2002, p.43

(9*) $P^* = \alpha^* P_T^* + (1-\alpha^*) P_T^* \frac{q_T^*}{q_N^*}$ or

(10*) $P_T^* = P^* \frac{1}{\alpha^* + (1-\alpha^*) \frac{q_T^*}{q_N^*}}$

The exchange rate results the in:

(11) $e = \frac{P_T}{P_T^*} = \frac{P(\alpha^* + (1-\alpha^*) q_T^* / q_N^*)}{P^*(\alpha + (1-\alpha) q_T / q_N)}$.

Is the domestic labour productivity at the tradables greater than at the non-tradables, is therefore $q_T > q_N$ (their ratio is greater than one), as the productivity of the tradables has increased compared to a earlier state, then the exchange rate is lower as in the state of the lower productivity of tradables (q_T). This means that the value of domestic currency has increased. The revaluation is the greater, the greater the share of the tradables on all goods (α).

This signifies:

1. Countries, which have a comparably small share of tradables in GDP have a higher exchange rate. This means that their currency is undervalued. You live cheap there, when making a holiday.

2. If the productivity of tradables is increasing more than the one of non-tradables, so the exchange rate is lower in the new state. The currency has revalued compared to the first state