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Collisions of localized convection structures in binary fluid mixtures

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Abstract. Collisions of a localized traveling wave structure with a localized stationary structure are investigated. In ethanol–water mixtures with appropriately chosen negative separation ratios, both exist bistably in the unstable quiescent surrounding for a range of supercritical heating rates. Depending on the Rayleigh number, we observe different evolution scenarios of the convection structures that appear as a result of the collision. The incident localized traveling wave can be absorbed by the stationary structure and then the latter expands: either both of its fronts get unpinned and propagate into the quiescent fluid or only the one that is hit propagates while the opposing one remains pinned. For smaller Rayleigh numbers, the stationary structure is destroyed while the incident localized traveling wave survives and a second one is created that moves ahead of the incoming one, both being coupled together. The mechanisms involved in these scenarios are analyzed and elucidated with the help of finite difference numerical simulations that are carried out subject to realistic boundary conditions.

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1. Introduction

Nonlinear systems that are driven away from equilibrium often show the phenomenon of self-organized structure formation. For example, a homogeneous state undergoes a symmetry breaking instability and a spatially extended structure appears [1]. Some systems also support spatially confined structures that are embedded in and connected to the surrounding homogeneous state via fronts. Such so-called localized structured states have been seen in a wide variety of systems: in fluids, optical media, granular material, colloidal suspensions, chemical reactions, e.g., on surfaces, and in various model equations.

In this paper, we deal with convection in a horizontal layer of a liquid binary mixture that is heated from below. This system can bistably support two different kinds of localized structures: stationary ones and slowly moving packets consisting of traveling waves (TW). Thus, one can easily investigate the collision of such a localized traveling wave (LTW) with the localized stationary structure. An experimental setup might consist of a narrow annular or straight channel as in earlier experiments [2–9].

Convection in binary mixtures such as, e.g., ethanol–water or $^3$He–$^4$He shows already close to onset a much richer spatiotemporal complexity than in the case of a pure, one-component fluid. Furthermore, the spectrum of possible convective states and their bifurcation properties are much richer. The reasons for this are the characteristic and largely dominating effects of concentration fluctuations. Being generated by temperature gradients via the Soret effect, they mostly experience an advection-dominated dynamics since their diffusive equilibration is typically very small. Most importantly, these strongly nonlinear fluctuations influence the local buoyancy forces which drive the convective flow in the first place. So there is a feedback between the Soret generated concentration variations, the resulting changes in the buoyancy and the mixing by the flow.
This coupling between the degrees of freedom of the temperature, velocity and concentration fields can cause a first-order, subcritical onset of convection. Then, developed convection amplitudes appear in the vicinity of the critical heating rate that marks the threshold for such a subcritical bifurcation. Furthermore, convection structures in binary mixtures with developed flow amplitudes tend to be strongly nonlinear because of the complex spatiotemporal behavior of the concentration field: Péclet numbers measuring the strength of its nonlinear advective transport relative to its diffusive one tend to be large so that large spatial variations and boundary layers appear. On the other hand, the balance equations for momentum and heat remain weakly nonlinear close to onset as in one-component fluids. Thus, the velocity and temperature fields remain smooth with variations that are basically harmonic in space as the linear critical modes.

Here we investigate convection in binary fluid mixtures with negative Soret coupling between temperature and concentration gradients, i.e. with a negative separation ratio, $\psi < 0$ [10]. Then the lighter component of the mixture migrates to the colder regions. This then stabilizes the density stratification in the quiescent, laterally homogeneous, conductive state against growth of convective perturbations. Consequently, the critical Rayleigh number for onset of convection is increased relative to that in a one-component fluid.

Furthermore, the above-described feedback interaction between the fields generates oscillations in them. In fact, the buoyancy difference between regions with different concentrations has already been identified in [11] as the cause of the traveling convection waves. However, the oscillatory behavior [3, 6, 8, 9, 12–22] appears not only in the form of spatially extended, fully relaxed, nonlinear TW convection, but also in TW fronts moving into the quiescent fluid. Also, a mostly unstable standing wave solution branches out of the conductive state at the common Hopf bifurcation threshold. Furthermore, traveling and standing oscillations can be easily observed in the transient growth of convection at supercritical heating. Last but not least, there are the above-mentioned, robust LTW states that have already been observed in early experiments [2, 3].

Not only oscillations can be found in binary mixtures with negative separation ratios, but also the so-called stationary overturning convection (SOC). This state consists of straight rolls such as the ones that appear in one-component fluids in a forward bifurcation out of the conductive state. Here, however, the Rayleigh number of the SOC bifurcation threshold quickly moves to infinity when $\psi$ becomes negative. Then the backwards bending, low-amplitude SOC solution branch becomes detached from the conductive state. Thus, it requires finite initial perturbations to reach the large-amplitude SOC state. Therein the fluid is well mixed and consequently the convection properties are largely similar to those in one-component fluids.

Spatially localized stationary overturning convection (LSOC) states have been found in numerical simulations of $^3$He–$^4$He mixtures [23] and of ethanol–water mixtures [24–26] and seem to have been seen in earlier experiments with ethanol–water mixtures [30, 31]. The pinning mechanism of the stationary LSOC fronts and their bistability with moving ones were elucidated in [25].

In this work, we consider collisions of a drifting LTW with an LSOC that both consist of straight parallel convection rolls. Such structures are easily generated experimentally in narrow

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4 For 5 wt% of ethanol mixed into water at $T = 20 ^\circ C$, the separation ratio measuring the Soret coupling strength [11] is $\psi \simeq -0.3$ [10].
5 The LSOC states are called convectons in [23, 26–29].
6 Collisions of two drifting pulse solutions of extended complex Ginzburg–Landau equations were investigated in [32].
rectangular and annular channels: the channel walls orient the roll axes perpendicular to the walls, say, in the y-axis direction. Thus, the convection structures have relevant phase variations only in the x-direction. One can well describe them by ignoring variations in the y-axis direction and by considering only the two-dimensional vertical x–z cross section in the middle of the channel perpendicular to the roll axes [33].

Depending on control parameters, we found three different collision scenarios that are most easily classified by their outcomes: in scenarios I and II, the incident LTW is absorbed and destroyed by the LSOC and then the LSOC expands: in scenario I, both of its fronts propagate in the post-collision phase with constant, equal speed opposite to each other into the quiescent fluid. In scenario II, only the LSOC front that is hit by the incident LTW gets unpinned and moves into the conductive state, while the opposite LSOC front remains pinned. In scenario III the incident LTW survives; the LSOC is destroyed and a second LTW is created that moves ahead of the incoming one, both being coupled together.

The paper is organized as follows. In section 2, we describe the system, the governing equations and our diagnostic tools. In section 3, we give a brief review of the LTW and LSOC solutions being surrounded by the unstable quiescent fluid. Section 4 contains our results concerning the collisions. The last section is devoted to concluding remarks.

2. The system

We consider a horizontal plane layer of a binary fluid mixture that is placed in the Earth’s gravitational field with the acceleration \( g \). The layer is bounded by rigid, perfectly heat conducting and impervious parallel planes located at \( z = 0 \) and \( h \). Thus, \( h \) is the thickness of the layer. The fluid might be a mixture of water and the lighter component ethanol at a mean mass concentration \( \overline{C} \). The temperatures \( T_u \) and \( T_l \) of the upper and lower boundaries, respectively, are fixed. The temperature difference is \( \Delta T = T_l - T_u \). We use \( T_u \) as the reference temperature \( \overline{T} \) and consider the (small) variations of the fluid density \( \rho \) due to temperature and concentration variations to be governed by the linear thermal and solutal expansion coefficients \( \alpha = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} \) and \( \beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial C} \), respectively. Both are positive for ethanol–water.

2.1. Equations

To describe convection in this binary fluid mixture, we use the balance equations for mass, momentum, heat and concentration in the Oberbeck–Boussinesq approximation, which read in non-dimensional form [20, 34, 35]

\[
\nabla \cdot \mathbf{v} = 0, \tag{1a}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \sigma \nabla^2 \mathbf{v} + \sigma R (T + C) \mathbf{e}_z, \tag{1b}
\]

\[
\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T = \nabla^2 T, \tag{1c}
\]

\[
\frac{\partial C}{\partial t} + (\mathbf{v} \cdot \nabla) C = L \nabla^2 (C - \psi T). \tag{1d}
\]

Here, \( \mathbf{v} \) is the velocity field, \( p \) is the pressure and \( \mathbf{e}_z \) is the unit vector directed upward. \( T \) and \( C \) are scaled (see below) deviations of temperature and concentration from \( \overline{T} \) and \( \overline{C} \), respectively.
The Dufour effect [36] that provides a coupling of concentration gradients into the heat current is ignored in equation (1c) since it is observed only in a few binary gas mixtures [37].

The field equations (1) contain the Rayleigh number $R$ characterizing the thermal driving of the fluid and three other parameters describing the physical properties of a binary fluid mixture: the Prandtl number $\sigma$, the Lewis number $L$ and the separation ratio $\psi$

$$R = \frac{g \alpha \Delta T h^3}{\nu \kappa}, \quad \sigma = \frac{\nu}{\kappa}, \quad L = \frac{D}{\kappa}, \quad \psi = -\frac{\beta k_T}{\alpha T}.$$

Here $\nu$ is the kinematic viscosity, $\kappa$ is the thermal diffusivity and $D$ is the concentration diffusion constant of the mixture, while $k_T = T \tilde{C}(1 - \tilde{C})S_T$ is the thermodiffusion coefficient [34] and $S_T$ is the Soret coefficient. Furthermore, the following scales have been used in equations (1): length, $h$; time, $h^2/\kappa$; velocity, $\kappa/h$; temperature, $\Delta T$; concentration, $\alpha \Delta T/\beta$; pressure, $\rho \kappa^2/h^2$.

We solved the field equations (1) for two-dimensional roll convection with axes oriented in the $y$-direction. To this end, we introduced the stream function $\Psi$ and the vorticity $\varphi$, which are connected with the velocity field in the following way:

$$\mathbf{v} = \left( \frac{\partial \Psi}{\partial z}, 0, -\frac{\partial \Psi}{\partial x} \right), \quad \varphi = (\nabla \times \mathbf{v})_y.$$

Then the partial differential equations (1) are transformed into

$$\varphi = \nabla^2 \Psi,$$

$$\frac{\partial \varphi}{\partial t} + \frac{\partial \Psi}{\partial z} \frac{\partial \varphi}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \varphi}{\partial z} = \sigma \nabla^2 \varphi - \sigma R \frac{\partial (T + C)}{\partial x},$$

$$\frac{\partial T}{\partial t} + \frac{\partial \Psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial T}{\partial z} = \nabla^2 T,$$

$$\frac{\partial C}{\partial t} + \frac{\partial \Psi}{\partial z} \frac{\partial C}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial C}{\partial z} = L \nabla^2 (C - \psi T).$$

2.1.1. Boundary conditions and parameters. The horizontal boundaries at $z = 0, 1$ are taken to be no-slip ($\mathbf{v} = 0$) and impervious so that there

$$\Psi = 0, \quad \frac{\partial \Psi}{\partial z} = 0, \quad \frac{\partial C}{\partial z} - \psi \frac{\partial T}{\partial z} = 0.$$

The temperatures at the boundaries are

$$T(z = 0) = 1, \quad T(z = 1) = 0.$$

Laterally, we impose periodic boundary conditions,

$$f(x, z, t) = f(x + \Gamma, z, t),$$

on all fields $f = \Psi, \varphi, T, C$ with $\Gamma = 80$ throughout the paper.
In this paper, we consider mixtures with $L = 0.01$, $\sigma = 10$ and $\psi = -0.08$. This set of parameters is characteristic of and easily experimentally realizable with ethanol–water mixtures.

2.1.2. The solution method. For solving the system of equations (4) in the general case ($\Psi \neq 0$), an alternating-direction implicit scheme is used with central differences for the spatial derivatives and one-sided right differences for the time derivatives. This is a finite difference method of second order. The Poisson equation for the stream function was solved by means of an iterative method of successive over relaxation at each time step. All calculations were performed on a grid of $1600 \times 21$ nodes, which corresponds to a homogeneous grid of mesh size $\Delta x = \Delta z = 0.05$. Such a grid was used also for investigations of TW, LTW and LSOC properties [25, 38].

2.2. Diagnostic tools

To measure the intensity of the heating we use the reduced Rayleigh number $r = R/R_0^c$, where $R_0^c$ is the critical Rayleigh number for onset of pure-fluid convection with the critical wave number $k_0^c$. Linear stability theory predicts $R_0^c = 1708$ and $k_0^c = 3.116$. However, to compare our finite differences numerical results presented in this paper with experimental, analytical or numerical ones, we scale $R$ by the threshold $R_0^c = 1686$ of our numerical code.

To monitor the convection intensity we use the vertical velocity at mid-height, $z = 1/2$, and also the extrema of the streamfunction. In the collision simulations, we also keep track of the locations of the nodes in the vertical velocity field $w(z = 1/2, x, t)$ at mid-height. The time evolution of these nodes is then shown in world line plots. Here we note that these nodes also correspond to extrema of the stream function.

Furthermore, we found it useful to monitor the spatial variations of the concentration field in order to measure how much the fluid was mixed. To this end, we determined the local mixing number of the concentration field

$$M = \sqrt{\overline{C^2}/(C_{\text{cond}})^2}. \quad (8)$$

Here, the overbar implies a spatial average, vertically across the layer

$$\overline{f} = \int_0^1 f(z)dz. \quad (9)$$

In the quiescent ($v = 0$) layer, the Soret-induced conductive concentration profile $C_{\text{cond}}(z) = -\psi(z - 1/2)$ varies from $-\psi/2$ at the top boundary to $\psi/2$ at the bottom one with $(C_{\text{cond}})^2 = \psi^2/12$. In the conductive state, one has $M = 1$ by definition. When convection is present, the advective mixing reduces the mean square deviation $\overline{C^2}$ of the concentration—the better the fluid is mixed advectively, the smaller is $M$. A perfectly mixed fluid would have a mixing number of $M = 0$.

3. Localized structures in the unstable quiescent fluid

Here we briefly review the main bifurcation and spatiotemporal properties of the localized convection structures, LTWs and LSOC, which are for our parameters stably embedded in the surrounding linearly unstable conductive state of a quiescent fluid. For details, see

Figure 1. Driving ranges in which the collision scenarios described in the text are realized: scenario I—pink interval (two fronts move), scenario II—red interval (one LSOC front moves) and scenario III—gray interval (two LTWs). Lines show the bifurcation diagrams of maximal vertical velocity versus reduced Rayleigh number $r$ for spatially localized states, stable LSOC (red line with lozenges) and stable LTW (black line with dots) and for spatially extended stable SOC (blue) and unstable TW (dashed). The wave number of the latter two is $k = \pi$. For better orientation the Hopf bifurcation threshold at $r_{osc}$ for the subcritical TW solution is indicated. Its upper stable branch ends with zero frequency at $r^* = 1.0885$, i.e. at the left boundary of the plot. Parameters are $L = 0.01$, $\sigma = 10$ and $\psi = -0.08$. 

[12, 19, 23, 25, 38] and references cited therein. This is meant to provide the background on which we present our results for the collision processes in section 4.

3.1. Quiescent conductive state and spatially extended convection structures

The quiescent conductive state is characterized by a laterally homogeneous stratification of temperature that entails via the Soret effect also a concentration stratification and a mass density stratification. However, for negative $\psi$ the solutal contribution, $\sim C_{\text{cond}}$, to the buoyancy force reduces the thermal one so that the quiescent fluid is stabilized by the Soret-induced vertical concentration gradient.

For our parameters, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$, the conductive state loses stability at the reduced Rayleigh number $r_{osc} = 1.095$. There, a backwards Hopf bifurcation occurs into spatially extended oscillatory convection in the form of a traveling and a standing wave. The latter solution is of no relevance here. But the former is experimentally seen since the initially unstable TW solution branch (see the dashed line in figure 1) undergoes a saddle-node bifurcation into a stable, strongly nonlinear TW solution. This TW solution ends at $r^*$ by merging with zero frequency into the branch of extended SOC. Below $r^*$ the SOC solution is unstable against a phase drift that leads to a TW. Above $r^*$ the SOC solution is stable as indicated by the blue line in figure 1. There, at large enough $r$, the flow amplitudes increase and thus the
fluid gets well mixed, i.e. the mixing number $M$ decreases. This advective homogenization is strong enough to reduce and largely eliminate the adverse, Soret-induced solutal contribution to the buoyancy that drives oscillations.

It is worthwhile to keep the stability properties of the quiescent conductive state in mind: it is oscillatory unstable against extended perturbations of TW and SW character. Thus, these kinds of perturbations will grow beyond $r_{osc}$. However, in the driving range where we performed the collision experiments, the instability is only convective and not absolute, so that the large group velocity of localized TW-packet perturbations lets them move away faster than they can grow.

On the other hand, the conductive state is linearly stable against extended SOC perturbations: since the SOC solution branch is disconnected from the conductive solution it requires a finite SOC perturbation amplitude to be attracted to the stable, large-amplitude SOC solution that is marked by the blue line in figure 1.

3.2. Localized traveling waves

LTWs are spatially confined convective regions in which the roll vortices are traveling. These convective states are quite robust. They have been investigated in experiments [2, 3, 6, 8, 30] and in numerical simulations [12, 21, 39].

The flow amplitudes in the LTWs (thick black line in figure 1) are only slightly smaller than in the SOC states (blue line). Thus, the advectively mixed LTW regions are embedded with relatively small concentration gradients in the surrounding quiescent fluid. In the latter, on the other hand, the Soret-induced thermodiffusive separation creates a significant vertical concentration gradient. A predominantly advective, large-scale concentration redistribution across the LTW self-consistently sustains a characteristic buoyancy force distribution that allows convection in the mixed central part of the LTW but stabilizes conduction in the quiescent regions outside it. Thus, the LTW sustains specific large-scale concentration currents. They lead to the force distribution that allows the existence of the LTW in the first place: it is such that traveling rolls can grow under the trailing front out of the quiescent fluid that they reach maximal flow amplitude in the center and also that they are forced to decay back into the unstable(!) conductive state at the opposing leading front. Here, trailing and leading fronts refer to the direction of the phase propagation. In that sense, the relatively narrow LTWs that we use here in our collision simulations with relatively small distances between their two fronts can be seen as two-front states in which the two fronts are tightly coupled to each other.

The velocity $v_p$ of the phase of the LTW varies laterally and is typically significantly larger than the drift velocity with which the whole LTW is moving. So, the two LTW fronts move with $v_d$. The drift velocity and the frequency $\omega$ in the frame comoving with the drifting LTW are constant in space and time and are uniquely selected quantities. They depend for fixed parameters only on the Rayleigh number. The same holds true for the lateral profile of the intensity envelope and in particular also for its width $l$. All LTWs in the bifurcation diagram of figure 1 drift in the same direction as the phase propagates.

3.3. Localized stationary overturning convection (LSOC)

LSOCs are spatially confined convective regions in which the roll vortices are stationary. Furthermore, both fronts that delimit the LSOC are pinned and therefore stationary as well.
Figure 2. (a) LSOC front velocity $v_F$ (filled circles) versus reduced Rayleigh number $r$ observed in the final stage of the two-front (2F) scenario I and in the one-front (1F) scenario II. Open circles represent front velocities reported in [25]. Red arrows mark the beginning and end of the interval in which bistability between the solutions for pinned and propagating fronts was found [25]. (b) Concentration step $\Delta C$ (full circles) ahead of pinned LSOC fronts versus $r$ as identified in figures 4(a), 6(a) and 8(a). Open circles refer to the results reported in [25]. The latter were obtained with a different numerical scheme leading, e.g., to a value of $r^*=1.0935$ for the end of extended TWs and the beginning of stable SOC and LSOC states. We got, on the other hand, $r^*=1.0885$. Thus, we rescaled the results of [25] by shifting them in (a) and (b) by the amount of 0.005 along the abscissa. Parameters are $L=0.01$, $\sigma=10$, $\psi=-0.08$.

These states have been seen in early experiments [30, 31] and have been investigated in numerical simulations [23, 26]. They appear in the driving region where the extended SOC solution is stable, i.e. at $r > r^*$. Their flow amplitudes (thick red line in figure 1) are practically the same as in the SOC states (blue line).

In contrast to LTWs, there is multistability of these localized structures containing different numbers of convection rolls for the same Rayleigh number. Lateral reflection symmetry of the system allows then to classify LSOCs into even and odd states [23] with even and odd numbers of rolls, respectively. Furthermore, the LSOC solutions exist bistably with front propagating solutions in a driving interval that is marked for our parameters by two red arrows in figure 2(a). The stable front propagating solutions describe a situation where one or both LSOC fronts move with constant, uniquely selected speed $|v_F|$ into the unstable quiescent fluid.

The bulk LSOC regions are well mixed advectively so that the concentration variations and therefore the mixing numbers are small as in the SOC states. At the two fronts there exists a characteristic difference between the inside concentration level and a wall-like pile-up on the outer side of the front. The latter falls off to the flat conductive levels further away from the front. The stepwise concentration variation across the front on a sub-wavelength scale which
is absent at moving fronts entails a buoyancy profile that is characteristic of pinned, stationary fronts. It prohibits the growth of another vortex in the quiescent fluid ahead of the front so that the latter is pinned. It also prevents the decay of the front vortex so that also the conductive state cannot invade the LSOC region. Thus, the LSOC state that is bounded by such a peculiar, step-wise concentration distribution at its ends cannot invade the quiescent fluid. On the other hand, at a moving LSOC front the concentration step and pile-up is suppressed by the successive generation of rolls with alternating vorticity ahead of the moving front [25].

4. Collision of localized structures

In this section, we present our results concerning the collision of an LTW with a LSOC that we obtained for a separation ratio of $\psi = -0.08$. We focus here on an initial situation where an LTW drifts toward an even LSOC with an even number of convection vortices. Thus, such an ‘experiment’ requires parameter combinations for which LSOC and LTW solutions exist bistably.

We found three different collision scenarios. They appear also when an odd LSOC is hit by an LTW. The scenarios imply different, characteristic, collision-induced changes in the concentration distributions. They play a dominant role in the outcome of the collision process. However, for notational convenience we identify and characterize these scenarios by the final states, i.e. by the post-collision convection behavior. The details can be found in sections 4.1–4.3.

- In scenario I the LTW is destroyed and the LSOC expands. Here, the two LSOC fronts propagate with constant, equal speed opposite to each other into the quiescent fluid.
- Also in scenario II the LTW is destroyed. But here the LSOC expands only via the front that was hit by the LTW. The opposing LSOC front is pinned.
- In scenario III the incident LTW survives, the LSOC is destroyed and a second LTW is created ahead of the incoming one.

These scenarios are realized in the three adjacent intervals of the Rayleigh number that are shown in figure 1. So, what scenario will appear depends for given $\psi$, $L$ and $\sigma$ on the heating rate. For $\psi = -0.08$, we did not find other final states after the collision than the above three.

Note that the realization of scenarios I and II requires bistability of LSOC solutions with pinned fronts and of propagating-front solutions: initially, the LSOC fronts are pinned and then the perturbation by the incident LTW unbinds one of them or both. However, the details of the collision processes during the interaction time of LSOC and LTW are quite different in the three scenarios, see sections 4.1–4.3.

Thus, to summarize, LSOCs that are delimited by two stationary, pinned fronts are destroyed when a stable LTW drifts into them: in all three of the shaded $r$-intervals of figure 1 in which a stable, relaxed LTW is available for a separation ratio of $\psi = -0.08$ for a collision experiment with a (linearly) stable LSOC, the latter does not survive this large perturbation.

However, when the Soret coupling is stronger, we found, e.g., at larger $r$ for $\psi = -0.13$ also a scenario where the LSOC absorbs the LTW. Then the final state is a fully stationary LSOC with two pinned fronts that, however, consist of a larger number of convection vortices than in the initial state before the collision.
4.1. Scenario I: LSOC expansion via two propagating fronts

Collision scenario I appears at the large-$r$ end of the existence range of stable LSOC. The $r$-interval in which we found scenario I is marked by ‘2 fronts move’ in figure 1 and by ‘2F’ in figure 2(a). This interval lies slightly below the location of the rightmost arrow in figure 2(a). There, LSOCs with two pinned fronts were reported to become unstable via front propagation when the heating was increased. Below this stability threshold of LSOC, which was identified by $r_{\text{max}}^{\text{p}}$ in [25], there is bistability between the LSOC solutions with pinned fronts and those where one or (both fronts) propagate with a uniquely selected speed $|v_F|$ into the surrounding quiescent fluid.

Figures 3 and 4 show with different diagnostic tools the characteristic evolution from the pre-collision situation consisting of an LSOC and an incident LTW to the post-collision situation where an LSOC expands at both ends via front propagation. The bottom and top insets of figure 3 contain the color-coded concentration distribution in the layer in the pre- and post-collision stages, respectively, together with high-value isolines (solid lines) of the streamfunction $\Psi_1$ in order to display the more intensive roll vortices. For visualization purposes, only every second vortex, here the counter-clockwise turning ones with negative vorticity and positive $\Psi_1$, is shown.

The inset at time $t = 0$ indicates that the initial LSOC contained about ten pairs of roll vortices. The first vortex at the left front rotates clockwise and its mirror image at the right front rotates anticlockwise. Both these end vortices ‘suck’ negative (red coded) concentration out of the quiescent fluid with its large Soret-induced vertical concentration gradient and inject it advectively into the LSOC, cf the upwards oriented red plume-like structures at both ends. This lateral transport of negative $\delta C$ out of the quiescent fluid into the LSOC creates an enhancement of negative $\delta C$ on the convective sides of both fronts and simultaneously a positive $\delta C$-surplus on the conductive sides of the fronts, i.e. a marked lateral concentration difference between the inside and the outside. This peculiar change of the concentration distribution on a sub-wavelength scale across the fronts changes in turn the local buoyancy forces there in such a way that the growth of a further anticlockwise turning vortex ahead of the left front and similarly of a clockwise rotating one ahead of the right front is prohibited [25]. Thus, the fronts remain pinned: the LSOC state that is bounded by such a peculiar concentration distribution at its ends cannot invade the quiescent fluid despite the fact that the conductive state is unstable against spatially extended convective perturbations.

Lines in the central part of figure 3 are world lines of every second vortex center being identified by the node position of the vertical velocity field $w$ at mid-height, $z = 1/2$, of the layer. So, the lines capture the lateral motion of the convection rolls. The thin blue lines refer to low-intensity vortices with $0.01 < \Psi < 0.1$. The black lines, on the other hand, identify vortices with $\Psi \geq 0.1$. So, the blue lines allow us to better identify the decay into and the growth out of the quiescent fluid, respectively, of convection rolls.

In the LSOC well before and well after the collision, the roll vortices do not move laterally, i.e. their vertical world lines remain parallel to each other in figure 3. The LSOC expansion via front propagation in the late stages is realized by successive growth of new, spatially fixed vortices at the fronts out of the quiescent fluid.

The world lines of the vortices in the incident LTW at pre-collision times show that they move with a constant phase velocity of $v_p = -0.51$ from right to left. Simultaneously, this TW wave packet drifts as a whole to the left with the significantly smaller velocity $v_d = -0.1$.
Figure 3. Collision scenario I. Initially, at time \( t = 0 \), an LTW drifts with velocity \( v_d = -0.1 \) from the right toward an even LSOC. The LSOC absorbs the LTW and after the collision the LSOC expands at both ends behind fronts that propagate with velocity \( v_F = \pm 0.042 \) into the quiescent fluid. Bottom and top insets show the pre- and post-collision stages, respectively, with (i) the color-coded concentration distribution in the layer (blue, green and red denote high, mean and low concentration, respectively) and with (ii) isolines of the streamfunction \( \Psi \) (solid lines). Only the left turning vortices with positive \( \Psi \) are shown for visualization purposes. Lines in the central part of the figure show the location of these vortex centers, i.e. node positions of the vertical velocity field \( w \) at midheight of the layer. Thin blue and thicker black lines refer to vortices of lower intensity \( (0.01 < \Psi < 0.1) \) and higher intensity \( (\Psi \geq 0.1) \), respectively. Parameters are \( r = 1.112, L = 0.01, \sigma = 10, \psi = -0.08 \).

Note that the low-intensity leading tail of the LTW extends much further into the quiescent surrounding fluid than the trailing tail as indicated by the different horizontal extensions of the blue world lines. The decaying vortices ahead of the LTW are pushed with sizeable intensity.
Figure 4. Collision scenario I. Shown are snapshots of the vertical velocity $w(x, z = 1/2)$ at mid-height (black), of the concentration deviation $\delta C(x, z = 1/2)$ from its global mean (green) and of the local mixing number $M(x)$ (8) (red) at different times as indicated. See figure 3 for complementary details. The green arrow labeled $\Delta C$ ahead of the pinned front in (a) identifies the concentration step that is incorporated into figure 2(a). Parameters are $r = 1.112$, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$.

much further into the quiescent fluid than the range over which the LTW pulls phase out of the conductive state at its tail.

Complementary to figure 3 we use in figure 4 additional visualization tools in order to show the evolution of the colliding convection structures. We show snapshots of the vertical velocity $w(x, z = 1/2)$ at mid-height (black), of the concentration deviation $\delta C(x, z = 1/2)$ from its global mean (green) and of the local mixing number $M(x)$ (8) (red) at different times as indicated.

In the pre-collision situation at $t = 0$ (bottom inset of figure 3), the two convection structures practically do not yet interact: the velocity field is practically zero between them, and the local mixing number $M$ measuring in particular the advection generated concentration variations is still 1. However, there is already a slight large-scale concentration redistribution between the LSOC and the LTW, cf the inclination of, e.g., the boundary between green and blue in the bottom inset at $t = 0$. This deviation from horizontal iso-concentration levels (which characterize a genuine, laterally extended state of heat conduction) is mostly driven by diffusion. It equilibrates laterally the concentration bulge ahead of the incident LTW to the concentration step at the right front of the LSOC. A similar large-scale, diffusive concentration redistribution appears also when the two front-generated concentration steps of
an LSOC diffusively communicate with each other in a system with laterally periodic boundary conditions [25, 26, 28].

At $t = 0$ this step is like the one at the left LSOC front in figure 4(a)—at $t = 45$ the latter is still pinned and not yet influenced by the coming collision. It was shown that it is this step in the concentration distribution ahead of the LSOC that provides a pinning force: the concentration redistribution at the front relative to the bulk LSOC changes the local buoyancy forces on a sub-wavelength scale in such a way as to prevent convection growth there, so that expansion of a strongly nonlinear, stable SOC into the unstable quiescent fluid is impossible [25]. Propagating SOC fronts, on the other hand, do not have such a step-wise concentration redistribution.

Figure 4(a) shows that at time $t = 45$ the vertical velocity field $w$ (black) is still very small between the collision partners and that the mixing number $M$ (red) there is almost 1 as in the quiescent conductive state. Also the velocity field, the concentration distribution and the mixing number within the LSOC have not yet changed. However, the deviation $\delta C(x, z = 1/2)$ from its global mean (green) has increased dramatically between the two localized structures, since now the aforementioned LSOC concentration step and the LTW bulge overlap. Furthermore, the concentration distribution between the LSOC and the LTW is oscillating with the frequency that is imposed by the incoming LTW, see video 1, available in supplementary data (stacks.iop.org/NJP/14/093055/mmedia).

Thereafter, the strong interaction phase of the collision begins: the LSOC vortices at the right end start to oscillate and then merge with the incident LTW vortices. The LTW gets completely absorbed by the LSOC. New LTW vortices are no longer being produced at the trailing LTW front, the ones at the leading LTW front are annihilated and those in the center are slowed down and are transformed into SOC vortices, see the evolution of the LTW world lines in figure 3 during the interval $129 \lesssim t \lesssim 162$. All this happens during an interval of only about 33 thermal diffusion times.

Furthermore, the concentration step $\Delta C$ at the right LSOC front is destroyed and also the concentration distribution within the LSOC is changed—compare the green lines in figures 4(a) and (b) at times 45 and 162, respectively, with each other; see also video 1. As a result of this LTW-induced concentration current into the LSOC, the concentration level of $\delta C(x, z = 1/2)$ in the LSOC is raised also close to the left front. This reduces the step size $\Delta C$ there and also weakens the pinning force for the left front. Furthermore, the mixing on the convective sides of the front and with it the local buoyancy force there increases significantly: $M$ reduces there to about 0.06 while in the center of the LSOC it remains at a value of about 0.26 (figure 4(c)).

By the time $t \approx 180$ the inside concentration levels behind the two LSOC fronts have reached large enough height to unbind the fronts: the difference across the fronts between the inside and outside concentration levels, i.e. the step size $\Delta C$, has become too small to be able to stabilize the quiescent fluid against the invasive growth of vortices. Thus, new SOC rolls are successively generated ahead of the fronts. This is done with a constant rate which is uniquely selected for this particular Rayleigh number. Thus, the fronts move symmetrically into the quiescent fluid with constant front velocities, $v_F = \pm 0.042$. The successive generation of vortices with alternating vorticity annihilates the concentration step of pinned fronts since the associated time-periodic reversing of the advective concentration exchange between the convective and the conductive side of the front averages to zero.

So, here the collision with the concentration pulse that is characteristic of our narrow LTW changes the LSOC concentration distribution in such a way that both fronts are unbound.
4.2. **Scenario II: LSOC expansion via one propagating front**

In the collision scenario II, only the LSOC front that is hit by the LTW propagates in the post-collision situation into the quiescent fluid—here to the right, i.e. opposite to the direction of the incident LTW. The left front of the LSOC remains pinned in the sense that new vortices are not generated there. However, as a result of the impact of the LTW the vortices in the left half of the LSOC are shifted as a whole slightly to the left during the interval $160 \lesssim t \lesssim 220$. This can be clearly seen in figure 5 containing the world lines of the vortices. Also video 2 shows that new vortices are not created in this shift (see supplementary data, available at stacks.iop.org/NJP/14/093055/mmedia).

Scenario II appears in the central part of the $r$-range in which LSOC solutions with two pinned fronts bistably coexist with moving fronts, see figure 2(a). The parameter range where this scenario is realized is indicated in figures 1 and 2(a). Scenario I, on the other hand, with both fronts being unpinned as a result of the collision takes place closer to the large-$r$ unpinning instability for fronts, the right arrow of figure 2(a). The increased driving distance from the upper unpinning instability presumably explains partly that the LTW perturbation cannot unpin both LSOC fronts.

Here, at smaller $r$, the flow intensities of the collision partners, the drift velocity $v_d = -0.091$ of the incident LTW and also the front velocity $v_F = 0.03$ of the expanding LSOC in the final stage are smaller than the respective values in scenario I at larger $r$. Because of the smaller advection, the concentration step $\Delta C$ that is necessary to pin the LSOC fronts before the collision is smaller: at smaller $r$ the thermal buoyancy forces are smaller and thus already a smaller solutal contribution can ensure the force balance which prevents front motion occurring so that neither convection nor conduction expands. Nevertheless, here the concentration current injected by the incident LTW is not large enough to increase the concentration level in the LSOC behind its left front sufficiently, see figure 6. As a result, the pinning concentration step survives this perturbation. Consequently, the buoyancy balance at the left front that prevents the growth of an additional left turning roll in the quiescent fluid ahead of the first right turning one in the LSOC remains intact.

On the other hand, at the right LSOC front that is hit by the LTW the concentration changes evolve in practically the same way as in scenario I. The incident LTW destroys the concentration step at the right front and injects a concentration current into the LSOC. To see the close similarity, it is instructive to compare the right-hand parts of figures 3 and 4 with those of figures 5 and 6, respectively. Compare also the spatiotemporal evolution in the right halves of videos 1 and 2. Again, the LTW is completely absorbed by the LSOC: the traveling vortices of the LTW are annihilated or get slowed down to become SOC vortices. As in scenario I, new traveling vortices cease to be created at the trailing end of the LTW, here at $t \approx 168$. The last LTW roll gets transformed into an SOC roll at the location of the former right LSOC fronts. The destruction and transformation of the LTW happen within a time interval of about 30 vertical thermal diffusion times, that is of the same order as in the previous case. Shortly thereafter, say, at $t \approx 228$, new SOC vortices are generated at a constant rate ahead of the right front so that the latter moves with velocity $v_F = 0.03$.

4.3. **Scenario III: creation of a second localized traveling wave**

In collision scenario III the LSOC is transformed into a second LTW with the same uniquely selected LTW characteristics as those of the incident one. The new LTW drifts ahead.
Figure 5. Collision scenario II. Initially, at time $t = 0$, an LTW drifts with velocity $v_d = -0.091$ from the right toward an even LSOC. The LSOC absorbs the LTW and after the collision the LSOC expands with front velocity $v_F = 0.03$ to the right into the quiescent fluid. Bottom and top insets show the pre- and post-collision stages, respectively, with (i) the color-coded concentration distribution in the layer as in figure 3 and with (ii) isolines of the streamfunction $\Psi$ (solid lines). Only the left turning vortices with positive $\Psi$ are shown for visualization purposes. Lines in the central part of the figure show the location of these vortex centers, i.e. node positions of the vertical velocity field $w$ at mid-height of the layer. Thin blue and thicker black lines refer to vortices of lower intensity ($0.01 < \Psi < 0.1$) and higher intensity ($\Psi \geq 0.1$), respectively. Parameters are $r = 1.107$, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$.

of the incident LTW, which comes unchanged out of the collision process. The distance between the centers of the two final LTWs is rather small, say, about 16.5. Nevertheless, the fast traveling, low-amplitude concentration waves that are emitted by the leading front of the second LTW are absorbed at the tail front of the first LTW without changing the
Figure 6. Collision scenario II. Shown are snapshots of the vertical velocity $w(x, z = 1/2)$ at mid-height (black), of the concentration deviation $\delta C(x, z = 1/2)$ from its global mean (green), and of the local mixing number $M(x)$ (red) at different times as indicated. See figure 5 for complementary details. The green arrow labeled $\Delta C$ ahead of the pinned front in panel (a) identifies the concentration step that is incorporated into figure 2(a). Parameters are $r = 1.107$, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$.

spatiotemporal structure of this LTW couple anymore (see supplementary video 3, available at stacks.iop.org/NJP/14/093055/mmedia): it drifts with the same constant velocity $v_d = -0.047$ as the incident LTW in the pre-collision state.

Scenario III appears in the lower part of the $r$-range in which LTWs exist and thus are available for the collision experiment, see the gray interval in figure 1. In figures 7 and 8 and in...
Figure 7. Collision scenario III. Initially, at time $t = 0$, an LTW drifts with velocity $v_d = -0.047$ from the right toward an even LSOC. The LSOC is transformed into an LTW and after the collision there are two LTWs that drift, one behind the other, to the left. Bottom and top insets show the pre- and post-collision stages, respectively, with (i) the color-coded concentration distribution in the layer as in figure 3 and with (ii) isolines of the streamfunction $\Psi$ (solid lines). Only the left turning vortices with positive $\Psi$ are shown for visualization purposes. Lines in the central part of the figure show the location of these vortex centers, i.e. node positions of the vertical velocity field $w$ at mid-height of the layer. Thin blue and thicker black lines refer to vortices of lower intensity ($0.01 < \Psi < 0.1$) and higher intensity ($\Psi \geq 0.1$), respectively. The left inset shows a blow-up of the world lines in the right rectangle. Parameters are $r = 1.0995$, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$. 

**Figure 8.** Collision scenario III. Shown are snapshots of the vertical velocity $w(x, z = 1/2)$ at mid-height (black), of the concentration deviation $\delta C(x, z = 1/2)$ from its global mean (green) and of the local mixing number $M(x)$ (red) at different times as indicated. See figure 7 for complementary details. The green arrow labeled $\Delta C$ ahead of the pinned front in panel (a) identifies the concentration step that is incorporated into figure 2(a). Parameters are $r = 1.0995$, $L = 0.01$, $\sigma = 10$, $\psi = -0.08$.

In video 3, we show the details of the collision of an LSOC with an LTW at $r = 1.0995$, i.e. at the smallest possible $r$. Here the initial LSOC has 22 rolls instead of 20 as in the previous figures and videos. In fact, we performed simulations with initial LSOCs of different lengths to check that all the scenarios were reproduced.

If one compares the evolution of the vortex world lines in the two previous scenarios (figures 3 and 5) with that in scenario III (figure 7 and its central inset), one sees that here the
rightmost vortices of the LSOC are first modulated by the LTW and then they are successively destroyed. The vortices of the LSOC that survive the collision are being pushed forward to the left. Until $t \approx 750$ the vortices in this intermediate localized structure are moving relatively slowly to the left while being accelerated. Also the concentration distribution in this pushed wave packet still resembles that of an LSOC. But then this localized structure shrinks to the selected width appropriate for an LTW at $r = 1.0995$. Simultaneously, its concentration distribution develops the large-amplitude fluctuations between the vortices that are characteristic of an LTW and the phase of this wave packet begins to travel with the selected fast LTW velocity $v_p = -0.75$, see figure 8 and video 3.

On the other hand, the vortices of the incident LTW practically do not seem to be affected much during the collision. However, the drift speed with which its trailing front moves to the left in figure 3 increases to about $|\nu_d| = 0.056$ during the collision time $318 \lesssim t \lesssim 750$, but then resumes its pre-collision value in the post-collision phase. Also the concentration profile of the incident LTW in figure 8 is not altered.

But the fast-oscillating concentration wave that is emitted by the leading LTW front and that originates from the fast phase velocity of the LTW plays a decisive role. This holds true in particular when the distance between the right LSOC front and the incident LTW front has become small. Then, the emitted concentration wave hits the LSOC with a large amplitude. For example, at $t \approx 216$ the concentration step that stabilizes the LSOC at its right end is destroyed, see figure 8.

These large-amplitude concentration oscillations can be clearly seen in video 3 (see supplementary data) in the gap between the LSOC and the incident LTW once the latter has come close to the former, say, beyond $t \approx 200$. The large amplitude of the concentration wave can also be seen in the lateral concentration profiles at mid-height, see green lines in figure 8. And they show up also in the mixing number (red lines in figure 8) that reaches extremal values of 1 in the interval between the collision partners.

It seems worthwhile to compare also some characteristic numbers of the pre-collision phase of the three scenarios I–III for the target LSOCs ($M = 0.30, 0.25, 0.18$) and for the incident LTWs ($\nu_d = -0.1, -0.091, -0.047; v_p = -0.51, -0.61, -0.75; M = 0.26, 0.28, 0.32$). Thus, the LSOC of scenario III is best mixed with its mixing number being lowest. The drift velocity $\nu_d$ with which the LTW approaches the LSOC is smallest in scenario III, its phase velocity $v_p$ is largest and also its concentration variation, i.e. its mixing number $M$, is largest in scenario III.

All in all, the fast and large-amplitude concentration wave that the incident LTW emits at its leading front first annihilates one after the other the LSOC vortices at the right front while the remaining ones are pushed forward. Then, the concentration oscillations that the incident LTW drives into this wave packet lead to the second LTW. All this takes about 500 thermal diffusion times. Thus, the interaction time here is much longer than that in scenarios I and II where the transformation into the final state takes only about 50 thermal diffusion times.

### 4.4. Nusselt number changes

The collision of the LTW with the LSOC causes characteristic changes in the Nusselt number

$$ Nu = \frac{1}{L} \int_0^L dx \frac{\partial T}{\partial z}. \quad (10) $$

Here the temperature gradient is taken at either of the two no-slip boundaries $z = 0$ or $z = 1$. $Nu$ (10) is the quotient of the total vertical heat current through the layer of lateral extent $L$.
and the purely diffusive current through the quiescent fluid. The difference $\delta Nu = Nu - 1$ is a global measure of the intensity of convection.

Note first of all that in the pre-collision phase of all three scenarios with an LSOC and an LTW that are sufficiently separated, $\delta Nu$ is constant in time. For example, in scenario III the pre-collision value is $\delta Nu = 0.034$. On the other hand, at post-collision times when two LTWs are in the system, one has only $\delta Nu = 0.0066$, i.e. a value that is about five times smaller. The reason for this reduction is that the LSOC which is annihilated in scenario III has larger convection amplitudes and larger lateral extent than the LTW that is created instead.

In scenarios I and II, however, $\delta Nu$ increases in the post-collision phase at constant, albeit different rates since new SOC vortices are created. In scenario I both fronts move into the quiescent fluid, thus creating new vortices at both ends of the LSOC. Hence, the temporal growth rate of $\delta Nu$ in the post-collision phase of scenario I is twice as high as in scenario II where only one front moves.

Thus, the characteristic changes of $\delta Nu$ can be used also as a diagnostic tool for characterizing and monitoring the collision dynamics.

5. Conclusion

The bifurcation properties and the spatiotemporal behavior of colliding localized convective structures in binary fluid mixtures were investigated in a cross-section perpendicular to the roll axes. Finite difference numerical simulations were performed for parameters adapted to experiments that use ethanol–water mixtures with small negative Soret coupling. Various visualization and diagnostic tools were used to elucidate the rich and rather complex, strongly nonlinear response behavior that appears in the collision of an LTW with an LSOC. Thus, the investigations were performed in a parameter range where these two localized structures bistably coexist.

Three different scenarios were found with three spatiotemporally different convection properties in the post-collision phase depending on the Rayleigh number.

In scenarios I and II the LSOC absorbs the LTW. Then the LSOC expands into the surrounding quiescent fluid. In scenario I both of its fronts move with a constant, selected front velocity as new SOC rolls are generated with constant rates at the moving fronts in the post-collision phase. In scenario II only the LSOC front that is hit by the LTW gets unpinned and moves into the conductive state while the opposing front remains pinned. Thus, the pinned and the moving front solutions of LSOC states were observed in the bistability range of these solutions: before the collision the fronts were pinned and after the collision perturbation one or both of the LSOC fronts moved.

Scenario III is realized at lower heating rates. There the incident LTW is almost unaffected by the collision. But the LSOC is transformed into a second LTW that has the same selected characteristics as the incident LTW. In the post-collision phase the newly created LTW drifts at short distance ahead of the incident one. Both form a couple that is linked together via fast propagating concentration waves—the trailing LTW emits them and the leading LTW absorbs them.

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