Quantum computation with trapped ions and atoms

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Since the first preparation of a single trapped, laser-cooled ion by Neuhauser et al. in 1980, a continuously increasing degree of control over the state of single ions has been achieved, such that what used to be spectroscopy has turned into coherent manipulation of the internal (electronic) state, while laser cooling has evolved into the control of the external degree of freedom, i.e. of the motional quantum state of the ion in the trap. Based on these developments, Cirac and Zoller proposed in 1995 to use a trapped ion string for processing quantum information, and they described the operations required to realise a universal two-ion quantum logical gate. This seminal proposal sparked intense experimental activities in many groups and has led to spectacular results.

In these lecture notes, we will review the basic experimental techniques which enable quantum information processing with trapped ions and, much more briefly, with atoms. In particular, we will explain how the fundamental concepts of quantum computing, such as quantum bits (qubits), qubit rotations, and quantum gates, translate into experimental procedures in a quantum optics laboratory. Furthermore, the recent progress of quantum computing with ions and atoms will be summarised, and some of the new approaches to meet future challenges will be mentioned. It is intended to provide an intuitive understanding of the matter that should enable the non-specialist student to appreciate the paradigmatic role and the potential of trapped single ions and atoms in the field of quantum computation. More advanced topics are covered in the original literature and in other recent reviews, e.g. (Häffner et al., 2008a; Benhelm et al., 2009; Häffner et al., 2008b).

These notes are based on, and extend, an earlier compilation on the same subject (Eschner, 2006). The main addition is a more elaborate discussion of atom-photon-qubit interfaces which have shown significant progress in the last few years.
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Ion (and atom) quantum logic

In this chapter we describe the basics and the state of the art of quantum logic with single atomic systems, using trapped-ion implementations as the main reference, and addressing briefly some recent advances with neutral atoms.

1.1 Single ion and atom storage

1.1.1 Paul trap

To confine an ion, a focusing force in three dimensions is required. In an electric field a singly charged calcium ion (Ca\(^{+}\)) experiences very strong forces, corresponding to an acceleration on the order of \(2 \times 10^{8} \text{ m/s}^2\) per \(\text{V/cm}\). Although ions cannot be trapped in a static electric field alone, a rapidly oscillating field with quadrupole geometry, 

\[ \Phi(r) = (U_0 + V_0 \cos(\xi t)) \frac{x^2 + y^2 - 2z^2}{r_0^2} \]  

(1.1)

generates an effective (quasi-) potential which under suitably chosen experimental parameters provides 3-dimensional trapping (Paul et al., 1958). The potentials \(U_0\) and \(V_0\) generate a field that alternates too fast for the ion to reach the electrodes of the trap before it reverses its direction. The driven oscillatory motion at the frequency \(\xi\) of the applied field (micro-motion) has a kinetic energy which increases quadratically with the distance from the trap center, i.e. from the zero of the quadrupole field amplitude. By averaging over the micro-motion, an effective harmonic potential is obtained, in which the ion oscillates with a smaller frequency, the secular or macro-motion frequency \(\nu\); this is the principle of the Paul trap. The three frequencies \(\nu_{x,y,z}\) along the three principal axes of the trap may be different. The full trajectory of a trapped ion, consisting of the superimposed secular and micro-motion, is mathematically described by a stable solution of a Mathieu-type differential equation (Ghosh, 1995). Typical macroscopic traps for single ions are roughly 1 mm in size, with a voltage \(V_0\) of several 100 V and frequency \(\xi\) of a few 10 MHz, leading to secular motion with frequencies \(\nu\) in the low MHz range.

The linear variant of the Paul trap uses an oscillating field with linear quadrupole geometry in two dimensions, providing dynamic confinement along those radial directions, while trapping in the third direction, i.e. along the axis of the 2-d quadrupole, is obtained with a static field. This type of trap is advantageous for trapping several ions simultaneously: under laser cooling and with radial confinement stronger than the axial one, the ions crystallise on the trap axis, thus minimising their micro-motion.
which otherwise leads to inefficient laser excitation and unwanted modulation-induced resonances (micro-motion sidebands). A schematic image of a linear Paul trap set-up is shown in Fig. 1.1.

**Fig. 1.1** Schematic illustration of a linear ion trap set-up with a trapped ion string. The four blades are on high voltage (neighbouring blades on opposite potential), oscillating at radio frequency, thus providing Paul-type confinement in the radial directions. The tip electrodes are on positive high voltage and trap the ions axially. A laser addresses the ions individually and manipulates their quantum state. The resonance fluorescence of the ions is imaged onto a CCD camera. (Drawing courtesy of Rainer Blatt, Innsbruck.) In the lower part the CCD image of a string of eight cold, laser-excited ions is shown (Ca⁺ ions at ICFO). The distance between the outer ions is about 70 µm.

### 1.1.2 Quantized motion

The secular motion of a trapped ion can be regarded as a free oscillation in a 3-dimensional harmonic trap potential, characterised by the three frequencies \( \nu_{x,y,z} \). As described in (Stenholm, 1986; Wineland and Itano, 1979; Eschner \textit{et al.}, 2003), laser cooling reduces the thermal energy of that motion to values very close to zero. Therefore the motion must be accounted for as a quantum mechanical degree of freedom. For a single ion it is described by the Hamiltonians

\[
H_k = \hbar \nu_k \left( a_k^\dagger a_k + \frac{1}{2} \right), \quad (k = x, z, y)
\]
with energy eigenstates $|n_k\rangle$ at the energy values $\hbar \nu_k (n_k + 1/2)$. The quantum regime is characterised by $n$ being on the order of one or below.

For a string of $N$ ions, there are $3N$ normal modes of vibration, $2N$ radial ones and $N$ axial ones. The two lowest-frequency axial modes are the center-of-mass mode, where all ions oscillate like a rigid body, and the stretch mode, where each ion’s oscillation amplitude is proportional to its distance from the center (Steane, 1997; James, 1998). Their respective frequencies are $\nu_z$ and $\sqrt{3}\nu_z$, independently of the number of ions, where $\nu_z$ is the axial frequency of a single ion. Movies of ion strings with these modes excited can be found in (Nägerl et al., 1998). As will be explained below, coupling between different ions in a string for QIP purposes is achieved by coherent laser excitation of transitions between the lowest quantum states $|n = 0\rangle$ and $|n = 1\rangle$ of one of the axial vibrational modes, usually the center-of-mass or the stretch mode.

1.1.3 Laser cooling

Like trapping, laser cooling is a pivotal ingredient for preparing cold ions or atoms for quantum computing. Laser cooling relies on the photon recoil or, more generally, the mechanical effect of light in a photon scattering process (absorption-emission cycle), which is caused by the spatial variation of the electric field $e^{i k x}$. Since several comprehensive reviews of laser cooling exist (Stenholm, 1986; Wineland and Itano, 1979; Eschner et al., 2003), the description shall be limited to a brief summary of Doppler cooling in section 1.3.1, an initial cooling technique which is essential for ion storage and cooling to the Lamb-Dicke regime, and sideband cooling, which will be discussed in section 1.3.2.

1.1.4 Dipole trap

Individual neutral atoms are trapped in dipole potentials (Grimm et al., 2000; Adams and E.Riis, 1997), created by strongly focused laser beams (Schlosser et al., 2001; Frese et al., 2000). In order to trap arrays of atoms while keeping them optically addressable, one may combine several spatially separated laser beams (Beugnon et al., 2006; Yavuz et al., 2006); more sophisticated optical methods have also been demonstrated, such as lens arrays (Dumke et al., 2002) or spatial light modulators (Bergamini et al., 2004).

1.2 Qubits

The atomic qubit, in which quantum information is encoded, requires two stable levels which allow coherent laser excitation at Rabi frequencies much higher than all decay rates. Adequate states may be either a ground state and a metastable excited state connected by a forbidden optical transition ("optical qubit"), or two hyperfine sublevels of the ground state of an ion with non-zero nuclear spin ("hyperfine qubit"). Both cases are treated as atomic two-level systems $\{|g\rangle, |e\rangle\}$ or $\{|\downarrow\rangle, |\uparrow\rangle\}$, see sec. 1.3. Depending on the magnetic properties of the ion species, particular Zeeman substates of the levels are selected, like $S_{1/2}, m = 1/2$ and $D_{5/2}, m = 5/2$ in the optical qubit of Ca$^+$ (Fig. 1.2). An optical qubit transition is driven with a single strong laser with Rabi frequencies $\Omega \gg \Gamma$, while in a hyperfine qubit two lasers, far detuned from an
intermediate level, drive a Raman transition with $\Omega$ being the Raman Rabi frequency (Wineland et al., 2003).

The ion species which are suitable for QIP can thus be divided in two main classes depending on the implementation of the qubits. For an optical qubit one uses ions with a forbidden, direct optical transition, such as $^{40}\text{Ca}^+$, $^{88}\text{Sr}^+$, $^{138}\text{Ba}^+$, $^{172}\text{Yb}^+$, $^{198}\text{Hg}^+$, or $^{199}\text{Hg}^+$. In this case, the lack of hyperfine structure in the electronic level scheme makes the qubit levels sensitive to ambient magnetic fields and to laser phase fluctuations, which are sources of decoherence. Nevertheless, effective qubits can be constructed from multi-ion quantum states, which form a decoherence-free subspace (Häffner et al., 2005; Kielpinski et al., 2001; Langer et al., 2005).

For a hyperfine qubit one employs odd isotopes such as $^9\text{Be}^+$, $^{25}\text{Mg}^+$, $^{43}\text{Ca}^+$, $^{87}\text{Sr}^+$, $^{137}\text{Ba}^+$, $^{111}\text{Cd}^+$, $^{171}\text{Yb}^+$. Here, the effect of laser phase noise is reduced because usually the two fields for the Raman transition are derived from the same source, and their difference frequency is stabilised to a microwave oscillator. The magnetic field sensitivity can also be significantly decreased by using $m = 0$ magnetic sublevels (Haljan et al., 2005), or by applying a static magnetic field at which the qubit levels have the same differential Zeeman shift (Langer et al., 2005). The use of the hyperfine structure, however, comprises a limitation by spontaneous scattering during a Raman excitation pulse. This has been assessed in (Wineland et al., 2003), pointing at heavy ions like Cd$^+$ or Hg$^+$ as good qubit candidates.

Both methods are currently pursued in different groups. For example, the NIST group works with $^9\text{Be}^+$, the Michigan group with $^{111}\text{Cd}^+$ and $^{171}\text{Yb}^+$, and both the Innsbruck group and the Oxford group use $^{40}\text{Ca}^+$. Important progress, such as Quantum Teleportation, has been achieved with both types of qubits (Riebe et al., 2004; Barrett et al., 2004).

Ca$^+$ ions are attractive for comparing optical qubits ($^{40}\text{Ca}^+$) and hyperfine qubits ($^{43}\text{Ca}^+$) while using the same trapping and laser setup. Work with $^{43}\text{Ca}^+$ is carried out in Innsbruck (Kirchmair et al., 2009) and Oxford (Lucas et al., 2004). Direct excitation of hyperfine qubits by microwave radiation has also been considered (Mintert and Wunderlich, 2001). It requires the presence of high magnetic field gradients to obtain sufficient sideband coupling (through an effective Lamb-Dicke parameter, see section 1.3), which at the same time splits the frequencies of neighbouring ions, thus allowing for their addressing.

1.3 Laser interaction

Resonant interaction with laser light is used at all stages of QIP with trapped ions: for cooling, for initial state preparation, qubit manipulation, quantum gates, and for state detection. Various types of optical transitions are employed, and the laser interacts with both the electronic and motional quantum state. Photon scattering is either desired (for cooling and state detection) or unwanted (for qubit manipulations and quantum gates). The relevant processes are summarised in this section, approximately in the order in which they are applied in an experimental cycle: preparation, manipulation, i.e. logic operations, and measurement of the outcome. To give a specific example, Fig. 1.2 shows the relevant levels, transitions, and wavelengths of $^{40}\text{Ca}^+$ and explains how these processes are implemented in a real atom.
Fig. 1.2 Levels and transitions of $^{40}\text{Ca}^+$. They are employed in the following ways for the processes described in this section: Doppler cooling happens by driving the $S_{1/2}$ to $P_{1/2}$ dipole transition at 397 nm; a laser at 866 nm prevents optical pumping into the $D_{3/2}$ level. Sideband cooling is performed on the dipole-forbidden $S_{1/2}$ to $D_{5/2}$ line at 729 nm, with a laser at 854 nm pumping out the $D_{5/2}$ level to enhance the cycling rate. A Zeeman sublevel of $S_{1/2}$ and one of $D_{5/2}$ form the qubit (for example $m_S = 1/2$ and $m_D = 5/2$, see right-hand diagram), on which coherent operations are driven by the 729 nm laser. The final state is detected by the 397 nm laser which excites resonance fluorescence when the ion is in $S_{1/2}$, but does not couple to $D_{5/2}$.

The atom-laser interaction processes are described by a simple two-level atom with resonance frequency $\omega_A$ and states $|g\rangle$ (ground state) and $|e\rangle$ (excited state), and a single vibrational mode with frequency $\nu$, states $|n\rangle$ and creation (annihilation) operator $a^\dagger$ ($a$). The Hamiltonian of the interaction is

$$H_{\text{int}} = -\hbar \Delta |e\rangle\langle e| + \hbar \nu a^\dagger a + \hbar \frac{\Omega}{2} |e\rangle\langle e| e^{i\eta(a^\dagger + a)} + \text{h.c.}$$

(1.3)

where $\Delta = \omega_L - \omega_A$ is the detuning between laser frequency $\omega_L$ and atomic resonance and $\Omega$ is the Rabi frequency characterising the strength of the atom-laser coupling. The exponential in the bracket stands for the travelling wave of the laser, $\eta(a^\dagger + a) = k z$, with the Lamb-Dicke parameter $\eta = k z_0$ which relates the spatial extension of the motional ground state $z_0 = (0|z^2|0)^{1/2}$ to the laser wavelength $\lambda = 2\pi/k$. Although not strictly necessary, QIP experiments are typically performed in the Lamb-Dicke regime, characterised by $\eta \sqrt{\langle n \rangle} \ll 1$, which means that the ion’s motional wave packet is much smaller than the laser wavelength. In this limit transitions $|g, n\rangle \leftrightarrow |e, n'\rangle$ between motional states of different quantum numbers $n \neq n'$ are suppressed as $n - n'$ increases, such that only changes $n - n' = 0, \pm 1$ need to be accounted for. The corresponding laser-driven transitions are called carrier ($|g, n\rangle \leftrightarrow |e, n\rangle$), red sideband ($|g, n\rangle \leftrightarrow |e, n - 1\rangle$), and blue sideband ($|g, n\rangle \leftrightarrow |e, n + 1\rangle$). With respect to Eq. (1.3) this corresponds to expanding the exponential, $e^{i\eta(a^\dagger + a)} \rightarrow 1 + i\eta(a^\dagger + a)$, which will be employed in section 1.4.
1.3.1 Doppler cooling

Doppler cooling is performed by exciting the ion(s) on a strong (usually a dipole) transition of natural linewidth $\Gamma > \nu$, with the laser detuned by $\Delta \simeq -\Gamma/2$. It reduces the thermal energy $\hbar \nu \langle n \rangle$ to about $\hbar \Gamma$ in all vibrational modes which have a non-zero projection on the laser beam direction. With a typical dipole transition of 20 MHz width and a trap frequency of 1 MHz, the average residual excitation of the vibrational quantum states is $\langle n \rangle \sim 20$. Much higher trap frequencies allow for Doppler cooling to $\langle n \rangle \sim 1$ (Monroe et al., 1995b). Figure 1.3 shows an illustration of Doppler cooling.

1.3.2 Sideband cooling

![Diagram of Doppler cooling](image)

![Diagram of sideband cooling](image)

Fig. 1.3 Laser cooling in the Lamb-Dicke regime, $\eta \ll 1$. Only $\delta n = 0, \pm 1$ transitions are relevant. (a) Energy levels and schematic representation of an absorption-emission cycle which cools. (b) Spontaneous emission results in average heating because $|n\rangle \rightarrow |n+1\rangle$ transitions are slightly more probable than $|n\rangle \rightarrow |n-1\rangle$ transitions. (c) In Doppler cooling, when $\Gamma > \nu$, the laser excites simultaneously $|n\rangle \rightarrow |n\rangle$ and $|n\rangle \rightarrow |n \pm 1\rangle$ transitions. By red-detuning the laser to $\Delta \simeq -\Gamma/2$, excitation from $|g,n\rangle$ to $|e,n-1\rangle$ is favoured which thus provides cooling. The final state of Doppler cooling is an equilibrium between these cooling and heating processes. (d) In sideband cooling, when $\Gamma \ll \nu$, the laser is tuned to the $|g,n\rangle$ to $|e,n-1\rangle$ sideband transition at $\Delta = -\nu$, such that all other transitions are off-resonant. This provides a much stronger cooling effect than Doppler cooling, while the heating by spontaneous emission is the same. Thus the final state is very close to the motional ground state.

After the ion(s) is(are) cooled to the Lamb-Dicke regime by an initial Doppler
cooling stage, sideband cooling is used to prepare the vibrational mode in the motional ground state $|n = 0\rangle$, which is a pure quantum state. Fig. 1.3 also illustrates this cooling technique. It is accomplished by exciting an ion on a narrow transition, $\Gamma \ll \nu$, which in practical cases is either a forbidden optical line, such as the $S_{1/2} \rightarrow D_{5/2}$ transition in Hg$^+$ (Diedrich et al., 1989), Ca$^+$ (Roos et al., 1999), or Sr$^+$ (Sinclair et al., 2001), or a Raman transition between hyperfine-split levels of the ground state, as in Be$^+$ (Monroe et al., 1995b; King et al., 1998) or Cd$^+$ (Deslauriers et al., 2004). The laser is tuned into resonance with the red sideband transition at $\Delta = -\nu$. To increase the cycling rate, the upper level $|e\rangle$ may be pumped out by driving an allowed transition to a higher-lying state which then decays back to $|g\rangle$. Pulsed schemes involving coherent Rabi flopping ($\pi$-pulses) are also used (Monroe et al., 1995b; King et al., 1998). Usually only one mode is cooled to the ground state, which then serves to establish coherent coupling between ions in a string, see sections 1.2 and 1.5. Preparation of the motional ground state of the center-of-mass mode with $> 99.9\%$ probability is routinely achieved (Roos et al., 1999; Monroe et al., 1995b).

### 1.4 Motional Qubits

When ground state cooling has been achieved, the two lowest levels $|0\rangle$ and $|1\rangle$ of the vibrational mode form a motional qubit, in analogy to the atomic qubit discussed in section 1.2. Considering first a single trapped ion, then the basis for quantum logical operations, the “computational subspace” (CS), is formed by a combination of motional and atomic qubits, i.e. by the four states $\{|g,0\rangle, |g,1\rangle, |e,0\rangle, |e,1\rangle\}$. For coherent manipulation of the quantum state within the CS, the red sideband, carrier, or blue sideband transition may be excited by tuning the laser to $\Delta = -\nu$, 0, or $+\nu$, respectively.

![Fig. 1.4 Computational subspace of a single ion with carrier (C) and sideband (±) transitions.](image)

The interaction Hamiltonians for these three excitation processes are derived from Eq. (1.3) in the Lamb-Dicke regime and after dropping non-resonant terms,

\[
\begin{align*}
\text{(carrier)} & \quad H_C = \hbar \frac{\Omega^2}{2} (|e\rangle\langle g| + \text{h.c.}) \\
\text{(red sideband)} & \quad H_- = \hbar \eta \Omega \frac{\Omega^2}{2} (|e\rangle\langle g|a + \text{h.c.})
\end{align*}
\]
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\( H_+ = \frac{\hbar \eta \Omega}{2} \left( |e\rangle \langle g| a^\dagger + \text{h.c.} \right) \). \hspace{1cm} (1.6)

These Hamiltonians induce unitary dynamics in the CS, which are described by the operators

\begin{align*}
\text{(carrier) } & R_C(\theta, \phi) = \exp \left[ i \frac{\theta}{2} (e^{i\phi}|e\rangle \langle g| + e^{-i\phi}|g\rangle \langle e|) \right] \hspace{1cm} (1.7) \\
\text{(red sideband) } & R_-(\theta, \phi) = \exp \left[ i \frac{\theta}{2} (e^{i\phi}|e\rangle \langle g|a + e^{-i\phi}|g\rangle \langle e|a^\dagger) \right] \hspace{1cm} (1.8) \\
\text{(blue sideband) } & R_+(\theta, \phi) = \exp \left[ i \frac{\theta}{2} (e^{i\phi}|e\rangle \langle g|a^\dagger + e^{-i\phi}|g\rangle \langle e|a) \right] . \hspace{1cm} (1.9)
\end{align*}

where \( \theta = \Omega t \) (\( \theta = \eta \Omega t \) on the sidebands) is the rotation angle of a pulse of duration \( t \), and \( \phi \) is the phase of the laser. The laser phase is arbitrary when the first of a series of pulses is applied on one transition, but it has to be kept track of in all subsequent operations until the final state measurement. Fig. 1.5 shows an example how the quantum state evolves in the CS under the action of subsequent \( R_+ \left( \frac{\pi}{2}, 0 \right) \) pulses.

\[ \begin{array}{c|c|c|c|}
|S, 0\rangle & R^+ \left( \frac{\pi}{2}, 0 \right) & R^+ \left( \frac{\pi}{2}, 0 \right) & \ldots \\
\hline
\frac{1}{\sqrt{2}} (|S, 0\rangle + i|D, 1\rangle) & R^+ \left( \frac{\pi}{2}, 0 \right) & i|D, 1\rangle & \ldots \\
\hline
\frac{1}{\sqrt{2}} (|S, 0\rangle - i|D, 1\rangle) & R^+ \left( \frac{\pi}{2}, 0 \right) & -|S, 0\rangle & \ldots
\end{array} \]

Fig. 1.5 Example for the evolution of a quantum state under application of \( \pi/2 \)-pulses. \( |S\rangle \) and \( |D\rangle \) stand for \( |g\rangle \) and \( |e\rangle \) in the text, indication the implementation with \( \text{Ca}^{2+} \). Note the sign change of the wave function after a \( 2\pi \)-pulse.

An example for coherent state manipulation, or unitary qubit rotations, within the CS of a single ion is shown in Fig. 1.6. This result was obtained with an optical qubit in a single \( \text{Ca}^{2+} \) ion. Analogous operations with a hyperfine qubit in \( \text{Be}^{2+} \) were employed to realise the first single-ion quantum gate between an internal and a motional qubit (Monroe et al., 1995a).

1.5 Quantum gates

The central concept around which quantum gates with trapped ions are constructed are the common modes of vibration. Sideband excitation of one ion modifies the motional state of the whole string and has an effect on subsequent sideband interaction with all other ions, thus providing the required ion-ion coupling. Due to the strong Coulomb coupling between the ions, the vibrational modes are non-degenerate, therefore they can be spectrally addressed, by tuning the laser to the sideband of one particular mode. In particular, the center-of-mass mode at the lowest frequency \( \nu \) is separated
from the next adjacent mode, the stretch mode, by $(\sqrt{3} - 1)\nu$. Example spectra can be found in Refs. (Schmidt-Kaler et al., 2000; Rohde et al., 2001). Spectral resolution of the sidebands imposes a speed limit on gate operations, which will be discussed in section 2.3.

The generalisation of the computational subspace from the single-ion case is straightforward: the CS of two ions (these can be any two in a longer string) is the product space of their internal qubits and the selected vibrational mode. The resulting states and transitions are displayed in Fig. 1.7.

The role of the vibrational mode is illustrated by considering a basic entangling operation which can be realised with two laser pulses that address the ions individually (see also section 1.6.3). The pulses are defined analogously to Eqs. (1.8-1.9), with the additional superscript (1) or (2) indicating the addressed ion.

$$|g, g, 0\rangle \xrightarrow{R^{(1)}_{\pi} (\pi/2, 0)} \frac{1}{\sqrt{2}} (|g, g, 0\rangle + i|e, g, 1\rangle) \xrightarrow{R^{(2)}_{\pi} (\pi, 0)} \frac{1}{\sqrt{2}} (|g, g, 0\rangle - |e, e, 0\rangle)$$

The pulse sequence creates an entangled state $(|g, g\rangle - |e, e\rangle)/\sqrt{2}$ of the two internal qubits while leaving the motional qubit in the ground state.

Several specific gate protocols within the two-ion CS have been proposed and implemented. The classic Cirac-Zoller gate works in the following way (Cirac and Zoller, 1995): first the state of ion 1 is transferred into the motional qubit by a SWAP
Fig. 1.7 Computational subspace of two ions and one vibrational mode, with all possible transitions. Short, medium, and long arrows are red sideband, carrier, and blue sideband transitions, respectively.

operation (described below). Then a single-ion gate is carried out between the motional qubit and ion 2. Finally the motional qubit state is swapped back into ion 1, which completes the gate between the two ions. The experimental realisations of quantum gates are discussed in section 1.7.

Note that the motion, although it could be regarded as providing one qubit per vibrational mode, always serves only as a "bus" connecting the internal qubits of the ions. In particular, the motional qubits cannot be measured independently. Thus at the end of a logic operation, the motional state has to factorise from the state of the internal qubits, which itself can be entangled.

The SWAP operation exchanges the state of two qubits and is employed, e.g., to write the state of the internal into the motional qubit, like in

\[(a|g⟩ + b|e⟩)(c|1⟩ + d|0⟩) \rightarrow (c|g⟩ + d|e⟩)(a|1⟩ + b|0⟩)\].

It can be obtained by a π-pulse on the blue sideband, but only if the initial state is not \(|g, 1⟩\). Sideband pulses couple to states outside the computational subspace defined in section 1.4, namely \(|g, 1⟩\) to \(|e, 2⟩\) on the blue and \(|e, 1⟩\) to \(|g, 2⟩\) on the red sideband. Moreover the Rabi frequency on the \(|1⟩ ↔ |2⟩\) transitions is larger than on \(|0⟩ ↔ |1⟩\) by a factor of \(\sqrt{2}\). This problem can be solved by composite pulses. This technique is well-known in NMR spectroscopy (Levitt, 1986) and its application for ion traps has been proposed in Ref. (Childs and Chuang, 2001). A composite pulse sequence,

\[R_{SWAP} = R_+(\pi/\sqrt{2}, 0) \ R_+(\pi \sqrt{2}, \phi_{SWAP}) \ R_+(\pi/\sqrt{2}, 0)\ ,\]

where \(\phi_{SWAP} = \cos^{-1}(\cot(\pi/\sqrt{2})) \approx 0.303\pi\).
Fig. 1.8 Phase gate for Cirac-Zoller gate operation. Of the four input states \(|g, g\rangle, |g, e\rangle, |e, g\rangle\) and \(|e, e\rangle\) only the last one is affected by the pulse sequence which consists of a \(\pi\)-pulse on the red sideband on ion 1, a \(2\pi\)-pulse on the red sideband resonant with an auxiliary state on ion 2 and a second \(\pi\)-pulse on the red sideband on ion 1. A dashed arrow indicates a pulse that has no effect, either because the ion is in the wrong initial state or because there is no resonant final state. States \(|g, g\rangle\) and \(|g, e\rangle\) do not couple to any of the three pulses and hence stay unchanged. The phases acquired by \(|e, g\rangle\) with each pulse compensate each other, leaving also this state unchanged. Only \(|e, e\rangle\) acquires an overall phase of -1 and gets transferred to \(-|e, e\rangle\), thus completing the phase gate.

performs the SWAP operation for all states of the CS, by acting as a \(\pi\)-pulse on the \(|g, 0\rangle \leftrightarrow |e, 1\rangle\) transition and simultaneously as a \(4\pi\) rotation on \(|g, 1\rangle \leftrightarrow |e, 2\rangle\). Composite pulses have been used in the implementation of the Cirac-Zoller quantum gate with Ca\(^+\) ions (Schmidt-Kaler et al., 2003b).

The single ion gate that is performed between the motional qubit and ion 2 is a controlled-NOT gate (CNOT). In a CNOT gate the control qubit of the input state, in this case the motional qubit, decides if the target qubit, the atomic qubit of ion 2, is flipped or not. This operation can be realized by a pair of Ramsey pulses enclosing a phase gate. The details of the phase gate used in the Cirac-Zoller proposal are shown in Fig. 1.8. In the first experimental realization of this proposal a composite pulse sequence in analogy to the one used for the SWAP operation was applied (Schmidt-Kaler et al., 2003b).
1.5.1 State detection

At the end of a quantum logical process the state of the qubits has to be determined. This is accomplished by exciting the ions and recording the scattered fluorescence photons on a transition which couples to only one of the qubit levels; see Fig. 1.2 for the example of Ca$^+$. More technical details are given in the next section. State detection is a projective measurement which destroys all quantum correlations. The outcome of one individual measurement is a binary signal for each ion, full fluorescence or no photons, corresponding to either $|g\rangle$ or $|e\rangle$. For finding out the probabilities that the ion is in either of the two states after the logic operation, one has to repeat the measurement many times. Typically 100 or several 100 runs are used. In the example of Fig. 1.6, each data point is an average of 200 single measurements.

In a teleportation experiment, where part of the operations is conditional on the outcome of an intermediate measurement, it is necessary to measure the state of one ion while the other ions are in superposition states. Therefore these ions have to be protected from the resonant light. This is achieved either by separating the ions spatially, which has been shown to be possible without affecting internal-qubit superpositions (Rowe et al., 2002; Barrett et al., 2004), or by a hiding pulse, as has been demonstrated with Ca$^+$ (Riebe et al., 2004; Häffner et al., 2005b). Such a $\pi$-pulse transfers the population from the qubit level in $S_{1/2}$, which might couple to the resonant light on the neighbouring ion, to a Zeeman sublevel of $D_{5/2}$ which is different from the qubit level. After the measurement pulse on the adjacent ion, a $-\pi$-pulse reverses the hiding process and restores the superposition of the qubit.

Further examples for techniques which deal with experimental imperfections are spin echo pulses to counteract the effect of magnetic field fluctuations and gradients (Barrett et al., 2004), and the compensation of Stark shifts which occur during sideband excitation due to non-resonant interaction with the carrier transition (Häffner et al., 2003).

1.6 Experimental techniques

1.6.1 Initial state preparation

After the motional state of the ion has been prepared by the laser cooling techniques explained in section 1.1.3 and 1.3.2, the atomic state may have to be initialised, e.g. to a specific Zeeman sub-level of the ground state. This is achieved by applying a short optical pumping pulse of appropriate polarisation and beam direction, e.g. in Ca$^+$ by a $\sigma^+$-polarised pulse at 397 nm which propagates along the quantisation axis and pumps all atomic population into $S_{1/2}, m = +1/2$, see Fig. 1.2. Such optical pumping may heat up the ion and is therefore alternated with sideband cooling.

1.6.2 Laser pulses

The experimental conditions for qubit manipulations are stringent. Even for a basic operation like a two-ion quantum gate, a sequence of several pulses is required, during which phase-perturbing or decay processes must be negligible. Apart from stable atomic states, this requires the control of environmental magnetic fields, and in particular it puts challenging requirements on the stability of the laser sources. In the
example of the optical qubit, where spontaneous decay of the excited state happens on the 1 s time scale, the laser phase fluctuations are critical. A residual laser linewidth in the 100 Hz range or better is desired in order to maintain phase coherence over a significant number of pulses. To achieve this, sophisticated stabilisation techniques like those developed in the context of optical clocks have to be applied.

With hyperfine qubits, laser stability is still very important but less critical, because there the difference frequency between two light fields is relevant, which is in the GHz range and can be made very stable using microwave oscillators.

Once the phase-stable light field for driving the qubits is generated, the creation of laser pulses of desired amplitude, detuning, phase shift, and duration is comparatively simple. It is illustrated in Fig. 1.9.

Fig. 1.9 Creation of laser pulses. Continuous light from a stable laser (coherence time $\gg$ gate time) is sent through an acousto-optical modulator (AOM), which receives a radio frequency signal from a stable oscillator (RF). The RF frequency $\Delta$ is added to the light frequency, thus defining the laser detuning, the RF phase $\phi$ translates directly into the phase of the light, and the RF power $P$ controls the light intensity $I$, including switching it on and off. Intensity stability may be improved by monitoring the beam at the trap and compensating unwanted changes by a feedback to the RF power. The AOM process is based on Bragg diffraction from a sound wave in a crystal.

1.6.3 Addressing individual ions in a string

The conventional way to manipulate individual qubits in a string of ions, as envisioned in the original Cirac-Zoller proposal, is to use a well-focused laser beam which interacts with only one ion at a time, and which can be directed at any ion in the string. This kind of addressing was first demonstrated in (Nägerl et al., 1999). It is only possible if the distance between the ions is significantly larger than the diffraction limit of the focussing lens. This limits the range of possible trap frequencies and thus the speed of gate operations (Steane et al., 2000). With the 729 nm light exciting the optical qubit in Ca$^+$, focusing to $\sim 2.5 \mu$m can typically be achieved, leading to an ion spacing around 5 $\mu$m (Nägerl et al., 1999). The Rabi frequency on a neighbouring ion may
still reach about 5% of that on the addressed ion, introducing addressing errors in the quantum logic operations. Such systematic errors can, however, be compensated for by adjustments to pulse lengths and phases (Schmidt-Kaler et al., 2003a).

Addressing in gate operations is not indispensable, since protocols have been developed (Sørensen and Mølmer, 1999) and demonstrated (Sackett et al., 2000; Kielpinski et al., 2001) which employ simultaneous laser interaction with both participating ions. The gate used by the NIST group in their recent experiments relies on the action of spin-dependent dipole forces on both ions when they are simultaneously excited (Leibfried et al., 2003). Still, in general a process is required which allows for shielding one ion from the light which excites the adjacent one. The strategy which appears to be most promising is to separate and combine ions in segmented traps, see section 2.1.

1.6.4 State discrimination

The final state read-out on a string of ions, by a measurement as described in section 1.5.1, requires recording the resonance fluorescence of each ion in the string. At the same time, state read-out should happen fast to allow for a high experimental cycle frequency and to avoid state changes by decay processes. Thus high photon detection efficiency at low dark count rate is desired. A photon counting camera resolving the individual ions is the appropriate device. The suitable parameters are found by acquiring a histogram of photon counts corresponding to the positions of the ions on the camera image and to the two possible signal levels. The histogram will show Poisson distributions around the mean numbers of fluorescence and dark counts, and efficient state discrimination is achieved when the distributions do not overlap. As shown in the example of Fig. 1.10, a few 10 photons are sufficient. In the example of Ca\(^+\), in 10 ms a state discrimination efficiency above 99% can be reached (Riebe, 2005).

In experiments without individual addressing, the ions have to be separated using segmented traps for measuring their final states individually (Barrett et al., 2004). Recently it has been demonstrated that speed and fidelity of the state readout may be optimised by using time-resolved photon counting and adaptive measurement techniques (Myerson et al., 2008).

1.7 Milestone experiments

1.7.1 Trapped ions

After preliminary work which already implemented the relevant entangling operations (Sackett et al., 2000; Kielpinski et al., 2001), the first two-ion gates were realised in 2003, simultaneously in Boulder with a hyperfine (Leibfried et al., 2003) and in Innsbruck with an optical qubit (Schmidt-Kaler et al., 2003b; Schmidt-Kaler et al., 2003a). The optical implementation with Ca\(^+\) ions was very close to the original Cirac-Zoller proposal, while the hyperfine version with Be\(^+\) used a protocol based on state-dependent forces, similar to the scheme proposed by Sørensen and Mølmer (Sørensen and Mølmer, 1999).

A spectacular result in 2004 was the first deterministic quantum teleportation of the state of an atom, which again was achieved by the same two groups simultaneously (Barrett et al., 2004; Riebe et al., 2004). A diagram which illustrates the procedure
Milestone experiments

Fig. 1.10 Example histogram for state discrimination with a single Ca$^+$ ion (c.f. (Riebe, 2005)). The ion is prepared with $\sim 70\%$ probability in the $S_{1/2}$ state, which fluoresces, and with $\sim 30\%$ probability in $D_{5/2}$, which is dark. Resonance fluorescence photons are collected for 9 ms. The experiment is repeated several 1000 times. The separation between the two Poisson distributions allows to define a clear threshold that discriminates between the possible outcomes.

nicely is found in (Kimble and van Enk, 2004). The NIST experiment involved shuttling ions between zones of a segmented trap, in order to allow for both individual and simultaneous laser manipulation of the ions. This can be considered as a first step towards scalable trap architecture.

The first realisation of a decoherence-free subspace (DFS) was reported in Ref. (Kielpinski et al., 2001). Very long coherence times, of many seconds, of entangled states and states in DFSs have been demonstrated in Refs. (Langer et al., 2005) and (Häffner et al., 2005), and recently a quantum gate within such a DSF was realised (Monz et al., 2009b).

Several basic quantum logical operations were also demonstrated, such as quantum dense coding (Schaetz et al., 2004), quantum error correction (Chiaverini et al., 2004), the quantum Fourier transform (Chiaverini et al., 2005b), the Toffoli Gate (Monz et al., 2009a), and entanglement purification (Reichle et al., 2006). Specific entangled 2-ion (Roos et al., 2004a) and 3-ion (Roos et al., 2004b) quantum states have been deterministically created and characterised by quantum tomography. Also multi-ion entangled states, like an 8-ion entangled W state forming a “quantum byte” (Häffner et al., 2005a) or 6-ion GHZ states (Leibfried et al., 2005), have been created and characterized. Just recently the first quantum simulations with cold trapped ions were realized (Friedenauer et al., 2008; Gerritsma et al., 2010). Moreover, a programmable two-qubit quantum processor capable of performing arbitrary unitary transformations on a certain set of input states was realised (Hanneke et al., 2010).
Ion (and atom) quantum logic

1.7.2 Trapped atoms

For implementing neutral atom quantum gates, various proposals have been put forward including short range dipolar interactions (Brennen et al., 1999), ground state collisions (Jaksch et al., 1999), cavity QED coupling (Pellizzari et al., 1995), magnetic dipole-dipole interactions (You and Chapman, 2000), gates with delocalized qubits (Mompart et al., 2003), and Rydberg state mediated dipolar interactions (Jaksch et al., 2000). Promising progress has recently been reported with the latter type of approach. Rydberg atoms show long-range dipolar interaction which may be used to couple atoms in independent dipole traps by the so-called Rydberg blockade (Urban et al., 2009; Gaëtan et al., 2009). Two-atom gates (Wilk et al., 2010; Isenhower et al., 2010) have been demonstrated using this mechanism.
2 Scalability

After basic few-ion operations have been demonstrated and simple quantum algorithms have been implemented, the question of scalability has to be addressed seriously. One very important aspect of it are quantum error correction codes, which where discovered simultaneously by Shor (Shor, 1995) and Steane (Steane, 1996). Building an ion trap quantum computer which implements full error correction appears to be an overwhelming task. In this section we summarise ideas and results which are expected to contribute to meeting this challenge. Three main directions are considered, ion shuttling in segmented traps, sympathetic cooling, and fast gates.

2.1 Trap architecture

Addressing ions in a static string by steering a laser beam becomes technically difficult as the number of ions increases. Moreover, the conventional two-ion gate protocol employing a vibrational mode of a string becomes difficult to implement as the size of the string grows, because the increasing mass of the ion crystal reduces, through the Lamb-Dicke parameter $\eta$, the coupling on the sideband transitions. Therefore a trap architecture seems favourable which allows for moving the ions. In this scenario, ions are picked from a register and shuttled by time-dependent trap voltages into an interaction zone where two-ion gates are carried out of the kind used in (Barrett et al., 2004). A detailed scheme has been proposed by the NIST group (Kielpinski et al., 2002), and several other groups have started activities to simulate, build, and test various implementations (Metodiev et al., 2005; Steane, 2007; Home and Steane, 2006; Chiaverini et al., 2005; Microtrap, 2006). A notable experimental achievement has been reported by the Michigan group who shuttled an ion around a corner in a T-junction (Hensinger et al., 2006), and just recently the internal-state coherence of an ion could be preserved while transport through a X-junction at NIST (Blakestad et al., 2009). Some important issues of the transport and shuttling through junctions of ions in multidimensional trap arrays are covered in (Hucul et al., 2008). At the moment different strategies to build scalable micro traps are followed in parallel. Some experiments use fabrication techniques from semiconductor technology to build microfabricated traps integrated into silicon (Brownmatt et al., 2006), gallium-arsenide (Stick et al., 2006) or gold coated aluminium oxide (Schulz et al., 2008) based microchips. Another way is to keep fabrication procedure and trap design as simple as possible with regard to enabling large-scale quantum information processing. This can be achieved by employing 2D surface electrode structures which are realised by vapor deposited gold on glass substrates (Seidelin et al., 2006; Allcock et al., 2009) or vacuum
Scalability

compatible printed circuit boards (Brown et al., 2007; Splatt et al., 2009). The problem that all micro traps encounter is that, due to the vicinity of ion and electrodes, the heating rates are relatively high with respect to macroscopic traps. One attempt to overcome this problem is to go to cryogenic temperatures (Deslauriers et al., 2006; Labaziewicz et al., 2008). Recent progress in the fast moving field of trap technology includes the study of fast ion transport (Huber et al., 2008), which is important for the speed of quantum algorithms, and an experiment which shows coherent manipulation and sideband cooling close to the motional ground state of a single ion in a micro trap (Poschinger et al., 2009).

2.2 Sympathetic cooling

Part of the future trap technology will be to apply new means of cooling the ions without perturbing their quantum state, i.e. without scattering photons from them. This can be achieved by sympathetic cooling, when the collective modes of a string are cooled by scattering photons only from ions which do not participate in the quantum logic process. Sympathetic Doppler cooling was demonstrated some time ago with Hg$^+$-Be$^+$ mixtures (Larson et al., 1986) and with Mg$^+$ (Bowe et al., 1999). Ground state cooling of a two-ion crystal through one of the ions was reported in (Rohde et al., 2001). Further relevant results were obtained with mixtures of Cd$^+$ isotopes (Blinov et al., 2002) and with a Be$^+$-Mg$^+$ crystal which was cooled to the motional ground state (Barrett et al., 2003). More recently, sympathetic cooling of a Be$^+ – Al^+$ crystal was employed to perform spectroscopy on Al$^+$ using quantum logic (Schmidt et al., 2005; Rosenband et al., 2007). In the future this may lead to an accurate frequency standard based on Al$^+$. Combination of sympathetic cooling with other scalable methods has already been demonstrated (Home et al., 2009).

2.3 Fast gates

Another major contribution to scaling up ion trap QIP is expected from faster gate operations. The current limitation is the use of spectrally resolved sideband resonances, which requires individual laser pulses to be longer than the inverse trap frequency. Smaller traps with larger frequencies can improve the situation, but theoretical and numerical studies indicate that one could in fact use significantly shorter, suitably designed pulses or pulse trains (García-Ripoll et al., 2003; Duan, 2004; Zhu et al., 2006). Their action is, rather than to excite one mode, that they impart photon momentum kicks and thus interact with a superposition of many modes or even with the local modes of one ion, i.e. its local vibration as if all other ions were fixed (Zhu et al., 2006). It will be interesting to see how ideas of quantum coherent control can help advancing into this direction (García-Ripoll et al., 2005; Nebendahl et al., 2009). An interesting novel approach is to employ an optical frequency comb for fast coherent qubit control (Hayes et al., 2010).
An important field of activity related to QIP with trapped ions is qubit interfacing, i.e. linking static quantum information stored in ionic qubits to propagating quantum information stored in photonic quantum states. Such quantum interaction between single trapped ions and single photons is a central objective for many QIP schemes like quantum networking (Kimble, 2008), distributed quantum computing or quantum communication. At present the main limitation of photon-based quantum communication is the loss of photons in the quantum channel. This limits the distance of a single quantum link to about 250 km with present technology (Stucki et al., 2009). A solution to this problem is to subdivide larger distances into smaller sections over which photons can be faithfully transmitted to create remote atomic entanglement. A quantum repeater situated at each of the nodes of the quantum network would then extend the entanglement to longer distances (Briegel et al., 1998). To accomplish this, a fully coherent and efficient state transfer between flying and stationary qubits, as well as a faithful transmission between the specific locations is necessary 1.

First progress was achieved using atomic ensembles as quantum memory (Julsgaard et al., 2001; Julsgaard et al., 2004; Chou et al., 2005; Chanielière et al., 2005; Eisaman et al., 2005). Compared to single-qubit implementations in ions these systems have the disadvantage that there is no scheme for (scalable) local processing of the stored information.

Atom-photon qubit interfacing can be achieved through a variety of different techniques. The first proposal of coherent coupling of ionic qubits to photonic modes involved a high-finesse resonator (Cirac et al., 1997). Direct photon exchange between remote ions or deterministic gate operations can be realized by such a strongly coupled atom-cavity system. A potentially less complex alternative for efficient atom-light coupling are high numerical aperture optics in free space. There are a number of interesting approaches towards this technique (Tey et al., 2009; Tey et al., 2008; Maiwald et al., 2009; Wrigge et al., 2008; Zumofen et al., 2008; Stobinska et al., 2009).

As mentioned, one application of this interfacing is a quantum network of small ion trap processing units, coupled with each other by photons. The currently most promising method to realize a quantum network is to create entangled ion-photon states by a projective measurement of spontaneously emitted photons. Subsequently, the interference of such single photons can generate heralded entanglement of remote

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1 The QIPC Strategic Report (Zoller et al., 2005) highlights the development of quantum interfaces and repeaters as an integral part of full scale quantum information systems and identifies their realization as one of the key tasks for the future.
ions (Cabrillo et al., 1999; Bose et al., 1999; Feng et al., 2003; Duan and Kimble, 2003; Simon and Irvine, 2003). Deterministic quantum gate operations between two remote logic ions would then be realized by probabilistically entangling two distant ancilla ions and performing deterministic local gates between neighboring logic and ancilla ions (Duan et al., 2004).

3.1 Experiments

In this section we will briefly review some existing approaches and results in qubit interfacing and quantum networking with single atoms, including both trapped ions and trapped neutral atoms.

The direct coupling of a trapped ion to a high-finesse optical cavity was realised in two groups. The MPQ group demonstrated coupling on a dipole transition (Guthöhrlein et al., 2001; Keller et al., 2003), which was then applied to the creation of single photons with tailored wave packets (Keller et al., 2004). In Innsbruck, Rabi oscillations on a qubit transition induced by a coherent cavity field were observed, and cavity sideband excitation was demonstrated (Mundt et al., 2002; Mundt et al., 2003). In the same group, cavity-assisted Raman spectroscopy on the D to P dipole transition was realised (Russo et al., 2009). Here, the vacuum field of the cavity together with a driving laser stimulate a Raman transition from the D\(_{3/2}\) to the S\(_{1/2}\) state in \(^{40}\)Ca\(^+\). Moreover, with the same system a single-photon source from a single ion was realized recently (Barros et al., 2009).

Earlier, a single photon source was implemented with neutral atoms falling through a high finesse cavity (Kuhn et al., 2002; McKeever et al., 2004). Due to the difficulty of integrating ion trap electrodes and a resonator setup, neutral atom experiments with cavities have been historically more accessible. In state of the art experiments, neutral atoms are trapped within the cavity mode by far off-resonant dipol traps (FORT), which is either formed by modes of the cavity itself (Boozer et al., 2007) or by laser beams transverse to the cavity axis (Nussmann et al., 2005). With such a system a coherent state of light was reversibly mapped onto the hyperfine states of an atom trapped within the mode of a high-finesse cavity (Boozer et al., 2007). In an ideal qubit interface the coherent state of light would have to be replaced by a single-photon state.

Neutral atoms have also been used in attempts to reach efficient atom-photon coupling by the use of high numerical aperture optics (Schlosser et al., 2001; Weber et al., 2006; Tey et al., 2008). An example of what can be achieved by this method is the observation of the attenuation and the phase shift of a weak coherent beam induced by a single atom (Tey et al., 2008; Tey et al., 2009). The central challenge of this approach is to mode-match the incoming light to the emission pattern of a single atom (Quabis et al., 2000; van Enk, 2004; Sondermann et al., 2009). One interesting proposal is to use a single trapped ion inside a parabolic mirror (Sondermann et al., 2007). The inward-moving wave is transformed into a time reversed, radially polarized dipole wave which in principle could excite a single ion, trapped in the focus of the mirror with a new type of Paul trap (Maiwald et al., 2009), with an efficiency of up to 100%. There is a number of promising ideas of how to maximize the numerical aperture of lenses for ion-photon coupling, e.g. it has been proposed to use parabolic integrated mirror-ion-trap structures (Luo et al., 2009) or spherical mirrors in combination with
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aspheric correction optics (Shu et al., 2009a; Shu et al., 2009b). A technique applied with quantum dots and single molecules, but less convenient to combine with atom or ion traps is the use of index-matched GaAs immersion lenses (Vamivakas et al., 2007; Wrigge et al., 2008).

An important step towards quantum networking was the demonstration of the entanglement between an ion and the polarisation of an emitted photon (Blinov et al., 2004). The ion decays under spontaneous emission of a photon from an exited state into one of two different hyperfine ground states. By selecting a certain emission direction with an aperture, the photon polarization is entangled with the final state of the deexcited ion. This result was reproduced with neutral atoms in a single atom dipole trap (Volz et al., 2006) and in a high finesse cavity (Wilk et al., 2007). Using two ions entangled with their emitted photons, it was then possible to postselectively create remote entanglement of two $^{171}\text{Yb}^+$ ions (Moehring et al., 2007). The quantum interference of indistinguishable photons was used in this experiment to perform a Bell measurement which projects the ions into an entangled state. This seems to be the most

![Fig. 3.1](image_url)  
Fig. 3.1 Setup for entangling the internal states of two distant atoms by measurement of two photons (Feng et al., 2003; Duan and Kimble, 2003; Simon and Irvine, 2003). The atoms are prepared in the state $|r_1, r_2\rangle$ and then decay spontaneously. The photon wavepackets overlap at a 50:50 beam splitter (BS). The detection apparatus at the output ports of the BS involves two polarizing beam splitters (PBS) and four detectors. Coincident clicks at $D_{+,e}$ and $D_{+,g}$ or $D_{-,e}$ and $D_{-,g}$ project the atoms into the state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|e_1, g_2\rangle + |e_2, g_1\rangle)$. The state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|e_1, g_2\rangle - |e_2, g_1\rangle)$ is found by coincident clicks at $D_{+,e}$ and $D_{-,g}$ or $D_{+,g}$ and $D_{-,e}$. Two indistinguishable photons will always leave the beam splitter through the same output port (Hong et al., 1987) and thus do not cause coincidence detection.
promising way to establish distant entanglement as a resource for quantum information processing. A generic scheme is explained in figure 3.1. Another method (Cabrillo et al., 1999), which can be more efficient for small photon detection efficiencies (Zippilli et al., 2008), is based on the indistinguishability of photon scattering from the two ions, as explained in Fig. 3.2. The drawback of this method is that it requires interferometric stability, which is experimentally challenging and most likely the reason why this scheme has not yet been realised. The latest highlight is the demonstration of a specific teleportation protocol based on the heralded entanglement of the atoms through two-photon interference, as described above (Olmschenk et al., 2009).

The combination of local quantum logic operations in ion or atom traps with atom-photon interfaces which establish connections between distant traps is certainly an exciting aspect of the future perspectives of quantum computing with single ions or atoms.

### 3.2 Entanglement transfer

The currently most efficient and practical resource of remote entanglement is pair photon generation by spontaneous parametric down-conversion. Using atom-photon qubit interfacing offers thus a promising alternative to create atomic remote entanglement by entanglement transfer. The first proposals for such entanglement distribution involved atom-light coupling via resonators (Lloyd et al., 2001; Kraus and Cirac, 2004). In con-

![Fig. 3.2 Schematic experimental set-up for postselective entanglement creation between distant trapped ions (Cabrillo et al., 1999). After preparation of both ions in a state $|$0$\rangle$, a laser pulse excites a state-changing transition $|$0$\rangle \rightarrow |1\rangle$ with small probability $\epsilon$, thus producing the state $|0,0\rangle + \epsilon( |0,1\rangle + |1,0\rangle ) + \epsilon^2 |1,1\rangle$. Part of the scattered light from both ions is coherently superimposed on a beam splitter. A photon detection event behind the beam splitter (of $\pi$ polarisation in the example) does not allow to determine which ion changed its state, and therefore signals creation of the entangled state $(|0,1\rangle + |1,0\rangle)/\sqrt{2}$.


Fig. 3.3 Scheme for transferring photon-photon to atom-atom entanglement. A spontaneous parametric down-conversion source creates the polarization-entangled state $\Psi^- = \frac{1}{\sqrt{2}}(|\sigma^-\rangle|\sigma^+\rangle - |\sigma^+\rangle|\sigma^-\rangle)$ of 854 nm photons, resonant with the D$_{5/2}$ to P$_{3/2}$ transition in $^{40}$Ca$^+$. Each partner photon is sent to one of two independently trapped $^{40}$Ca$^+$ ions, both of which have previously been prepared in a coherent superposition of the $m = -3/2$ and $m = 3/2$ Zeeman sublevels of the metastable D$_{5/2}$ state. If both ions absorb the photons, their population is transferred to either the P$_{3/2}$, $m = -1/2$ or the P$_{3/2}$, $m = +1/2$ state, depending on the polarization of the absorbed photon. The P$_{3/2}$ state decays rapidly into the S$_{1/2}$ ground state. From the corresponding spontaneous photons only those from $\pi$ transitions are filtered, such that a coincidence detection of two 393 nm photons projects the system into the entangled atom-atom state $\Psi^- = \frac{1}{\sqrt{2}}(|\pi^+\rangle_1|\pi^-\rangle_2 - |\pi^-\rangle_1|\pi^+\rangle_2)$ composed of the Zeeman sublevels of the S$_{1/2}$ ground state.

Contrast to the entangling protocol discussed in section 3.1, which relies on spontaneous emission of single photons by single ions and a subsequent projective measurement, entanglement transfer requires efficient absorption of single photons by single ions. Both processes are complementary and need to be integrated in the same system for efficient bi-directional qubit interfacing.

The scheme in figure 3.3 describes in detail one possible implementation of entanglement transfer from photons to $^{40}$Ca$^+$ ions. Two independently trapped ions each absorb one of the two entangled partner photons from a SPDC source, which transfers their population to an excited state. Both ions have been prepared such that coincidence detection of the subsequently spontaneously emitted photons projects the ions into an entangled state.

A number of groups are currently developing nonclassical light sources tailored to interact with atoms for quantum information applications (Bao et al., 2008; Wollgramm et al., 2008; Neergaard-Nielsen et al., 2007). In our group we have implemented a tunable narrowband entangled photon pair source which is designed to interact with $^{40}$Ca$^+$ and which is intended to be used to realise the scheme explained in figure 3.3.
(Haase et al., 2009; Piro et al., 2009).

As a first step towards entanglement transfer it was possible to show resonant interaction of a single ion with the single photons from this down-conversion source (Schuck et al., 2010). Employing a quantum jump scheme and using the temperature dependence of the down-conversion spectrum as well as the tunability of the narrow source, absorption of the down-conversion photons was quantitatively characterized.

In an extension of this work the efficiency of the atom-photon interaction was increased using a pulsed scheme on the D_{5/2} to P_{3/2} transition. The observation of a time correlation between absorption events and the detection of the filtered partner photons (Piro et al., 2010) indicate that entanglement transfer is within the realm of possibility.
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