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DENSITY FLUCTUATION SPECTRA OF DILUTE ^{3}He-^{4}He MIXTURES

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Density correlation functions of a 6% ^{3}He-^{4}He mixture are evaluated for wavenumbers in the roton vicinity in the zero temperature limit within a phenomenological theory which couples renormalized ^{4}He single modes to ^{3}He quasiparticle-quasihole excitations. Roton energy shifts are shown in agreement with experiments to be caused mainly by the different structure of ^{4}He in mixtures. Renormalization by emission of quasiparticle-quasihole excitations is of minor importance.

The neutron scattering intensity /1,2/ of ^{3}He-^{4}He mixtures of concentration $x=N_3/(N_3+N_4)$ is proportional to

$$S_{tot}(k,\omega)=(1-x)S_{44}(k,\omega)+x\sigma_3/\sigma_4\ S_{33}(k,\omega)$$
$$+2\sqrt{x(1-x)\sigma_3/\sigma_4}\ S_{34}(k,\omega) \qquad (1)$$

if incoherent scattering ($\sigma_3^{inc} \simeq 0.25\ \sigma_3$) is neglected. We use the zero temperature limit of the fluctuation dissipation theorem to relate the dynamic structure functions $S_{ij}(k,\omega)$ to the absorptive parts $\chi''_{ij}(k,\omega)$ of the 2x2 matrix of density response functions

$$\chi(k,z) = -[z^2-\Omega^2(k) + z\Sigma(k,z)]^{-1}\ k^2m^{-1}. \qquad (2)$$

Here m is the diagonal matrix of bare masses. The off-diagonal elements of the matrix $\Omega^2(k)=k^2m^{-1}\chi^{-1}(k)$ of characteristic frequencies

$$\Omega^2(k) = \frac{1}{1-W^2(k)} \begin{pmatrix} \omega_3^2(k); & -W(k)\sqrt{\frac{4}{3}}\ \omega_3(k)\omega_4(k) \\ -W(k)\sqrt{\frac{3}{4}}\ \omega_3(k)\omega_4(k); & \omega_4^2(k) \end{pmatrix} \qquad (3)$$

couple density fluctuations with a strength

$$W(k) = \chi_{34}(k)/\sqrt{\chi_{33}(k)\chi_{44}(k)}\ . \qquad (4)$$

We approximated this quotient of static susceptibilities within a generalized Feynman approximation (Σ=O) — $k^2\chi_{ij}(k)\simeq 4[S(k)mS(k)]_{ij}$ — in terms of structure functions $S_{ij}(k)$. As input we use S_{33} of the ideal Fermi gas and S_{34} taken from /3/. S_{44} is extracted from the quotient S/S_{44}^o of total X ray intensities of the mixture /4/ and of pure ^{4}He /5/. This leads to a coupling strength of 0.074 for roton momenta.

Density fluctuations of a 6% mixture differ from those in pure ^{4}He as a result of at least 3 effects listed in the order of their importance: (i) The restoring force ω_4 differs from ω_4^o in pure ^{4}He since static properties of ^{4}He are changed. This structural effect estimated within a Feynman model by

$$\omega_4(k) \simeq \omega_4^o(k)\ S_{44}^o(k)/S_{44}(k) \qquad (5)$$

leads to a frequency shift $d\omega_4=\omega_4-\omega_4^o=0.26$ °K at $k=2$ Å^{-1}. (ii) The coupling W to a second characteristic frequency ω_3 induces resonance shifts. (iii) In the mixture density fluctuations decay via additional dissipative mechanisms (e.g. into quasiparticle-quasihole excitations) described by the imaginary parts $\Sigma''_{ij}(k,\omega)$ of the self-energies causing via the real parts a different renormalization of fluctuation frequencies.

Since the (additional) roton decay rate in a 6% mixture at T=0.25 °K is only about 0.06 °K /6/ and since we are mostly interested in the roton vicinity we neglect Σ''_{44} (as well as Σ_{34} and Σ_{43}). The real part Σ'_{44} is accounted for by replacing ω_4 by a renormalized frequency ε_4. Whereas in pure ^{4}He ω_4^o is renormalized by $\Sigma_{44}^{o\prime}$ to ε_4^o /7/ one expects in a mixture $\varepsilon_4(k)=\varepsilon_4^o(k)+d\varepsilon_4(k)$ caused at least by the different restoring forces even if the variation of Σ'_{44} is neglected

$$d\varepsilon_4(k) = \frac{\partial\varepsilon_4^o(k)}{\partial\omega_4^o(k)}\ d\omega_4(k)=2m_4\ \frac{\omega_4^o(k)}{k^2}\ Z_4^o(k)d\omega_4(k). \qquad (6)$$

Evaluating the above derivative from the resonance condition /7/ $z^2+z\Sigma_{44}^o(z)=\omega_4^{o2}$ for $z=\varepsilon_4^o$ in terms of the single mode intensity $Z_4^o(k)$ of pure ^{4}He one obtains for roton momenta $d\varepsilon_4 \simeq 0.66\ d\omega_4$.

^{3}He in superfluid ^{4}He can well be described as a gas of quasiparticles moving without friction with a dispersion $\varepsilon_3(k)=k^2/2m_3^*(k)$ determined by a slightly wavenumber dependent effective mass /8/. Hence we approximate the characteristic frequency ω_3 and the self energy Σ_{33} by the corresponding quantities of an ideal Fermi gas of particles with mass 2.65 m_3 being appropriate for wavenumbers around the roton.

Were it not for off-diagonal elements of $\Omega^2(k)$ our approximations would describe undamped single mode excitations of ^{4}He and an ideal gas of ^{3}He quasiparticles. However, the coupling W (4) induces a k,z dependent roton exchange potential between the ^{3}He quasiparticles leading to a RPA density response $\chi_{33}(k,z)$. Moreover, ^{4}He density fluctuations decay into (interacting) quasiparticle-quasihole excitations if $\varepsilon_3(k)-kv_F\leq\omega\leq\varepsilon_3(k)+kv_F$: In our approximation

$$\chi_{44}(k,z)=-[z^2-\Omega_{44}^2(k)-\Omega_{44}^2(k)\pi_{44}(k,z)]^{-1}\ k^2/m_4 \qquad (7)$$

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the polarization operator for ^{4}He density fluctuations

$$\pi_{44}(k,z)=\gamma^2(k)\pi_{FG}(k,z)[1-\gamma^2(k)\pi_{FG}(k,z)] \qquad (8)$$

is given in terms of the particle-hole polarization "bubble" $\pi_{FG}(k,z)=-\chi_{FG}(k,z)/\chi_{FG}(k)$ of the ideal Fermi gas and the effective coupling $\gamma(k)=W(k)/\sqrt{1-W^2(k)}$. The decay rate is only 0.13 °K for k = 2 Å^{-1}. Also the renormalization of $\Omega_{44}=\varepsilon_4/\sqrt{1-W^2}$ by π'_{44} is quite small, about -0.05 °K, if compared with the roton shift $d\varepsilon_4$ (6) caused by structural effects.

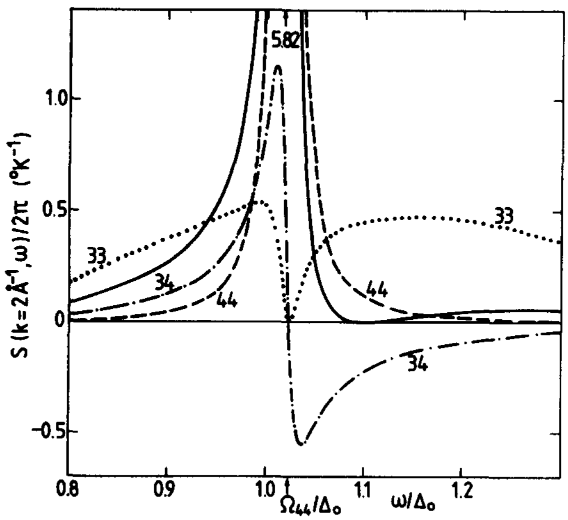

Fig. 1: Fluctuation spectra $S_{ij}(k=2\text{Å}^{-1},\omega)/2\pi$ as functions of ω reduced by roton energy Δ_0 of pure ^{4}He. Full curve denotes $S_{tot}(k,\omega)$ (1); upper arrow the peak position thereof.

Since a quasiparticle-quasihole excitation band coupled weakly to undamped rotons produces only a linewidth but not a peak splitting of the latter $S_{44}(k,\omega)$ shows a peak at roughly the frequency Ω_{44} which lies in Fig. 1 well within the band. That $S_{33}(k,\Omega_{44})=0$ is a result of our approximation. More realistic Σ_{ij} yield only a dip in $S_{33}(k,\omega)$ and other changes /9/ in $S_{ij}(k,\omega)$. Note that S_{34} should not be neglected in comparison with S_{33} in particular since the former's weight in (1) is 3.8 times larger than the latter's: The asymmetry and the shift of the peak of $S_{tot}(k,\omega)$ (full curve in Fig. 1) with respect to that of $S_{44}(k,\omega)$ which is for k=2 Å^{-1} about -0.02 °K is mainly caused by $S_{34}(k,\omega)$.

The shift of the peak in $S_{tot}(k,\omega)$ with respect to the single mode energy ε_4^o of pure ^{4}He is in semiquantitative agreement with experiments /2/ done at 0.6 °K, 0.75 °K and in marked contrast to previous theories /10/ yielding large negative shift in the wavenumber-range of Fig. 2. The renormalization of ^{4}He fluctuation frequencies by quasiparticle-quasihole emission is indeed negative but by far smaller in size than the static restoring force change. Thus our investigation supports an earlier conjecture /2/ by quantitatively showing the shift of Fig. 2

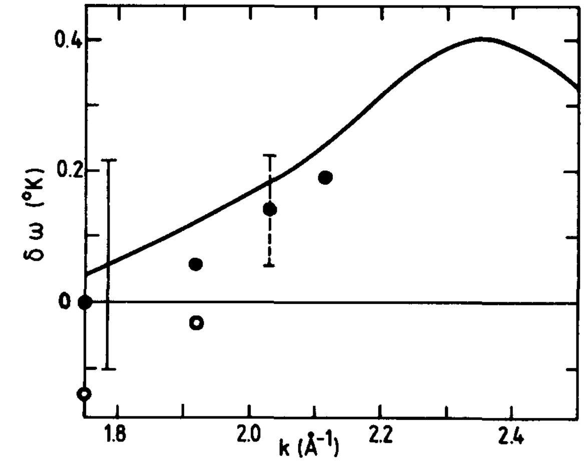

Fig. 2: Peak shift of $S_{tot}(k,\omega)$ (1) with respect to $\varepsilon_4^o(k)$ of pure ^{4}He. Experimental results /2/ obtained at 0.6 °K and 0.75 °K are shown as open and full circles with a typical error bar (vertical dashed line). Full line error bar is explained in the text.

to be mostly due to the change in the static restoring force $\omega_4(k)$ (5) which in turn is caused by the structural change of ^{4}He in the mixture.

Fig. 2 shows a typical error (full vertical line) of our calculated shifts resulting from the experimental error /4/ of $S_{44}(k)/S_{44}^o(k)$ and from a surmised uncertainty in $S_{34}(k)$ /3/ of about ± 0.02. The peak widths of $S_{tot}(k,\omega)$ compare favorably with experimental widths /2/ of peaks inside the quasiparticle-quasihole band. Since however the above discussed input uncertainties lead to relative errors of our widths of about 150% we did not plot them.

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/10/ For a discussion see ref. 2.