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The dispersion of parametrically excited surface waves in viscous ferrofluids

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Abstract

Surface waves on a ferrofluid, which is exposed to a normal magnetic field, may exhibit a non-monotonous behavior. Stationary standing waves can be excited mechanically by a vertical vibration of the vessel, or magnetically by a modulation of the applied field. A linear stability analysis for the onset of these parametrically excited waves is presented. It will be shown that a careful choice of the filling depth allows for a detection of the anomalous dispersion branch. Furthermore, a theoretical confirmation is provided for the synchronous wave response, recently observed in a magnetic Faraday experiment. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Magnetic liquids; Interfacial instability; Viscous instability

1. Introduction

The surface of a ferrofluid, which is exposed to a sufficiently strong normal magnetic field, may undergo a spontaneous instability (Rosensweig instability) [1,2]. For fields slightly below the Rosensweig threshold, the dispersion of surface waves becomes non-monotonous. Recent experiments [3,4] give evidence of this feature; however, the branch of anomalous dispersion ($\partial\omega/\partial k < 0$) has not been directly measured yet. Stationary standing surface waves are conveniently produced by either vertically shaking the vessel (gravity modulation) [4] or by a modulation of the applied magnetic field [5–9]. Surface waves generated in this way, occur beyond a finite modulation amplitude, indicating the parametric drive mechanism.

For a constant susceptibility magnetic liquid ($M = \chi H$) the dispersion of *free* (= *unforced*) inviscid surface waves, $\zeta \propto e^{i(kx+\omega t)}$, is of the form [6]

$$\omega^2 = \omega_0^2(k) = \tanh(kh)\left[gk - \frac{\mu_0}{\rho}\frac{1+\chi}{2+\chi} \times \left(1 - \frac{2\chi}{(2+\chi)^2 e^{2kh} - \chi^2}\right)\chi^2 H^2 k^2 + \frac{\gamma}{\rho}k^3\right]. \tag{1.1}$$

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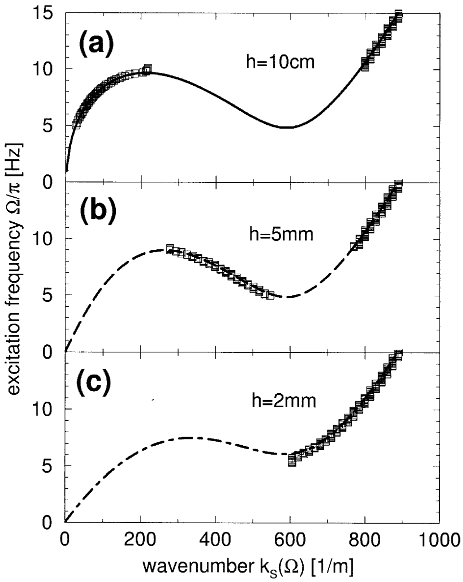

Fig. 1. Squares denote the dispersion of parametrically driven surface waves at different filling levels h as obtained from the solvability condition. Lines indicate the dispersion for free inviscid surface waves according to $\Omega = \omega_0(k)$.

Here g denotes the gravitational acceleration, μ_0 the permeability of free space, ρ the density of the fluid, and γ the surface tension. Plots of the frequency $\omega_0(k)/\pi$ for different values of the filling depth h are shown by the lines in Fig. 1. If the viscosity ν is taken into account the dispersion becomes more complicated [10,11]. However, the limit of weak viscous dissipation, $\nu k^2/\omega \ll 1$, is often applicable and yields approximately [12]

$$-\omega^2 + (-1+\mathrm{i})\frac{\sqrt{2\nu k^2}}{\sinh(2kh)}\omega^{3/2} + \mathrm{i}\omega\nu k^2[3+\coth^2(kh)] + \omega_0^2(k) \simeq 0. \tag{1.2}$$

The third term in Eq. (1.2) accounts for the energy dissipation in the bulk of the fluid, while the second one arises from the viscous boundary layer along the bottom of the vessel [13]. It contributes appreciably only if $kh \lesssim 1$. By varying the filling level h the relation between bulk and bottom damping is under external control.

A sinusoidal mechanical or magnetic drive is imposed by $g \to g[1 + a\cos(2\Omega t)]$ or $H \to H[1 + a\cos(2\Omega t)]$, respectively. As these quntities enter into ω_0^2, the drive is parametric and thus couples neighboring temporal Fourier modes separated by a frequency increment 2Ω. The surface displacement can be represented by the Floquet-ansatz $\zeta \propto \mathrm{e}^{\mathrm{i}\beta\Omega t}\sum_{n=-\infty}^{\infty}\zeta_n \mathrm{e}^{2\mathrm{i}n\Omega t}$. The case $\beta = 1 (\beta = 0)$ corresponds to the subharmonic (harmonic) surface response. The solvability condition for the ζ_n yields the neutral stability curves $a^{(\mathrm{S})}(k)$ and $a^{(\mathrm{H})}(k)$, respectively. A subsequent minimization with respect to k determines the onset amplitudes $a_c^{(\mathrm{S})}$ and $a_c^{(\mathrm{H})}$ and the critical wave numbers k_S, k_H. The latter ones, considered as a function of Ω, represent the dispersion of parametrically driven surface waves. The computations presented in the following sections are performed for the ferrofluid EMG 909 (Ferrofluidics Corporation), which has been used in recent experiments [3,4,9]. The parameters are $\nu = 6\ \mathrm{mm^2/s}$, $\rho = 1.02\ \mathrm{g/cm^3}$, $\gamma = 0.0265\ \mathrm{N/m}$, and $\chi = 0.8$.

2. Mechanical excitation

Here we discuss the subharmonic onset $a_c^{(\mathrm{S})}(\Omega)$ and the dispersion $k_\mathrm{S}(\Omega)$ for surface waves driven by a vertical vibration of the vessel. Within the considered parameter range the *harmonic* instability is suppressed, as it leads always to a higher threshold. To guarantee a considerable hysteresis in the wave dispersion, a static magnetic field H of only 1% below the Rosensweig threshold is adopted. The squares in Fig. 1 show the critical wave number $k_\mathrm{S}(\Omega)$ as obtained by minimizing the neutral curve $a^{(\mathrm{S})}(k)$.

By virtue of the boundary layer dissipation, modes with $k \lesssim 1/h$ are strongly inhibited. On reducing h, an increasing wave number range is overdamped and thus eliminated from the spectrum of available modes. Fig. 1 compares the computed dispersion $k_\mathrm{S}(\Omega)$ (squares) with Eq. (1.1) (lines). The filling level $h = 10$ cm corresponds to the infinite depth limit, where boundary layer dissipation is negligible. At $h = 5$ mm and $h = 2$ mm, wave numbers of $k \lesssim 200\ \mathrm{m}^{-1}$ and respectively $k \lesssim 500\ \mathrm{m}^{-1}$ are suppressed. Obviously, by tuning h, either of the dispersion branches can be probed. Wave number jumps occur in Fig. 1a and b at a drive frequency of $\Omega/\pi \simeq 10$ Hz. This bi-critical coexistence of two unstable wave vectors may induce the "twin peak" patterns observed recently [4] at an excitation frequency of 9.6 Hz.

3. Magnetic excitation

Since the magnetic field $H(t)$ enters quadratically into Eq. (1.1) the effective drive signal consists of two frequencies. On confining to the case $a \ll 1$ the excitation is almost monochromatic. A recent magnetic Faraday experiment [9] demonstrates a succession of alternating subharmonic and harmonic instability resonances similar to what has been predicted [14] for a thin layer ($h \ll \sqrt{\nu/\Omega}$) of a non-magnetic fluid under a mechanical vibration. However, in the latter case the necessary vibration amplitudes are so large, that the higher resonances are difficult to observe [13]. This difficulty is less crucial for the magnetic excitation: Fig. 2 compares the critical quantities for the mechanical and the magnetic drive. The wave number $k \sim 600\,\mathrm{m}^{-1}$ is associated with the stationary Rosensweig instability and acts as an attractor for k_c. The magnetic excitation favors shorter wavelengths and thus the higher resonances. This is because the effective magnetic drive amplitude is proportional to k^2, while it is only linear in k for the vibration mechanism. Taking into account that the experiment of Ref. [9] (fat symbols) has been performed in a narrow channel (while our calculation is done for a laterally infinite layer), the agreement is favorable.

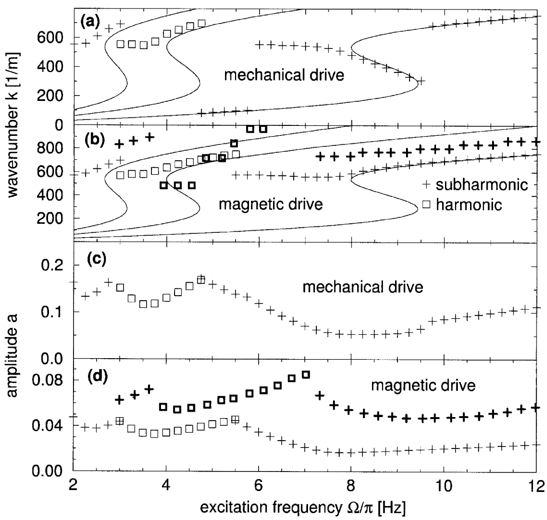

Fig. 2. Thin symbols denote the dispersion and onset amplitude for mechanically (a,c) and magnetically (b,d) excited surface waves in a ferrofluid layer of depth $h = 5$ mm. Thin lines correspond to the inviscid dispersion relation according to Eq. (1.1) with $\omega = \Omega$, 2Ω 3Ω, respectively. Fat symbols give the empiric results of Ref. [9].

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