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Magnetization of Rotating Ferrofluids: Predictions of Different Theoretical Models

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We consider a ferrofluid cylinder, that is rotating with constant rotation frequency $\mathbf{\Omega} = \Omega \mathbf{e}_z$ as a rigid body. A homogeneous magnetic field $\mathbf{H}_0 = H_0 \mathbf{e}_x$ is applied perpendicular to the cylinder axis $\mathbf{e}_z$. This causes a nonequilibrium situation. Therein the magnetization **M** and the internal magnetic field **H** are constant in time and homogeneous within the ferrofluid. According to the Maxwell equations they are related to each other via $\mathbf{H} = \mathbf{H}_0 - \mathbf{M}/2$. However, **H** and **M** are not parallel to each other and their directions differ from that of the applied field $\mathbf{H}_0$. We have analyzed several different theoretical models that provide equations for the magnetization in such a situation. The magnetization **M** is determined for each model as a function of Ω and H_0 in a wide range of frequencies and fields. Comparisons are made of the different model results and the differences in particular of the predictions for the perpendicular components $H_y = -M_y/2$ of the fields are analyzed.

1. Introduction

There are several theoretical equations for the dynamics of the magnetization $\mathbf{M}(\mathbf{r}, t)$ of a ferrofluid that is flowing with velocity $\mathbf{u}(\mathbf{r}, t)$ in an externally applied magnetic field $\mathbf{H}_0$ [1–5]. Here we compare their predictions for a simple special case that is experimentally accessible. We consider a ferrofluid cylinder of radius R of sufficiently large length to be approximated as infinite in a homogeneous applied field $\mathbf{H}_0 = H_0 \mathbf{e}_x$ in x-direction. The ferrofluid cylinder is enforced via its walls to rotate as a rigid-body around its long axis with constant rotation frequency $\mathbf{\Omega} = \Omega \mathbf{e}_z$ being oriented perpendicular to $\mathbf{H}_0$. The flowfield is thus $\mathbf{u}(\mathbf{r}) = \mathbf{\Omega} \times \mathbf{r} = \Omega r \mathbf{e}_\varphi$ where $\mathbf{e}_\varphi$ is the unit vector in azimuthal direction. In such a situation all aforementioned models allow for a spatially and temporally constant nonequilibrium magnetization **M** that

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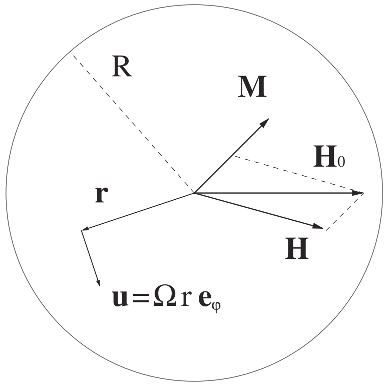

Fig. 1. Schematic plot of relevant vectors.

is rotated out of the directions of $\mathbf{H}_0$ and $\mathbf{H}$ by the flow. The Maxwell equations demand that the fields $\mathbf{H}$ and $\mathbf{M}$ within the ferrofluid are related to each other via

$$\mathbf{H} = \mathbf{H}_0 - \frac{1}{2}\mathbf{M} \tag{1.1}$$

as indicated schematically in Fig. 1 and that the magnetic field outside the ferrofluid cylinder

$$\mathbf{H}^{\text{out}} = \mathbf{H}_0 + \frac{1}{2}\frac{R^2}{r^2}\left(2\frac{\mathbf{r}}{r}\frac{\mathbf{M}\cdot\mathbf{r}}{r} - \mathbf{M}\right) \tag{1.2}$$

is a superposition of the applied field $\mathbf{H}_0$ and the dipolar contribution from $\mathbf{M}$. This result can be used for comparisons with experiments which measure the outside field.

2. Magnetization equations

The model equations that we compare here imply a relaxational dynamics either of $\mathbf{M}$ towards the equilibrium magnetization

$$\mathbf{M}_{\text{eq}}(\mathbf{H}) = \frac{M_{\text{eq}}(H)}{H}\mathbf{H} = \chi(H)\mathbf{H} \tag{2.1}$$

or of the "local equilibrium" or "effective" field

$$\mathbf{H}_{\text{eff}}(\mathbf{M}) = \frac{M_{\text{eq}}^{-1}(M)}{M}\mathbf{M} = F(M)\mathbf{M} \tag{2.2}$$

towards the internal field **H**. The equilibrium magnetization $M_{\text{eq}}(H)$ referring to the functional relation between internal field H and magnetization in the case of $\Omega = 0$ is a thermodynamic material property of the ferrofluid. The effective field $\mathbf{H}_{\text{eff}}$ lies parallel to **M** and can be seen as the inverse of the defining requirement

$$\mathbf{M} = \mathbf{M}_{\text{eq}}(\mathbf{H}_{\text{eff}})\,. \tag{2.3}$$

In equilibrium, $\Omega = 0$, one has $\mathbf{H}_{\text{eff}} = \mathbf{H}$ and $\mathbf{M} = \mathbf{M}_{\text{eq}}$.

We consider here the relations

$$\text{Debye:} \qquad \boldsymbol{\Omega} \times \mathbf{M} = \frac{1}{\tau}(\mathbf{M} - \mathbf{M}_{\text{eq}}) \tag{2.4}$$

$$\text{S'72 [2]:} \qquad \boldsymbol{\Omega} \times \mathbf{M} = \frac{1}{\tau}(\mathbf{M} - \mathbf{M}_{\text{eq}}) + \frac{\mu_0}{4\zeta}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}) \tag{2.5}$$

$$\text{FK [3]:} \qquad \boldsymbol{\Omega} \times \mathbf{M} = \gamma_H(\mathbf{H}_{\text{eff}} - \mathbf{H}) + \frac{\mu_0}{4\zeta}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}) \tag{2.6}$$

$$\text{S'01 [4]:} \qquad \boldsymbol{\Omega} \times \mathbf{H}_{\text{eff}} = \frac{1}{\tau}(\mathbf{H}_{\text{eff}} - \mathbf{H}) + \frac{\mu_0}{4\zeta}\mathbf{H}_{\text{eff}} \times (\mathbf{M} \times \mathbf{H}) \tag{2.7}$$

$$\text{ML [5]:} \qquad \boldsymbol{\Omega} \times \mathbf{M} = \xi(\mathbf{H}_{\text{eff}} - \mathbf{H}) \tag{2.8}$$

resulting for the rotating cylinder from the above 5 models. Here τ denotes a magnetic relaxation time, γ_H and ξ relaxation rates, ζ the vortex viscosity and μ_0 the vacuum permeability. In ML we use the weak field variant of Ref. [5]. These equations have to be solved numerically in combination with the Maxwell equation (1.1).

As an aside we mention that the above equations can be written in the common form

$$\mathbf{M} \times (\boldsymbol{\Omega} + \alpha_3 \mathbf{M} \times \mathbf{H}_0) = \alpha_1(\mathbf{H}_0 - \alpha_2 \mathbf{M}) \tag{2.9}$$

with coefficients:

$$\text{Debye:} \qquad \alpha_1 = \frac{\chi}{\tau}, \quad \alpha_2 = \frac{1}{\chi} + \frac{1}{2}, \quad \alpha_3 = 0$$

$$\text{S'72:} \qquad \alpha_1 = \frac{\chi}{\tau}, \quad \alpha_2 = \frac{1}{\chi} + \frac{1}{2}, \quad \alpha_3 = \frac{\mu_0}{4\zeta}$$

$$\text{S'01:} \qquad \alpha_1 = \frac{1}{F\tau}, \quad \alpha_2 = F + \frac{1}{2}, \quad \alpha_3 = \frac{\mu_0}{4\zeta}$$

$$\text{FK:} \qquad \alpha_1 = \gamma_H\,, \quad \alpha_2 = F + \frac{1}{2}, \quad \alpha_3 = \frac{\mu_0}{4\zeta}$$

$$\text{ML:} \qquad \alpha_1 = \xi\,, \quad \alpha_2 = F + \frac{1}{2}, \quad \alpha_3 = 0\,.$$

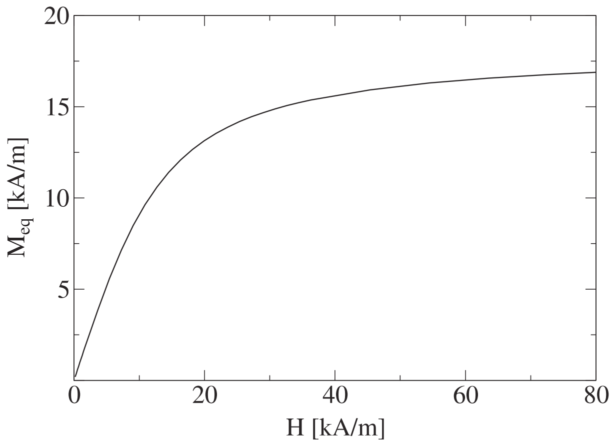

Fig. 2. Equilibrium magnetization $M_{\text{eq}}(H)$ used as input into the models compared here.

3. Results

In order to make the comparison of the theoretical results easier we replace the equilibrium magnetization $M_{\text{eq}}(H)$ by the Langevin expression $M_{\text{eq}}(H) = M_{\text{sat}}\mathcal{L}\,(3\chi_0 H/M_{\text{sat}})$ with the initial susceptibility $\chi_0 = \chi(H = 0)$. We use $\chi_0 = 1.09$ and $M_{\text{sat}} = 18\,149$ A/m for the saturation magnetization which is appropriate for the ferrofluid APG 933 of FERROTEC. The resulting curve is shown in Fig. 2. Furthermore, we replace the relaxation time $\tau(H)$ by $\tau_{\text{B}} = 6 \times 10^{-4}$ s. For $\zeta \simeq \frac{3}{2}\Phi\eta$ we use the values $\eta = 0.5$ Pa s and $\Phi = 0.041$ and for γ_H we use $\gamma_H = \chi_0/\tau_{\text{B}}$ [6]. For the parameter ξ of ML [5] we investigate two different choices: Either the low-field variant, $\xi = \chi_0/\tau_{\text{B}}$, as in FK that is denoted here by ML(F). Or the variant $\xi = 1/[F(M)\tau_{\text{B}}]$ as in S'01 that is denoted here by ML(S).

Especially the perpendicular component $H_y = -\frac{1}{2}M_y$ of the magnetic field is suited for a comparison of the different models with each other and with experiments. Before doing the former we should like to draw the attention to the frequency behavior of $M_y(H_0, \Omega)$. We mentioned already that M_y vanishes for zero vorticity, $\Omega = 0$. Furthermore, one finds that M_y as well as M_x vanishes also in the limit $\Omega \to \infty$. And since one can rewrite the solution of Eq. (2.9) in the form $M_y = \frac{\Omega\tau}{\alpha_1+\alpha_3 M^2}\frac{M^2}{H_0}$ one sees that $M_y(\Omega)$ has a maximum as a function of Ω as in Fig. 3. There we show H_y versus Ω.

The differences in the results for the different models are easily captured by comparing their predictions for the maximum values of $|H_y|$, the locations of these maxima at $\Omega^{\max}$, and the initial slopes $\frac{\mathrm{d}|H_y|}{\mathrm{d}\Omega}$ at $\Omega \to 0$, each as a function of applied field H_0. This is done in Fig. 4.

The maximal values of $|H_y|$ of Debye and S'72 are the same while their locations, $\Omega^{\max}$, differ. The models S'01, FK, and ML formulated in terms of

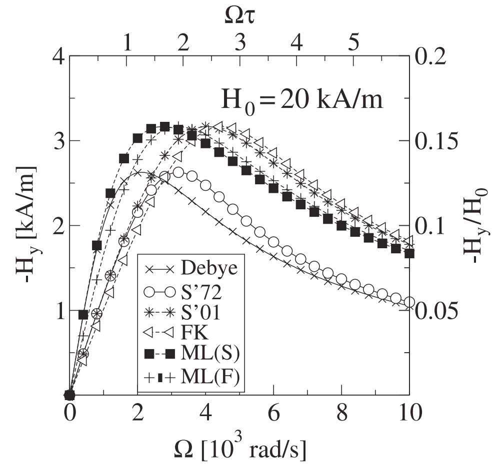

Fig. 3. Comparison of the predictions of the different theoretical models for the transverse internal field H_y versus rotation frequency Ω.

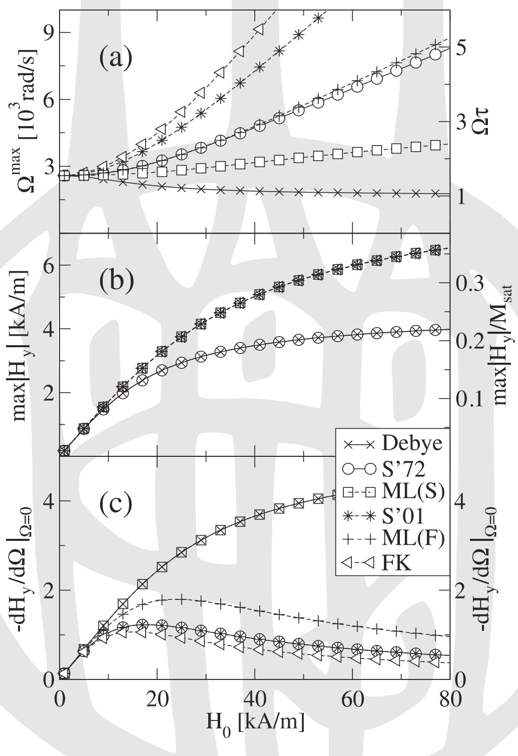

Fig. 4. (a) Frequency $\Omega^{\max}$ leading to maximal transverse field, (b) largest transverse field, and (c) initial slope $\frac{-\mathrm{d}H_y}{\mathrm{d}\Omega}$ at $\Omega \to 0$.

the effective field also share a common maximal value of $|H_y|$ being larger than that of Debye and S'72 while the location, $\Omega^{\max}$, differ partly substantially. Hence the magnetic torque, $\mathbf{M} \times \mathbf{H}$, entering into S'72, FK, and S'01 only shifts the frequency $\Omega^{\max}$. It remains to be seen whether experiments can be performed with sufficient accuracy to discriminate between the different theoretical predictions.

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