INITIAL-VALUE DEPENDENCE OF THE GROWTH OF CONVECTION PATTERNS IN PURE FLUIDS AND IN BINARY MIXTURES

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The growth dynamics of convection patterns in incompressible one-component and binary fluid layers heated from below are discussed. The investigated system is two-dimensional with periodic lateral and no-slip impermeable horizontal boundary conditions. Starting from different initial states the transients are investigated. In pure fluids the further temporal dynamics is independent of the structure of the initial state and depends only on one critical mode. In binary mixtures, however, the whole transient dynamics is strongly influenced by the initial condition due to two critical modes that finally interact nonlinearly.

Influence des valeurs initiales sur la croissance de la convection dans les fluides purs et dans les mélanges binaires

La dynamique de croissance des structures convectives dans des fluides incompressibles, simples ainsi que binaires, qui sont soumis à un gradient de température est discutée. Le système est à deux dimensions avec des parois latérales périodiques et horizontales réalistes. En outre, l'influence de l'état initial sur les transitoires est étudiée. Dans des fluides purs la dynamique ne dépend pas de l'état initial et posséde seulement un mode critique. Dans des mélanges binaires les transitoires sont largement influencés par les conditions initiales dues à l'interaction de deux modes critiques.

Symbols	
σ	Prandtl number (viscosity)
L	Lewis number (concentration diffusion)
ψ	separation ratio = influence of temperature on concentration field
r	reduced Rayleigh number Rayleigh number(critical Rayleigh number of pure fluid)
d	distance of plates
κ	heat diffusion
Γ	aspect ratio = heightwidth
$\mathbf{u}(x, z, t)$	velocity field
$w(x, z, t)$	component of velocity field
$T(x, z, t)$	temperature field
$T_1(z = 0, t)$	first lateral Fourier component of temperature field
$C(x, z, t)$	concentration field
$\phi(x, z, t)$	stream function

1. INTRODUCTION

Convection in binary miscible fluids like ethanol - water, ^{3}He - ^{4}He, or gas mixtures reveals many additional patterns in comparison to simple fluid convection.

Futher it shows interesting properties as subcritical bifurcation, oscillatory instabilities, stationary states, complex spatiotemporal behaviour, and turbulence [1, 2, 3]. It is a paradigmatic system for research on systems far from equilibrium because the equations are wellknown, experiments are sufficiently simple, and the computing power is strong enough to perform simulations. So it is feasible to attack this system from many sides. On the other side it is also of great practical interest : most fluids in nature are mixtures.

We use a finite-difference algorithm to simulate a two dimensional convection cell of height d and length $2d$ with realistic horizontal and periodic lateral boundary conditions. The x-axis is parallel and the z-axis is perpendicular to the horizontal boundaries. In contrast to the final convection patterns the transient growth of structures in binary mixtures has not been investigated intensively so far.

The pure fluid is controlled by the Prandtl number $\sigma = \nu/\kappa$ that is the ratio of velocity diffusion over temperature diffusion. Here sigma is fixed to $\sigma = 10$. For binary mixtures additional parameters come into play : the Lewis number $L = D/\kappa$ is the ratio of concentration diffusion over temperature diffusion and is fixed to $L = 0.01$; concentration diffusion is much smaller than the thermal diffusion and causes longtime diffusive mixing. The Soret number ψ couples the temperature field into the concentration

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equation and is negative here, $\psi = -0.25$. The control parameter in both cases is the reduced Rayleigh number r. It is the quotient of the Rayleigh number itself and the critical Rayleigh number of the pure fluid. It controls the temperature difference between the upper and lower plate which is responsible for the heat flow that actually drives the system out of equilibrium. We use a small supercritical value $r = 1.42$ that is about 6.2% beyond the critical value of the onset of convection in the binary mixture, $r_c = 1.3348$. For the pure fluid case we take the same value $r = 1.42$ that is 42% beyond the critical one. Time is scaled by the thermal-diffusion time and length by the layer thickness d^2/κ - and length by the layer thickness d.

With these parameters the pure fluid reveals exponentially growing convection that saturates in stationary-overturning-convection (SOC) rolls. The binary mixture shows exponentially and oscillatory growing standing waves (SWs) and ends with travelling waves (TWs). The Soret coupling is crucial for this oscillatory behaviour [1, 4].

The aim of this paper is to investigate the growth of convection, mainly the initial behaviour, if starting from the conductive state with superimposed noise. It should be noted that in our system of length $\Gamma = 2$ with lateral periodic boundary conditions only lateral wave numbers with multiples of $k = 2\pi/\Gamma$ appear. Thus defects, grain boundaries, and large scale modulations of the final pattern are suppressed.

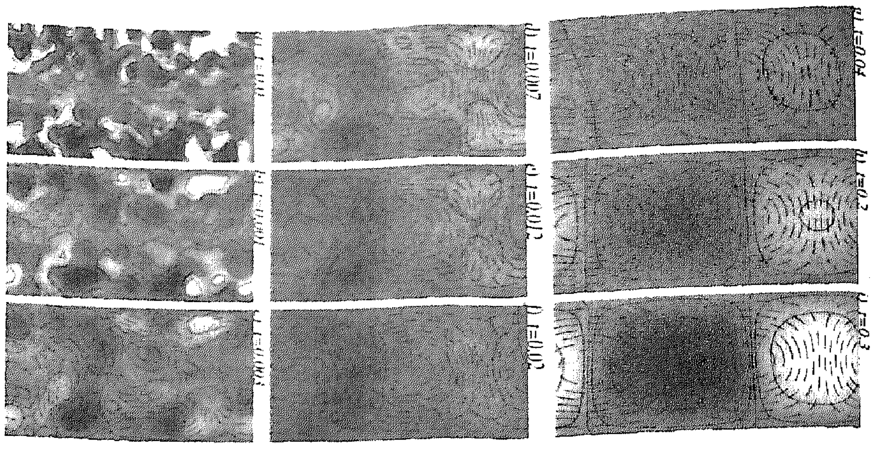

Fig. 1 Sequence of snapshot of the temperature field [closed lines and grey levels in a)-i)] and of the stream function [dashed lines in c)-i)] if starting with a noisy conductive-temperature field. The starting temperature field in a) shows a complicated strucutre reflecting the initial noise. The stream function is also omitted for the sake of clarity. The following pictures show both fields and the complex growth and decay that leads to the growing mode in i) that results finally in a SOC convection pattern at about $t \approx 1$.

2. CONVECTION GROWTH IN PURE FLUIDS

The fields in the conductive state are $T(x, z) = -z$ and $w(x, z) = 0$ and this state is the starting point for the computer experiments discussed here.

In Fig. 2 random numbers were added to the conductive temperature profile and the spatio-temporal development is shown as a sequence of pictures. The grey levels as well as the solid contour lines represent the temperature field. The dashed lines are contour lines of the stream function $\phi(x, z, t)$ with $(-\partial_z, 0, \partial_x)\phi(x,z,t) = \mathbf{u}(x,z,t)$ which visualizes the flow of the fluid. The flow always connects regions of high temperature with regions of low one and consequently the complexity of the temperature field is connected with large gradients. In a very short time a)-g) the fine structure coarse-grains dramatically and after a while the convection rolls come up. In h) and i) the typical pure-fluid SOC that is growing exponentially appears.

If starting with other randomly disturbed conduction states one gets very similar pictures except at the very first instants. Subsequently the same dynamics can always be observed : the damping of all those modes that cannot grow according to the linear analysis, the exponential growth of the critical mode, $k = \pi$, and the slightly delayed growth of nonlinear modes like, e.g., $\hat{T}_0$ and $\hat{T}_2$, that, however, are slaved by the critical mode.

To show this behaviour in a more quantitative manner the modulus of the first lateral Fourier coefficient of the temperature field taken at the midplane T_1 $(z = 0)$ is shown in Fig. 2. For three different noisy initial states in the inset the three curves are brought to complete coincidence if ignoring the beginning by shifting them in time.

Also the higher lateral Fourier modes are brought to coincidence by the same shift. However the initial relaxation interval during which the influence of the different initial conditions decays is longer.

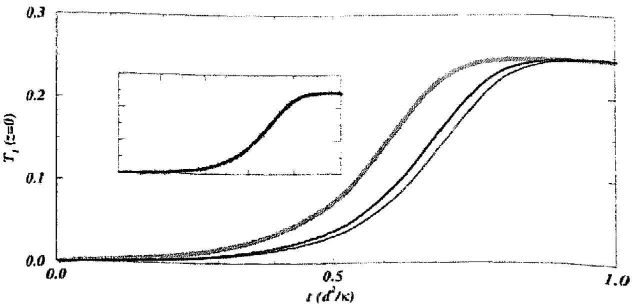

Fig. 2 Growth of pur-fluid convection with three different noisy initial states. The curves represent the modulus of the first lateral Fourier mode of the temperature field at mid height, z = 0, of the layer. They coincide after shifting them along the time axis (inset) except at the very beginning.

3. CONVECTION GROWTH IN BINARY FLUIDS

The convection growth in binary mixtures depends more sensitively on the initial state. In Fig.3 the growth out of two noisy conductive states is shown. The initial state influences the evolution much stronger than in the pure-fluid case. A simple shift does not bring the two solutions to coincidence. Only the oscillatory exponential growth ($t \approx 1$-7 for the solid curves) and the relaxation period ($t > 15$ for the solid curves) are in these cases the same. But the middle part is rather different.

Since the instability of the conductive state is of Hopf type there are two different critical modes that can grow : a left travelling wave $e^{i(kx+\omega t)}$ and a right travelling wave $e^{i(kx-\omega t)}$ with amplitudes determined by the initial state.

In a noisy state the two waves are of almost equal weight so that their superposition gives a standing wave (SW). Even if starting from other non-noisy states, e.g., if an arbitrarily oriented stripe pattern [6] or a large spot is added to the conductive temperature field, one observes similar behaviour except one hits just one of the two critical modes. So in such generic cases the initial amplitudes of the two critical modes are comparable in size. This then leads to initial SW behaviour like in Fig.3.

But it is also possible to calculate the critical modes [7] and to choose the weights of these waves in a controlled way as it was done in Fig. 3 (see also [5]). Three cases are shown there : (i) The pure TW : only a right-travelling wave perturbation was added to the conductive profile. It remains a pure TW when the nonlinear terms come into play. Thus no oscillation of the modulus appears. (ii) The generic case : starting from a noisy initial state one observes the generic modulus oscillation due to the almost equally weighted two waves. Nevertheless the very small difference in the two amplitudes is amplified when the nonlinear terms grow and the weaker wave is finally suppressed. Here also the final state is a TW. (iii) The pure SW : both weights are of exactly equal strength and remain so in the simulation since our code does not break the right-left symmetry. With mirror symmetry breaking being suppressed the system runs finally into an unstable mirror symmetric SOC fixpoint.

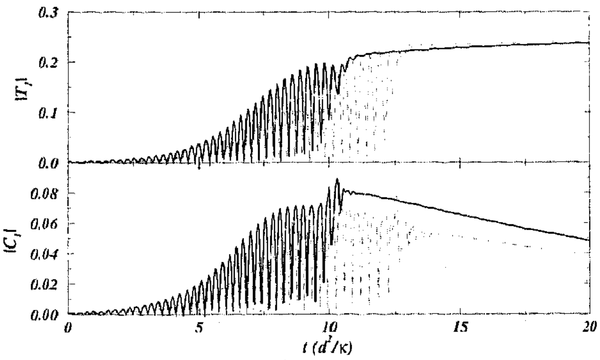

Fig.3 Modulus of the first and strongest Fourier component of the temperature field $|T_1|$ as well of the concentration field $|C_1|$ vs. time for two different realisation fo noisy conductive states. A time shift cannot get the curves to coincidence.

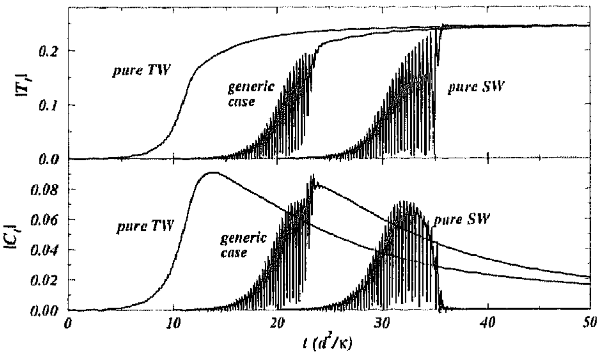

Fig. 4 Time evolution from specially selected (TW, SW) and generic initial states.

SUMMARY

In the pure-fluid convection the initial state influences only the initial transient. The further dynamics depends only on one critical mode that slaves the other modes.

In the binary-mixture case the initial state influences the whole transient dynamics : two modes are allowed to grow exponentially. Their weights depend on the initial state and consequently the whole interaction scenario in the nonlinear regime depends strongly on these weights and thus on the initial state.

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