Comment on “Disorder Induced Quantum Phase Transition in Random-Exchange Spin-1/2 Chains”

In a recent Letter Hamacher, Stolze, and Wenzel [1] studied the disordered spin-1/2 antiferromagnetic XXZ chain

\[ H = J \sum_{i=1}^{L-1} \left[ \lambda_i \left( S_i^z S_{i+1}^z + S_i^y S_{i+1}^y \right) + \Delta S_i^y S_{i+1}^z \right] \]  

(1)

with \( \lambda_i \) independent identically distributed random variables uniformly distributed over the interval \([1 - W; 1 + W]\), with the parameter \( W \) controlling the strength of the disorder. The authors claim that for \( \Delta < 1 \) they found numerical evidence for nonuniversal behavior for weak disorder (\( W < 1 \)) manifested in a continuously varying exponent \( \eta(W) \) describing the asymptotic decay of the transverse spin correlations

\[ C_{xx}(r) = \langle S_i^x S_{i+r}^x \rangle \propto r^{-\eta(W)}, \]

(2)

where \( \langle \cdot \cdot \cdot \rangle \) denotes the ground state expectation value averaged over the disorder and the sites \( i \). They concluded that there is no universal infinite randomness fixed point (IRFP) as predicted by Fisher [2].

In this Comment we show that these conclusions are inadequate because the numerical data presented in [1] are not in the asymptotic regime. We demonstrate that the observed behavior is very compatible with the IRFP scenario due to the existence of a \( W \)-dependent crossover length scale \( \xi_W \) that describes the crossover from the pure fixed point to the only relevant IRFP: For \( L < \xi_W \) one observes the critical behavior of the pure system (\( \lambda_i = \text{const} \)), and only for \( L > \xi_W \) the true asymptotic critical behavior \( \eta(W) = 2 \), independent of \( W > 0 \) of the disordered chain becomes visible. Even for strong disorder \( W = 1.0 \) \( \xi_W \) is of the same order of magnitude as the system sizes considered in [1].

In order to be able to reach sufficiently large system sizes we restrict ourselves to \( \Delta = 0 \) in which case (1) reduces to a free fermion model and the ground state computations are done following [3]. In Fig. 1 we show the averaged bulk correlation function \( C_{xx}(L/2) \) for different strengths of the disorder. We observe that asymptotically (i.e., for \( L \to \infty \)) the data follow the behavior \( C_{xx}(L) \propto L^{-2} \) as predicted by the real space renormalization group [2]. Only for small \( L \) do the data appear to follow a nonuniversal (i.e., \( W \)-dependent) power law, and this is the region on which [1] reports.

Because of the presence of a crossover length scale \( \xi(W) \), the correlation function obeys the scaling form

\[ C_{xx}(L/2) = L^{-1/2} \xi(W), \]

(3)

where \( \xi(W) \) is constant for \( x \to 0 \), and \( \xi(W) \to x^{-3/2} \) for \( x \to \infty \). This implies \( C_{xx}(L/2) \propto L^{-1/2} \) for \( L < \xi_W \) (the pure behavior) and \( C_{xx}(L/2) \propto L^{-2} \) for \( L > \xi_W \) (the IRFP behavior). In the inset of Fig. 1 we show such a scaling plot of the data in the main figure. We have chosen \( \xi_W \) for \( W = 1 \) such that the crossover region is centered around \( L/\xi_W = 1 \); the other estimates for \( \xi_W \) are then chosen to give the best data collapse. We see that for all disorder strengths the maximum system sizes used in [1] are still well within the crossover region and not in the asymptotic regime. We do not expect that this situation will improve for \( \Delta > 0 \). In general \( \xi_W \) diverges when \( W \to 0 \), and we found that our estimates for \( \xi_W \) obey \( \xi_W \propto \delta_W^{-0.8} \) where \( \delta \) is the \( W \)-dependent variance of the random variable \( \ln \lambda_i \). We obtain \( \Phi = 1.8 \pm 0.2 \).

To conclude we have shown that the asymptotic behavior of model (1) belongs to the universality class described by an IRFP also for weak disorder as predicted by [2].

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Nicolás La florence\(^1\) and Heiko Rieger\(^2\)
\(^1\)Laboratoire de Physique Théorique, CNRS-UMR5152
Université Paul Sabatier
F-31062 Toulouse, France
\(^2\)Theoretische Physik, Universität des Saarlandes
66041 Saarbrücken, Germany

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