

## Flow control of magnetic fluids exposed to magnetic fields

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**Abstract.** The description of flow in ferrohydrodynamics (Rosensweig, 1985) is based on a combination of equations, namely the continuity equation, the Navier-Stokes equation, the Maxwell equations and particular equations for the magnetization. Since the different models to describe the relaxation of magnetization differ, the adequate one has yet to be identified. By comparing experimental and simulation data of a model system, this goal may get achieved. As a model system, a Taylor-Couette apparatus was chosen. In this paper, experimental results concerning the transition from circular Couette flow to Taylor vortex flow at different field strengths of an axial magnetic field are compared to a linear stability analysis. The relaxation equation established by Shliomis (Shliomis, 1972) and the Debye-Model with a field dependent relaxation time showed to give qualitative accordance with the experimental data.

### 1. Motivation

The magneto-rotational effect as a cause for an increase of viscosity of ferrofluids exposed to magnetic fields has been the subject of many investigations [1-6]. In a ferrofluid being subject to shear flow, the particles are forced to rotate with their axis of rotation parallel to the orientation of vorticity. When a magnetic field is applied, the suspended particles try to align themselves with their magnetic moment along the orientation of the magnetic field. Assuming magnetically hard particles (i. e. the Brownian relaxation time is shorter than the Néel relaxation time), a (magnetic) torque is exerted on them in case the vorticity of the shear flow and the magnetic field are not collinear. This magnetic torque counteracts the viscous torque exerted by the carrier liquid, causing an increase of the fluid's viscosity. If vorticity and magnetic field are collinear, no increase of the fluid's viscosity will be observable. A flow field, with a not vanishing component of vorticity perpendicular to the magnetic field, turns the magnetic dipoles of magnetically hard particles out of the magnetic field direction. The magnetization of the fluid is now not only dependent on the magnetic field, but also on the flow field. It is thus not describable by the equations for equilibrium magnetization anymore. Models to describe the magnetization outside equilibrium are necessary.

Shliomis [2] was the first to describe the magnetization of a ferrofluid considering a not vanishing component of vorticity. Since then, other equations for the magnetization were developed, all of which are mainly based on the relaxation of magnetization against equilibrium, which is determined by a relaxation time. The existing equations to describe the relaxation of magnetization are different in their physical concepts and the way they predict experimental results. In order to determine the right

one, a comparison of analytical and experimental data of a model system is needed. In Section 2 some equations for the relaxation of magnetization are introduced, in section 3 the experimental apparatus is outlined shortly and in section 4 some results are presented.

## 2. Theoretical

The simplest model to calculate the magnetization outside the equilibrium is the Debye model, denoted here as *Debye*:

$$d_t \mathbf{M} = -\frac{1}{\tau} (\mathbf{M} - \mathbf{M}^{eq}) + \boldsymbol{\Omega} \times \mathbf{M} \quad (1)$$

This model implies a relaxation of the magnetization  $\mathbf{M}$  towards the equilibrium magnetization  $\mathbf{M}^{eq}$  at a given external magnetic field  $\mathbf{H}$  with a relaxation time  $\tau$  in a frame rotating with the vorticity  $\boldsymbol{\Omega} = \frac{1}{2} \nabla \times \mathbf{u}$  of the flow  $\mathbf{u}$ . Shliomis [2] developed the relaxation equation (denoted as *S'72*)

$$d_t \mathbf{M} = -\frac{1}{\tau} (\mathbf{M} - \mathbf{M}^{eq}) + \boldsymbol{\Omega} \times \mathbf{M} + \kappa (\mathbf{M} \times \mathbf{H}) \times \mathbf{M} \quad (2)$$

With an additional constant  $\kappa = (6\Phi \tilde{\eta})^{-1}$ , where  $\Phi$  is the volume fraction of the magnetic material and  $\tilde{\eta}$  the dynamic viscosity of the carrier liquid. Niklas et al. [4] considered stationary magnetizations ( $d_t \mathbf{M} = 0$ ) near the equilibrium ( $\mathbf{M} - \mathbf{M}^{eq} \ll 1$ ), which appear at not too high rotation rates ( $|\boldsymbol{\Omega}| \tau \ll 1$ ). In this case the magnetization equations presented above can be simplified with (denoted as *Niklas approach*)

$$\mathbf{M} - \mathbf{M}^{eq} = c_N \boldsymbol{\Omega} \times \mathbf{H} \quad (3)$$

$$c_N = \frac{\chi(H)\tau}{(1 + \kappa\tau\chi(H)H^2)} \quad (4)$$

$\chi$  is the susceptibility of the ferrofluid. The relaxation time  $\tau$  may either be assumed to be constant or dependent on the external magnetic field

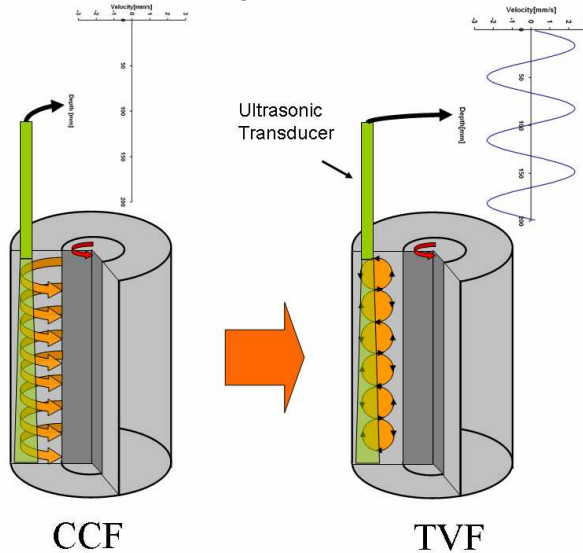
$$\tau(H_{ext}) = 2\tau_0 \frac{\mathbf{L}(\alpha)}{\alpha - \mathbf{L}(\alpha)} \quad (5)$$

with  $\alpha = \alpha_\tau \cdot H_{ext}$  and  $\mathbf{L}(x) = \coth(x) - \frac{1}{x}$ .

## 3. Experimental setup

A Taylor-Couette-Setup, consisting of rotary inner and outer cylinders, has been built with a gap height of 20 cm, a radius of the inner cylinder of 1 cm and a gap width of 1 cm, resulting in an aspect ratio of  $\Gamma=20$  and a radius ratio of  $\eta=0.5$ . Both cylinders may rotate in either direction, whereas they get driven by synchron-servo motors. The magnetic field is generated by four Helmholtz-coils cooled from three sides in Fanselau arrangement. The coils have 1500 windings of copper wire of 1.5 mm in diameter. Powered at around 2350W, a field of about 43kA/m in the homogeneous area (20x20 cm<sup>2</sup>) is achieved. The orientation of the magnetic field might get arranged axial or transversal relative to the fluid cell axis. Due to the fact that ferrofluids are opaque, the different flow states and their transitions are measured by ultrasound doppler velocimetry. As a velocimeter, the DOP2125 manufactured by Signal Processing (Lausanne, Switzerland) is used. According to the Doppler-Effect, ultrasound echo reflected from particles immersed in the fluid carries a higher (lower) frequency in case the particles move towards (away from) the transducer. By convention, the velocity of the particle is displayed as positive when the target is moving away from the transducer. Due to geometrical limitations, the ultrasound transducer was placed at the outer cylinder wall. Thereby appearing sinusoidal velocity

profiles (**Figure 1**) reveal phase transitions and may be analyzed by spatial and temporal FFT. A 4 MHz ultrasound transducer gave the best results under experimental conditions. Due to the fact that ferrofluid particles are too small to give a usable ultrasound echo, 0.05 vol% tracer particles with 10  $\mu\text{m}$  in diameter consisting of melamin resin had to be added.



**Figure 1.** The transition from the ground state (Circular Couette flow, CCF) to Taylor vortex flow (TVF). The ultrasound transducer on top of the fluid cell may evaluate only axial velocity components. Hence, in CCF the velocity signal is zero. In TVF, the velocity signal is sinusoidal, following the Taylor vortices streaming upwards and downwards at the outer cylinder wall.

#### 4. Results and discussion

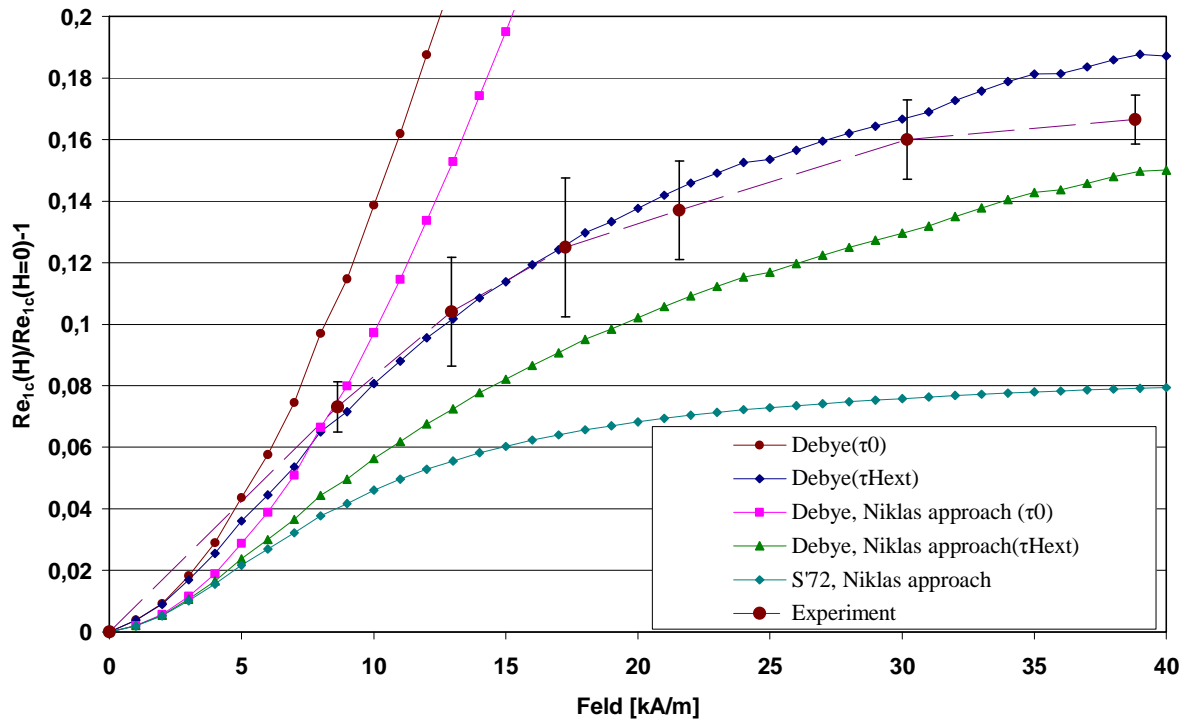
First measurements were carried out with a kerosene based ferrofluid (Table 1) provided by L. Vékás, Timisoara, and concerned the transition from CCF to TVF at different axial magnetic field strengths.

**Table 1:** Properties of the ferrofluid provided by L. Vékás, Timisoara.

Property	Value
Carrier Liquid	Kerosene
Coating	Oleic acid without excess surfactant
Material	Magnetite
$\Phi$	0.07
$M_s$	34 kA/m
$\chi_0$	0.93
Viscosity of carrier liquid	0.002 Pas (Lit. datum)
Viscosity	0.0068 Pas

In Figure 2, the experimental data is compared to a linear stability analysis conducted using different equations for the relaxation of magnetization as introduced in section 2. For the given fluid parameters,  $\tau_0$  might be calculated to approx. 0.01 ms, assuming a typical mean particle diameter for ferrofluids of 12 nm [1]. However, in order to also consider such crucial parameters like polydispersity and shape anisotropy of the particles [8],  $\tau_0$  was chosen to be 0.03 ms. For the analysis, the relaxation time  $\tau_0$  was either assumed to be constant or dependent on the external magnetic field  $H_{\text{ext}}$  which  $\alpha_\tau = 0.3 \text{ m/kA}$ .  $\chi(H)$  was determined by magnetization measurements.

The experiments show that  $Re_{lc}$  increases strongly until approx. 25 kA/m, when saturation behavior sets in. Theories describing the magnetization outside equilibrium should also reflect this behavior, which is not accomplished by Debye, calculated exactly, or in a Niklas approach with  $\tau_0 = \text{const}$ . Debye, calculated with a relaxation time depending on the external magnetic field  $\tau(H_{\text{ext}})$ , and S'72 get closer to the experimental observations. However, the significant deviation of the models is a demand for further investigations, including measurements of the relaxation rate and the influence of polydispersity and shape anisotropy effects.



**Figure 2.** Comparison of experimental data at  $20 \pm 0.25^\circ\text{C}$  and data obtained by a linear stability analysis using the equations for the relaxation of magnetization as introduced in section 2. The relaxation time is assumed to be  $\tau_0 = 0.03$  ms and the transition is calculated for an axial wave number of  $k = 2.8274$  (18 vortices).

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