Phase relaxation of Faraday surface waves

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Surface waves on a liquid-air interface excited by a vertical vibration of a fluid layer (Faraday waves) are employed to investigate the phase relaxation of ideally ordered patterns. By means of a combined frequency-amplitude modulation of the excitation signal a periodic expansion and dilatation of a square wave pattern is generated, the dynamics of which is well described by a Debye relaxator. By comparison with the results of a linear theory, it is shown that the measured relaxation time allows a precise evaluation of the phase diffusion constant.

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Our understanding of spatiotemporal pattern formation in nonequilibrium fluid systems has greatly benefited [1] from recent quantitative experiments in combination with the development of new theoretical concepts. One of them is the so-called amplitude equation approach [2], which is based on the linear instability of a homogeneous state and leads naturally to a classification of patterns in terms of characteristic wave numbers and frequencies. A different but equally universal description, the phase dynamics [3], applies to situations where a periodic spatial pattern experiences long-wavelength phase modulations. This approach, originally introduced in the context of thermal convection, has proven to be useful to understand the stability and the relaxation of periodic patterns, wave number selection, and defect dynamics. In many paradigmatic pattern forming systems such as thermal convection in a fluid layer heated from below (Rayleigh-Bénard convection, RBC) or the formation of azimuthal vortices in the gap between two rotating cylinders (Taylor-Couette flow, TCF) the dominating wave number is dictated by the geometry and thus inconvenient to be changed in a given experimental setup (for instance by a mechanical ramp of the layer thickness [4]).

Faraday waves are surface waves on the interface between two immiscible fluids, excited by a vertical vibration of the container. Beyond a sufficiently large excitation amplitude the plane interface undergoes an instability (Faraday instability) and standing surface waves appear, oscillating with a frequency one-half of the drive. This type of parametric wave instability is attractive as the wavelength of the pattern is dispersion rather than geometry controlled. Just by varying the drive frequency the wave number can be tuned in a wide range. In that sense the Faraday setup is well suited for the study of phase dynamics.

Nevertheless recent research activity in this field was mainly dedicated to the exploration of the processes underlying the selection of patterns with a fixed wavelength. Faraday [5] was the first to provide a quantitative study of this system, revealing that a sinusoidal vibration may induce a periodic array of squares. Later on, more complicated patterns with up to a 12-fold rotational symmetry (quasiperiodic structures) have been observed [6,7]. Here the amplitude equation technique contributed considerably to unfold the governing spatiotemporal resonance mechanisms. Applied to a set of modes $k$, with different orientations but fixed wavelength, $|k| = k$, the resulting set of Landau equations lead to a semiquantitative understanding [8] of pattern selection in this system. Motivated by these advances the idea came up to apply more complicated drive signals composed of two or more commensurable frequencies [9]. That way the simultaneous excitation of distinct wavelengths gave rise to novel surface patterns in the form of superlattices [9–11]. Only recently, the phase information carried by the participating modes was found to have a crucial influence on the visual appearance of the convection structures [12].

In comparison to other classical pattern forming systems such as RBC or TCF, the phase dynamics in the Faraday system is much less explored. In usual Faraday experiments the drive frequency (or frequency composition including relative amplitudes) is held fixed while the overall drive amplitude is ramped in order to record the bifurcation sequence of appearing structures. To our knowledge none of the previous investigations used the excitation frequency $\omega$ as the primary control parameter rather than the drive amplitude $a$. That way it is particularly simple to impose phase perturbations on ordered patterns and to study their relaxation dynamics. Moreover, doing phase dynamics on the Faraday system has the additional advantage of rather quick relaxation times, which in typical setups are one and two orders of magnitude faster than for instance in RBC.

The present paper reports a systematic investigation of phase relaxation on Faraday surface waves. Our study is focused on the relaxational dynamics of an ideal surface pattern with a square tessellation. By evaluating the relaxation time of the pattern in response to small changes of the frequency, the phase diffusion coefficient has been measured. The experimental results are found to be in good agreement with the predictions of the linear theory [3,4], which we evaluated for a system of infinite lateral extension.

The experimental setup consists of a black cylindrical container built out of anodized aluminum, and filled to a height $h$ of 4.2 mm with a silicone oil (kinematic viscosity $v = 21.4 \times 10^{-6}$ m$^2$/s, density $\rho = 949$ kg/m$^3$, surface ten-
The lateral boundaries of our container have a beachlike shape; the angle of which was adapted such as to avoid the formation of a meniscus. Phase pinning effects were thus avoided, actually we found no experimental evidences for it. In order to study finite size effects, we used three different containers, with inner diameters $L_1 = 265$ mm, $L_2 = 185$ mm, and $L_3 = 125$ mm. A glass plate covering the container was used to prevent evaporation, pollution, and temperature fluctuations of the liquid. Furthermore, to avoid uncontrolled changes of the viscosity, density, and surface tension of the liquid, all the measurements have been performed at a constant temperature of 30 ±0.1°C. The Faraday waves were excited by an electromagnetic shaker vibrating vertically with an acceleration allowing for simultaneous amplitude and frequency modulations in the form $a(t) \cos \omega t$. The corresponding input signal was produced by a wave form generator via a digital to analog converter. The instantaneous acceleration was measured by a piezoelectric sensor. In a preparatory experiment undertaken with a sinusoidal (i.e., unmodulated) drive $a \cos \omega t$, the critical acceleration amplitude $a_c(\omega)$ for the onset of the Faraday instability was determined by visual inspection of the interface while quasi-statically ramping $a$ at fixed $\omega = 2 \pi f$ (see Fig. 1). Throughout the investigated frequency interval $70 \text{ Hz} < f < 110 \text{ Hz}$ the surface patterns, which appear at a supercritical drive of less than about $1.1 \times a_c$, always consisted of an ordered square wave pattern, which—after some healing time—was free of defects [Fig. 1(b)]. In order to study the dynamics of phase-perturbed patterns we have carried out measurements of the average wave number $k(t)$ of the Faraday pattern in response to small changes of the drive frequency $\omega(t)$ around a mean value $\omega_0$. The $\omega$ modulation has been accomplished in two different ways. (i) By discontinuous jumps (back and forth) between frequencies $\omega_0 \pm \Delta \omega/2$ and $\omega_0 + \Delta \omega/2$ with a repetition period $T$ between 100 and 300 s, sufficiently large for the pattern to relax. (ii) By a sinusoidal modulation of the drive frequency according to $\omega(t) = \omega_0 + \Delta \omega \sin(\Omega t)$, with $T = 1/F = 2 \pi/\Omega$ between 2 and 1000 s (within the frequency range of our study, the response time of the shaker to small changes of the drive frequency is less than 10 ms and thus negligible). In both cases the vibration amplitude $a(t)$ was modulated in such a way that the instantaneous supercritical drive $\varepsilon = a(t)/a_c(\omega(t)) - 1$ remained constant [see Fig. 1(a)]. The bandwidth $\Delta \omega$ of the modulation needed to be confined to a few Hz in order to avoid the occurrence of dislocation-type defects. Under such conditions the square pattern remained practically ideal without perturbations [see Fig. 1(b)], just expanding and contracting (breathing) in a homogeneous manner with the modulation period $T$. To obtain the temporal wave number dependence a full frame CCD camera surrounded by a set of four incandescent lamps was mounted some distance above the container. About 100 pictures of the light reflected from the surface were taken at consecutive instances of maximum surface excursion from which the spatially averaged wave number $k(t)$ was extracted by evaluating the position of the principal peaks in a two-dimensional fast Fourier transform.

The wave number $k(t)$ followed the modulation in a relaxational manner. For the discontinuous modulation (i) this is directly apparent from Fig. 2. Here the relaxation time $\tau$ has been derived by fitting the exponential decay of the data. In the type (ii) experiment $k(t)$ oscillates around a mean value $k_0$ with an amplitude $\Delta k_m$ and a temporal phase lag $\delta$ (see Fig. 3). Introducing the complex wave number increment $\Delta k^* = \text{Re}[\Delta k^*] + i \text{Im}[\Delta k^*] = \Delta k_m e^{i\delta}$, its real and imaginary parts are plotted in Fig. 4 as a function of the modulation frequency $F$. The solid curves of this figure are fits of a linear Debye relaxator [13], where $\Delta k^* = \Delta k(\Omega = 2 \pi F)$.

![Fig. 1](image1.png)

**Fig. 1.** (a) Symbols denote the critical acceleration amplitude $a_c$ as measured at the onset of the Faraday instability at constant drive frequency $\omega = 2 \pi f$. The solid line indicates the theoretical prediction for the material parameters given. Along the dashed line the reduced drive amplitude is $\varepsilon = 9\%$. The phase relaxation experiments were performed by imposing simultaneous small changes of $f$ and $a$, as indicated for an example by the solid bar. (b) and (c) show a square pattern and the associated two-dimensional power spectrum as observed during the modulation experiments.

![Fig. 2](image2.png)

**Fig. 2.** Temporal decay of the wave number $k(t)$ in response to a steplike [type (i)] change of the drive frequency $f$. Also the drive amplitude $a$ was changed to keep the reduced amplitude $\varepsilon = a(t)/a_c(\omega(t)) - 1$ constant. The decay of the wave number $k(t)$ could be fitted by an exponential. (Parameters: $L = 0.265$ m, $\varepsilon = 9\%$, $f = 90$ Hz.)
The theoretical prediction Eq. (2) also implies a slight depen-
dence of $\tau$ on the drive amplitude $\varepsilon$. However, checking for this feature requires to account for the fact that the mean wave number $k_0$ is also affected by the drive strength. We deduced the empiric dependence $k_0 / k_c = 1 - \beta \varepsilon$ with $\beta = 0.264$ from a control run where $\varepsilon$ was slowly ramped at fixed $f$. Inserting this result into the last term on the right-hand side of Eq. (2) leads to the following expression:

$$\frac{\tau(\varepsilon)}{\tau(\varepsilon = 0)} = 1 - \frac{\varepsilon^2 k_c^2 \beta^2}{1 - \frac{3}{2} \varepsilon^2 k_c^2 \beta^2}.$$  

(3)

Although this relation [see solid line in Fig. 5(c)] gives a reasonable estimate for the $\varepsilon$-dependence of $\tau$ it does not correctly reflect the empiric dependence. Apparently this is a finite size effect, which is expected to become significant at small values of $\varepsilon$. Roth et al. [14] recently demonstrated that the effective phase diffusion constant, measurable in RBC and TCF relaxation experiments, depends sensitively on the aspect ratio $\alpha = \sqrt{6} L / \xi_0$, defined by the quotient between the container dimension and the linear correlation length. Taking TCF as an example, a decrease of $\alpha$ from 50 down to 15 (which in our experiment corresponds to a reduction of $\varepsilon$ form 9% to 2%) implies a decay of the relaxation time by 20–30%. This is of the same order of magnitude as the value observed in our measurements [see Fig. 5(c)].

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