Hendrik Baers

University of Bonn

Reducing stability questions for a nonlocal to a local Calderón type problem

The Calderón problem is one of the classic examples of an inverse problem. It is about determining the conductivity of a medium by making voltage and current measurements on its boundary. Some of the main questions of interest are about uniqueness and stability of this reconstruction. In this talk, we will consider the fractional (or nonlocal) and the local formulation of a Calderón type problem and discuss how these two are related. In particular, we will show that stability estimates for the local problem can be transferred to stability estimates for the nonlocal problem. The results presented in this talk are based on joint work with Angkana Rüland.

Leon Bungert

University of Würzburg

Sparsity and robustness in learning: insights from inverse problems

In this talk I will discuss how the training of sparse and robust neural networks benefits from techniques developed in the context of inverse problems. First, I will present a method to train sparse neural networks in an inverse scale space manner based on Bregman iterations, originally developed in the context of compressed sensing and image processing. For this we embed Bregman iterations into a stochastic optimization framework and propose novel momentum-based and Adam-type algorithms. Second, I will explain how adversarial training, a robust training method, can be recast as variational regularization problem with a non-local and data-dependent total variation regularizer. Combining this with Gamma-convergence techniques, one can prove that for vanishing adversarial budget the method converges to a Bayes classifier with minimal total variation.

Sarah Eberle-Blick

KU Eichstätt-Ingolstadt

Reconstructions of inclusions in elastic bodies based on experimental data

We solve the inverse problem of the time-harmonic wave equation by means of monotonicity methods. In more detail, we start with the standard monotonicity method for shape reconstruction of inclusions in elastic bodies by using time-harmonic elastic waves. In order to accelerate the standard monotonicity method, we derive a linearized version. The linearized method provides an efficient version of the method, drastically reducing computation time. In addition, we present reconstructions based on experimental data.

Ole Loseth Elvetun

Norwegian University of Life Sciences

Weighted group sparsity for an inverse EEG problem

A classical inverse problem in EEG is to recover one, or a few, dipole source(s) from voltage recordings on the scalp. This problem is well-suited for weighted sparsity techniques. However, since each dipole involves both a spatial and a orientational component, it will require three "basis" dipoles at each position to represent a freely oriented dipole, which makes it difficult to apply standard sparsity regularization.
We therefore suggest to solve this problem using a group sparsity method - also known as group LASSO - where each group consists of the three "basis" dipoles, which provides a rotational invariant penalty at each position. Using this approach, we prove that any single dipole - independent of rotation - can be recovered, and that under a certain disjointness condition, also several dipoles can be recovered. This condition essentially says that the recoverable dipoles must be far apart.
We illuminate the theoretical findings with several numerical examples.

Simon Hackl

University of Linz

Sound speed and layer adapted focusing methods in medical ultrasound

Focused ultrasound is a widely used non-invasive diagnostic and therapeutic tool in modern medicine. A crucial assumption in of its all applications is a constant sound speed in the observed medium. Non-constant sound speeds lead to actual times of flight of the ultrasound waves through the medium differing from calculated times of flight, which are accounted for in focusing algorithms. This leads to an aberrated focus, blurring ultrasound images. As real-time ultrasound imaging is computationally expensive, a fast aberration correction method is needed. In this talk, we present adapted ultrasound focusing algorithms based on geometrical acoustics that make a step into this direction. In a known layered medium setting, it is possible to calculate the correct times of flight. The resulting adapted focusing algorithms correct for the aberrations caused by the different sound speeds in the medium layers. Numerical simulations to determine the precision of our methods are conducted. And finally, the improvements obtained by our Methods in reconstructing Ultrasound Images are demonstrated.

Thorsten Hohage

University of Göttingen

Phase contrast imaging with partial coherence: Uniqueness and reconstructions from intensity correlations

X-ray phase contrast imaging is a powerful tool yielding nanometer scale resolution of large volumes of biological tissue, opening new applications in digital histopathology. Traditionally phase contrast imaging data are analyzed under the assumption of a perfectly coherent incident beam, an approximation well justified for synchrotron radiation. For high-throughput

experiments required in clinical applications, however, one must rely on laboratory sources with significantly reduced coherence properties, entailing a substantial loss of resolution. To mitigate this limitation, we investigate the use of time-resolved intensity measurements, which provide access to intensity correlations. The purpose of this talk is to examine and exploit the information contained in such correlations, particularly in comparison with standard mean-intensity data. Whereas in the case of perfect coherence intensity correlations carry no additional information compared to mean intensities at all, our numerical experiments on synthetic data demonstrate a substantial increase in resolution in the case of partial coherence. On the theoretical side we establish a uniqueness result for this imaging modality under a geometric assumption ruling out certain symmetries between the supports of the incident beam and the sample. On the algorithmic side we propose an algorithm avoiding the computations of intensity correlations from raw data in a preprocessing step while using the full information content of these correlations. Whereas the size of intensity correlations grows quadratically with the number of pixels, the complexity of our algorithm scales linearly with the number of pixels and quadratically with the rank of the covariance matrix of the incident beam. In view of typical pixel numbers in the order of millions, this linear complexity is crucial for applications to real data.

Simon Hubmer

University of Linz

The SCD semismooth* Newton method for the efficient meinimization of Tikhonov functionals

We consider the efficient numerical minimization of Tikhonov functionals with nonlinear operators and non-smooth and non-convex penalty terms, which appear e.g. in variational regularization. For this, we consider a new class of SCD semismooth* Newton methods, which are based on a novel concept of graphical derivatives, and exhibit locally superlinear convergence. We present a detailed description of these methods, and provide explicit algorithms in the case of sparsity (ℓ_p, $0 \leq p < \infty$) and TV penalty terms. The numerical performance of these methods is then illustrated on a number of tomographic imaging problems.

Tim Jahn

TU Berlin

Convergence of heuristic parameter choice rules for statistical inverse problems

Choosing the regularization parameter typically requires an a priori upper bound on the noise level. Heuristic rules such as the heuristic discrepancy principle and generalized cross-validation (GCV) avoid this requirement, but Bakushinskii's veto shows that in a worst-case (deterministic) error analysis they cannot guarantee convergence. Despite this, they work well in practice. In this talk I consider statistical linear inverse problems and show that both the heuristic discrepancy principle and GCV are in fact convergent.

Ming Jiang

University of Peking

Summary of the Sino-German Mobility Programme (M-0187)

This presentation is a summary about the Sino-German mobility programme (M-0187) funded by Sino-German Center for Research Promotion, from 2021.01.01 – 2025.12.31. The Sino-German Center is an agency under the National Science Foundation of China (NSFC) and Deutsche Forschungsgemeinschaft (DFG). The aim of this mobility programme is to facilitates bilateral cooperation and to build up an active collaboration on a longer timescale. Our project is entitled "Inverse Problems: Theories, Methods and Implementations". This summary, jointly prepared by Peter Maass and Ming Jiang, will cover the organization and progress of the project, and focus more on preparing the final report of this project.

Marvin Knöller

University of Helsinki

A computational method for the inverse Robin problem with convergence rate

In this talk we consider the inverse Robin problem, which is the determination of the Robin parameter a appearing in an elliptic partial differential equation's boundary condition. Let $\Omega \subset \mathbb{R}^2$ be bounded and sufficiently regular. Suppose that we know the solution to the Robin problem

$$\begin{aligned} \Delta u &= f \quad \text{in } \Omega\,, \\ \partial_\nu u + au &= g \quad \text{on } \partial\Omega \end{aligned}$$

only on a small subdomain $\omega \subset \Omega$ as well as the right hand sides $f \in L^2(\Omega)$ and $g \in H^{1/2}(\partial\Omega)$. Under the main assumption that the Robin parameter lies in a finite dimensional space of continuously differentiable functions, we establish a Newton-type algorithm for its reconstruction. This reconstruction algorithm requires first order piecewise continuous finite elements only. We show that the algorithm converges quadratically in the finite element's mesh size to the unknown Robin parameter. Moreover, we establish a perturbation analysis. We study several numerical examples that highlight the efficacy of our approach. Furthermore, we verify our theoretical convergence rates numerically.
This talk is based on joint work with Erik Burman (University College London) and Lauri Oksanen (University of Helsinki).

Jaakko Kultima

RICAM Linz

Wavelet-based reconstruction for Bayesian inverse problems in photoacoustic tomography

Photoacoustic tomography (PAT) is an imaging technique that combines the strong contrast of optical imaging with the high resolution of ultrasound. To fully exploit the resolution of the acoustic part, a large-scale inverse problem must be solved to obtain a reconstruction. In this talk, we approach the acoustic inverse problem in PAT from a Bayesian perspective. This framework allows us not only to reconstruct images but also to quantify the uncertainty in the reconstructions. To promote smoothness, we consider priors from the Matérn class of covariances, and highlight a wavelet-based implementation of these covariance functions. The benefit of this approach is demonstrated numerically through improved resolution.

Peter Maass

ZeTeM, University of Bremen

Status of the osteoporosis project

Osteoporosis is one of the most widespread common diseases, with significant economic implications and considerable restrictions in the lives of those affected. The planned FRACTURE project aims to improve osteoporosis diagnostics. Based on data from the DVO Osteoporosis Registry, patients with manifest osteoporosis will be analyzed. We aim to assess the risk of subsequent fractures over the next three, five, or ten years, also depending on the type of therapy (antiresorptive, bone-forming, or no therapy).
This presentation will report on the current status of data processing and the planned next steps.

Volker Michel

University of Siegen

Motivating children and adults for inverse problems – suggestions and experiences

As we all know, inverse problems occur in many applications and are central tools in science and engineering. However, major parts of the general public are not aware what an inverse problem is. In the same sense, they do not realize that, for example, computerized tomography would be impossible without advanced mathematics.
However, for motivating pupils to learn mathematics, inverse problems are useful, because they make it relatively easy to demonstrate why mathematics is needed for practical purposes. Moreover, in view of the current crisis in the acceptance of science, people of all ages can be shown how scientific results, for instance about climate change, are obtained or, more precisely, calculated.
As a matter of fact, many people are aware that mathematics is a rigorous science and they would not dare to doubt the validity of a mathematical theorem. Therefore, showing how mathematics is hidden in scientific work (which is often connected to inverse problems) can also increase the confidence in science.
For many years, the speaker has obtained experiences in science communication by giving talks to different audiences (from very young to very aged). He reports about his experiences and shows examples of inverse problems which can be used in lectures for the general public or children in particular.

Diana-Elena Mirciu

University of Klagenfurt

Higher order error estimates for regularization of inverse problems under non-additive noise

This talk focuses on higher order error estimates in terms of Bregman distances for inverse problems distorted by non-additive noise. The results are obtained by means of a novel source condition, inspired by the dual problem. Specifically, we focus on variational regularization having the Kullback-Leibler divergence as data-fidelity, and a convex penalty term. In this framework, we provide an interpretation of the new source condition, and present error estimates also when a variational formulation of the source condition is employed. We show that this approach can be extended to variational regularization that incorporates more general convex data fidelities.

Bjorn Fredrik Nielsen

Norwegian University of Life Sciences

Weighted TV-regularization

We consider inverse problems involving large null spaces. This is relevant for many applications, e.g., for the inverse ECG and EEG problems. Standard regularization techniques typically yield solutions which are either in, or close to, the orthogonal complement of the null space of the (linear) forward operator. This will often not provide adequate solutions, and in many cases internal sources are erroneously misinterpreted as being located close to where the data is recorded, e.g., close to the boundary (head and chest surfaces for EEG and ECG, respectively).
We have previously investigated how this phenomenon can be rectified, by introducing suitable weighting schemes, when Tikhonov or sparsity regularization are employed. The purpose of the present work is to extend this methodology to TV-regularization since this technique is well suited for identifying larger regions of (constant) anomalies.
In our talk we define the weighted TV-regularization method, present some

analysis and discuss a series of numerical experiments. Even though the standard approach fails to recover sources far from the boundary, its weighted counterpart handles such cases reasonably well: The location and size of the source are approximately identified but the shape cannot be accurately computed.
We will also present some theoretical and computational results for a hybrid method, combining TV-regularization and weighted sparsity regularization.

Alice Oberacker

Saarland University Saarbrücken

Reducing motion artifacts in nano-CT imaging with a learned RESESOP method

Tomographic X-ray imaging at the nano-scale helps to reveal the structures of materials like alloys and biological tissue. However, environmental perturbances during data acquisition can cause motion between the object and scanner. To reduce noise in the back-projection, a learned version of the RESESOP method was investigated. The deep network architecture unrolls the iterative reconstruction process and was trained with simulated imaging data, allowing the network to learn the back-projected image for a fixed number of iterations.

Florian Oberender

University of Göttingen

Stability of inverse problems in PINEM quantum tomography

Recent advancements in photon induced near-field electron microscopy (PINEM) enable the preparation, coherent manipulation and characterization of free-

electron quantum states. The available measurement consists of electron energy spectrograms and the goal is to reconstruct a density matrix which represents the quantum state. This requires the solution of an ill-posed inverse problem where a positive semi-definite trace-class operator is reconstructed given its diagonal in different bases. We analyze the stability of this problem and exploit the regularizing effect of the positive semi-definiteness constraint. Additionally we investigate numerical methods for the solution of inverse problems posed on positive semi-definite hermitian operators.

Lauri Oksanen

University of Helsinki

Optimality of stabilized finite element methods for elliptic unique continuation

We consider finite element approximation in the context of the ill-posed elliptic unique continuation problem and introduce a notion of optimal error estimates that includes convergence with respect to a mesh parameter and perturbations in data. The rate of convergence is determined by the conditional stability of the underlying continuous problem and the polynomial order of the finite element approximation space. We present a stabilized finite element method satisfying the optimal estimate and discuss a proof showing that no finite element approximation can converge at a better rate. The talk is based on joint work with Erik Burman and Mihai Nechita.

Amine Othmane

Saarland University Saarbrücken

Identification of cost functionals and stabilizing feedback laws in optimal control problems

Many natural processes and engineered systems can reasonably be assumed to exhibit optimal behavior. For example, the process of learning to walk in early childhood, the diversity of individual driving behaviors observed in traffic, and the decision-making strategies employed by experienced medical professionals in minimally invasive surgery planning all suggest an underlying optimization principle. However, the specific criterion governing this optimization is not readily apparent from observing the data from the state trajectory. Consequently, reconstructing the associated cost functionals constitutes an inverse problem of significant complexity. Understanding these costs functionals is crucial for both theoretical and practical applications, as it enables the prediction, control, and improvement of system performance in diverse domains, from robotics and neuroscience to transportation and healthcare.

In this work, we consider a general control system and assume access to observed optimal state trajectories, without requiring direct measurements of the control inputs. We formulate the problem within an inverse optimal control framework, where the objective is to infer the underlying value function, feedback law, and cost structure.

For linear systems, this formulation is closely related to identifying the optimal feedback law. To ensure stability, we employ optimization techniques on manifolds, which guarantee that the learned feedback policy is globally asymptotically stabilizing.

For nonlinear systems, our methodology utilizes structure-preserving neural network approximations to model both the value function and the state-dependent weighting of a control cost. The stability and optimality properties are enforced by incorporating constraints derived from Hamilton-Jacobi-Bellman and Lyapunov stability theory. The learning process is designed to minimize violations of these fundamental conditions while remaining consistent with observed data.

We validate the proposed framework through numerical simulations, demonstrating its ability to reconstruct value functions and cost structures that yield stable optimal control strategies. This approach offers insights into the underlying decision-making process of an observed controlled system and is particularly relevant for applications in robotics, autonomous systems, and data-driven control synthesis.

Tim-Jonas Peter

University of Siegen

Modelling of the earth's gravitational response to surface loading: an overview of available methods for Love number calculation

Gravitational data from satellites in Earth's orbit can be used to reconstruct the secular and periodic movements of mass on the surface, these effects can be the tides caused by the moon, seasonal variations in rainfall (a wet season) or the melting of glacial ice on the poles. All of these mass transports of course cause variations in the gravitational signal, but they also have to be considered as loads on the planet's surface, which will cause an elastic deformation of the Earth's body on short timescales. The so called Love numbers quantify the effect of this deformation in terms of spherical harmonic coefficients. In this presentation, multiple methods for calculating Love numbers are presented and evaluated. We also propose our own way of calculating these coefficients, involving a weak formulation of the equations of motion and a finite element approach.

Jan-Frederik Pietschmann

University of Augusburg

Transport map regularization for source identification

We study inverse source identification problems where the source is described by means of the push forward of a reference measure under a transport map. Our approach parametrizes this map and then solves a Tikhonov-type optimization problem. The reference measure encodes a priori information and an additional regularization term penalizes the distance between it and the reconstructed source.

Robert Plato

University of Siegen

Regularization of linear ill-posed problems with smooth solutions of low order in Banach spaces

In this presentation we consider, in a Banach space framework, the regularization of linear ill-posed problems. Our focus is on the recovery of solutions that allow a logarithmic source representation which is also known as low order smoothness. Such cases typically occur in exponentially ill-posed problems like the backwards heat equation. For a class of regularization schemes, convergence rates are deduced, both for a priori and a posteriori parameter choice strategies. The considered class includes the iterated version of Lavrentiev's method and the method of the abstract Cauchy problem.

E. Todd Quinto

Tufts University Medford

Spherical Radon transforms in tomography

We will analyze a spherical Radon transform, $\mathcal{R}$, which integrates a function over spheres in Euclidean space with arbitrary centers $\mathbf{y}$ and radii $r(\mathbf{y})$ that vary smoothly with $\mathbf{y}$. We prove conditions under which the normal operator, R^*R, (or a localized version) is an elliptic pseudodifferential operator. This gives us Sobolev regularity and stability for inversion. We explain the implications of our theorems and apply it to problems including Compton Scatter Tomography and Ultrasound Reflection Tomography.
This is joint work with James W. Webber (Cleveland Clinic).

Lisa Schätzle

Aalto University

Far field operator splitting for inhomogeneous medium scattering

We consider scattering of time-harmonic acoustic waves by an ensemble of compactly supported inhomogeneous objects, called scatterers, in a homogeneous background medium. The scatterers are illuminated by incident plane waves along all possible illumination directions and, each time, the resulting scattered waves are detected far away along all possible observation directions. This data can be described by the far field operator, which uniquely determines the scatterers in inverse medium scattering. Let the positions of some scatterers be known a priori, but not of others. We then consider the inverse problem of reconstructing the individual far field operators of those scatterers with known positions from the far field operator of the whole ensemble of scatterers. We refer to this as far field operator splitting. Provided the given data is exact this is a uniquely solvable but ill-posed problem. It turns out that the far field operator components of the scatterers with known positions can be approximated by sparse operators with respect to known modulated Fourier bases. The remaining far field operator components are of low rank. However, they are no longer sparse with respect to these modulated Fourier bases. This observation motivates to approximate the solution of the original splitting problem by solving a coupled nuclear norm and $\ell^1 \times \ell^1$ norm minimization problem. Using techniques from the field of compressed sensing, we present a stability analysis for this problem formulation that requires explicit assumptions on the wavenumber and on the positions and sizes of the scatterers. Furthermore, we verify our theoretical predictions by numerical examples.

Bjorn Sprungk

TU Freiberg

Bayesian inverse problems – stability and noise-level robust sampling

The Bayesian approach to inverse problems allows quite elegantly to quantify the uncertainty in data-based reconstructions of an unknown ground truth, and yields to well-posedness. In particular, one can show a local Lipschitz dependence of the solution of the Bayesian inverse problem – the so called posterior, a probability distribution in high- or infinite-dimensional function space – on the noisy data, numerical approximations of forward map and the choice of the prior distribution where the latter can be seen as choice of a probabilistic regularization. However, the price to pay comes in terms of the computationally challenging task of sampling the posterior. In this talk we also present our contributions to Metropolis-Hastings algorithms for Markov chain Monte Carlo sampling in function spaces, discuss the convergence of these algorithms in terms of geometric ergodicity as well as their robustness with respect to decaying noise in the data.

Anne Wald

University of Göttingen

Inverse problems for the identification of forces in biophysics

Cells are the building blocks of living beings. They mostly consist of different interacting proteins, enabling them to fulfil diverse tasks. A central property of cells is their ability to organise themselves, i.e., they execute active processes by means of conversion of energy. For instance, cells can attach to and migrate along surfaces, contract, or divide. In all these tasks, the cytoskeleton plays an important role. It consists mostly of protein filaments that shape and stabilise the cell, but also serves as a structure along which nutrients are transported. It is thus a major research question in cell physics to understand the mechanical properties of the cytoskeleton. This talk is focused on inverse problems that aim at identifying the forces exerted

by the cytoskeleton. Two experiments as well as their mathematical analysis are presented. The first experiment investigates droplets of actomyosin, i.e., a simple system consisting mainly of two types of protein that form a dynamic network, which causes a fluid flow inside the droplet as a consequence of its dynamic behaviour. The second experiment is called traction force microscopy, in which the forces exerted by a cell onto a surface of a substrate are reconstructed from measurements of the substrate's deformation. In both cases, we have to deal with parameter identification problems and their regularization, which will be addressed in this talk.

Simon Weissmann

University of Mannheim

Stability and convergence for stochastic gradient descent with decaying Tikhonov regularization

In this talk we study the minimization of convex, L-smooth functions defined on a separable real Hilbert space. We analyze regularized stochastic gradient descent (reg-SGD), a variant of stochastic gradient descent that uses a Tikhonov regularization with time-dependent, vanishing regularization parameter. We prove strong convergence of reg-SGD to the minimum-norm solution of the original problem without additional boundedness assumptions. Moreover, we quantify the rate of convergence and optimize the interplay between step-sizes and regularization decay. Our analysis reveals how vanishing Tikhonov regularization controls the flow of SGD and yields stable learning dynamics, offering new insights into the design of iterative algorithms for convex problems, including those that arise in ill-posed inverse problems. We validate our theoretical findings through numerical experiments on image reconstruction and ODE-based inverse problems.
This is joint work with Sebastian Kassing and Leif Döring.

Dean Zenner

Saarland University Saarbrücken

Learning a neural operator for inverse parameter estimation of an electric arc furnace

Understanding the melting process of a constructed Electric Arc Furnace is crucial in today's steel industry. To obtain that understanding, we utilize a controlled ODE system capable of simulating said process. Unknown parameters of the underlying furnace, that serve as input data to the ODE system, are needed to accurately simulate the melting process. Due to the complexity of the system, Inverse Parameter Estimation is not feasible. We therefore train a Neural Operator on simulation data which will then serve as a surrogate of the Forward Operator and thus make Parameter Estimation feasible.