Aufgabe 1

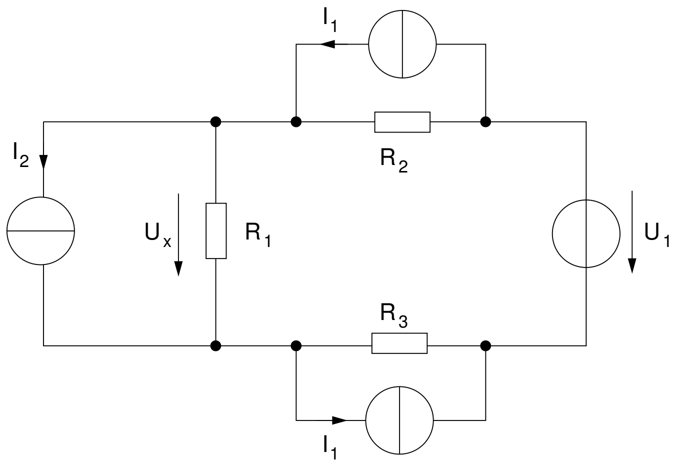
a)

Baum:

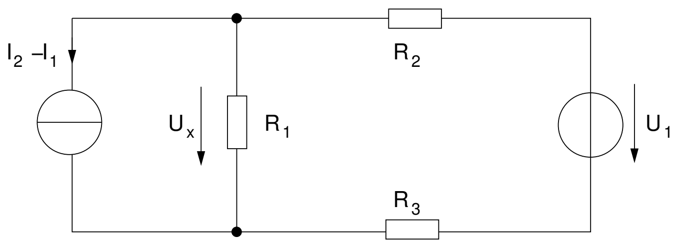
Kobaum:

b)

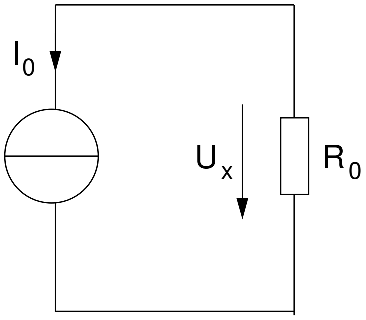
Umzeichnen des Netzwerks liefert:



Zusammenfassen der Stromquellen:



Umwandeln der Spannungs- in eine Stromquelle und weiteres Zusammenfassen:



mit:

$$I_0 = I_2 - I_1 - \frac{U_1}{R_2 + R_3}$$
$$R_0 = R_1 || (R_2 + R_3)$$

die gesuchte Spannung U_x ergibt sich somit zu:

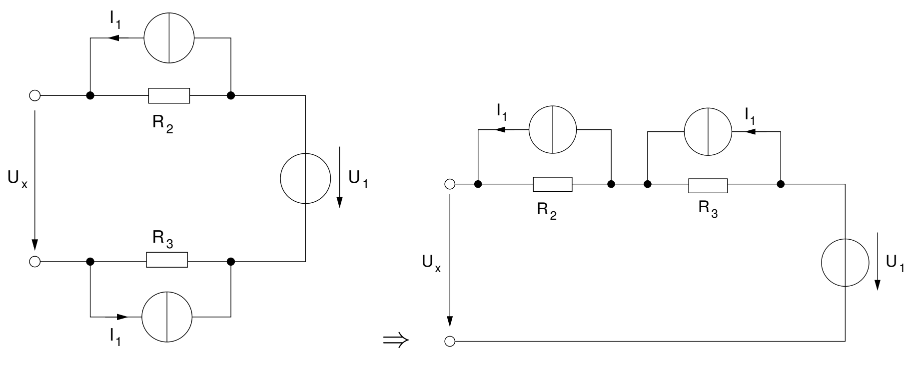
$$U_x = -I_0 \cdot R_0$$

c)

Da die Netzwerkumformungen aus Aufgabenteil b) nur ein bezüglich U_x äquivalentes Netzwerk liefern, muss die Berechnung der Verlustleistung am Originalnetzwerk erfolgen!

$$\begin{aligned} P_\Sigma &= P_{I_2} + P_{I_1,R_2} + P_{I_1,R_3} + P_{U_1} \\ &= -I_2 U_x - I_1 (U_1 - U_x) + I_1 U_1 + \frac{(U_1 - U_x) U_1}{R_2 + R_3} \end{aligned}$$

Wobei dies der Summe der von den Quellen abgegebenen Leistungen entspricht.

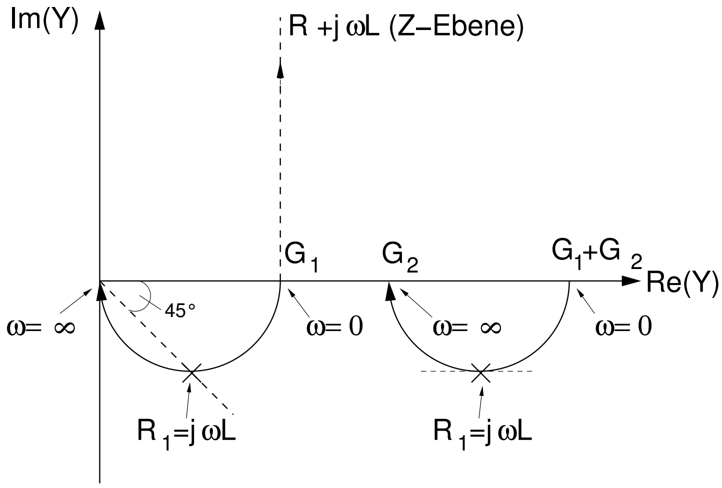


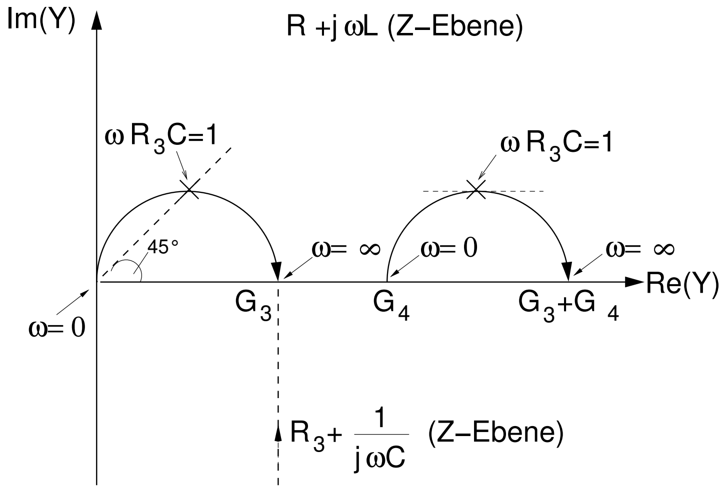
Aufgabe 2

a)

$$Y_1 = \frac{1}{R_1 + j\omega L} + G_2, Y_2 = \frac{1}{R_3 + \frac{1}{j\omega C}} + G_4$$

b)



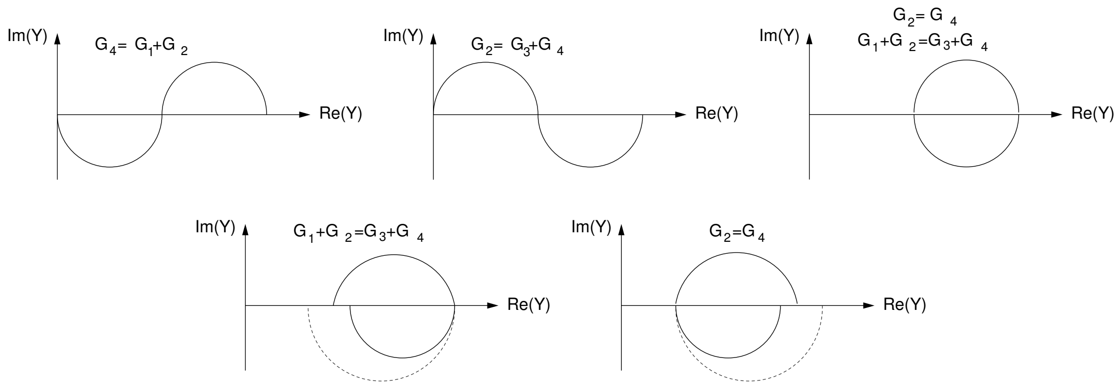


c)

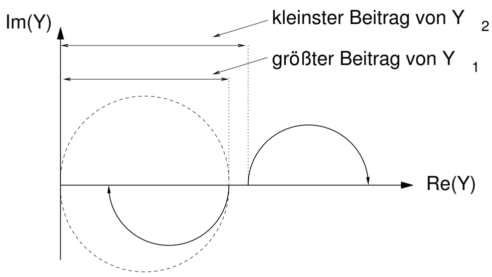
i)

Y_1: Die Kurve für Y_1 verläuft nur im 4. Quadranten. Die Punkte $G_2\,(\omega \to \infty)$ und $G_1 + G_2\,(\omega = 0)$ liegen auf der reellen Achse.
Y_2: Die Kurve für Y_2 liegt nur im 1. Quadranten. Die Punkte $G_4\,(\omega = 0)$ und $G_3 + G_4\,(\omega \to \infty)$ liegen auf der reellen Achse.

ii)



d)



$\Rightarrow$ In $\omega = 0$ sind sich beide Kurven im Betrag am nächsten: $G_1 + G_2 < G_4$

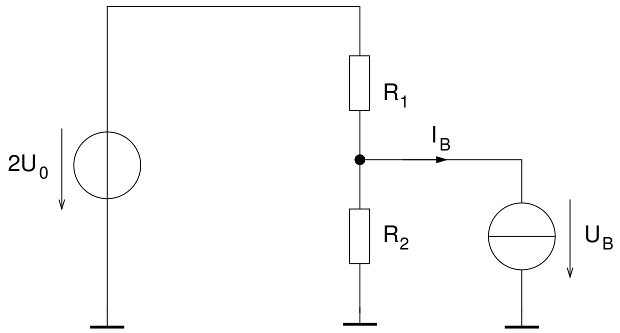
Aufgabe 3

a)

R_L stromlos:

$$I_E R_E = U_0$$
$$U_B = U_{BE} + I_E R_E = U_{BE} + U_0$$

Belasteter Basisspannungsteiler:



$$U_B = \frac{R_2}{R_1 + R_2} \cdot 2U_0 - I_B \left(R_1 || R_2\right)$$
$$U_{BE} + U_0 = \frac{R_2}{R_1 + R_2} \cdot 2U_0 - I_B \left(R_1 || R_2\right)$$
$$I_B = \frac{I_E}{B+1} = \frac{\frac{U_0}{R_E}}{B+1}$$
$$\Rightarrow U_{BE} + U_0 = \frac{R-2}{R_1 + R-2} \cdot 2U_0 - \frac{U_0}{B+1} \cdot \frac{R_1 R_2}{\left(R_1 + R_2\right) \cdot R_E}$$

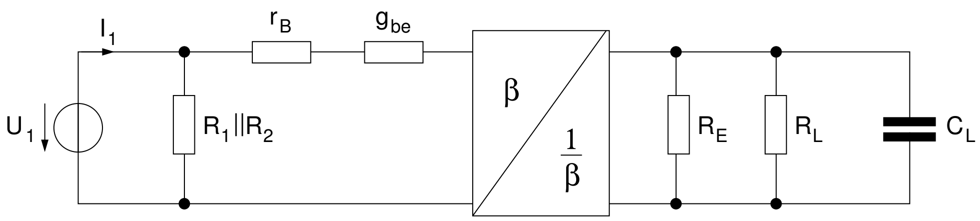
$$\left(R_1 + R_2\right)\left(U_{BE} + U_0\right) = R_2 2U_0 - \frac{U_0 R_1 R_2}{(B+1) \cdot R_E}$$
$$R_1 \left(U_{BE} + U_0 + \frac{U_0 R_2}{(B+1) R_E}\right) = R_2 2U_0 - R_2 \left(U_{BE} + U_0\right)$$
$$R_1 = \frac{2R_2 U_0 - R_2 \left(U_{BE} + U_0\right)}{U_{BE} + U_0 + \frac{U_0 R_2}{(B+1) R_E}}$$
$$= \frac{R_2 \left(U_0 - U_{BE}\right)}{U_0 + U_{BE} + \frac{U_0 R_2}{(B+1) R_E}}$$

b)

$$\begin{aligned} P &= U_{BE} I_B + U_{CE} I_C \\ &= U_{BE} \frac{I_E}{1+B} + U_0 I_E \frac{B}{B+1} \\ &= \left(U_{BE} \frac{1}{1+B} + U_0 \frac{B}{B+1} \right) \cdot \frac{U_0}{R_E} \end{aligned}$$

c)

$$\underline{Z}_{ein} = \frac{\underline{U}_1}{\underline{I}_1}$$

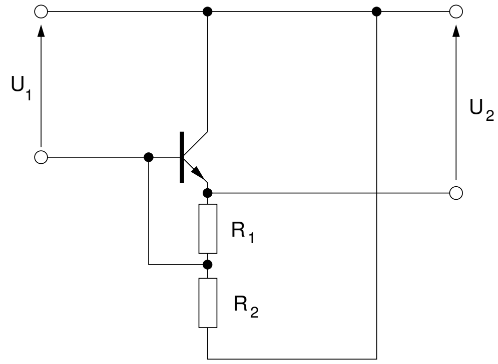


$$Z_{ein} = (R_1||R_2) \,||\, \left(r_B + g_{be} + \beta \cdot \left(R_E||R_L||\frac{1}{j\omega C} \right) \right)$$

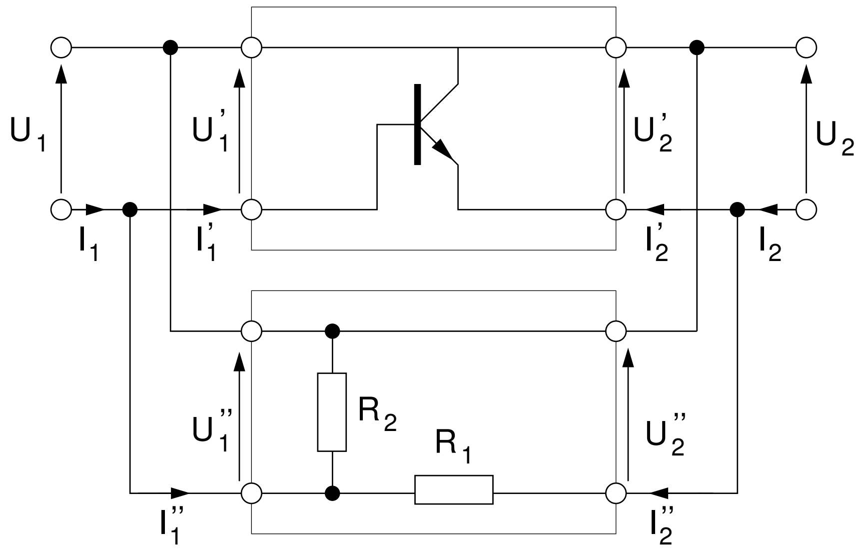
Aufgabe 4

a)

Kollektorgrundschaltung:



b)



c)

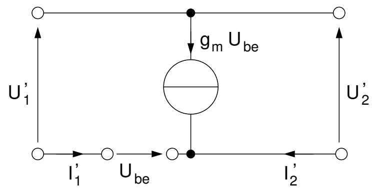
$$U_1 = U_1' = U_1''$$
$$U_2 = U_2' = U_2''$$
$$I_1 = I_1' + I_1''$$
$$I_2 = I_2' + I_2''$$

⇒ Y-Parameter, Parallel-Parallel-Kopplung (PPK):

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

d)

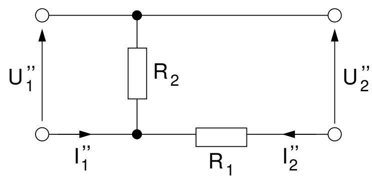
Hauptzweitor (HZT):



$$Y'_{11} = \left.\frac{I'_1}{U'_1}\right|_{U'_2=0} = 0 \qquad Y'_{12} = \left.\frac{I'_1}{U'_2}\right|_{U'_1=0} = 0$$

$$Y'_{21} = \left.\frac{I'_2}{U'_1}\right|_{U'_2=0} = -g_m \qquad Y'_{22} = \left.\frac{I'_2}{U'_2}\right|_{U'_1=0} = g_m$$

Rückkopplungszweitor (RZT):



$$Y''_{11} = \left.\frac{I''_1}{U''_1}\right|_{U''_2=0} = \frac{1}{R_1} + \frac{1}{R_2} \qquad Y''_{12} = \left.\frac{I''_1}{U''_2}\right|_{U''_1=0} = -\frac{1}{R_1}$$

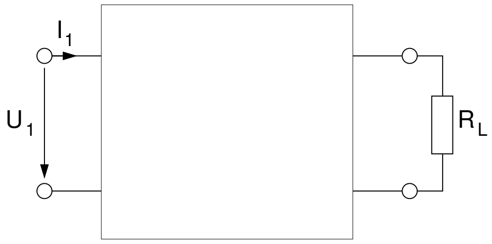
$$Y''_{21} = \left.\frac{I''_2}{U''_1}\right|_{U''_2=0} = -\frac{1}{R_1} \qquad Y''_{22} = \left.\frac{I''_2}{U''_2}\right|_{U''_1=0} = \frac{1}{R_1}$$

$$\Rightarrow Y = Y' + Y'' = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -g_m - \frac{1}{R_1} & g_m + \frac{1}{R_1} \end{bmatrix}$$

e)

Quelle hochohmig ⇒ Strom aus RZT fließt in HZT (nicht in die Quelle)
Last hochohmig ⇒ Strom aus HZT fließt in RZT (nicht in die Last)

f)



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$
$$\frac{U_2}{R_2} = Y_{21}U_1 + Y_{22}U_2$$
$$\Rightarrow U_2 = \frac{Y_{21}U_1}{\frac{1}{R_L} - Y_{22}}$$

$$I_1 = Y_{11}U_1 + Y_{12}U_2 = Y_{11}U_1 + \frac{Y_{21}U_1}{\frac{1}{R_L} - Y_{22}}$$

$$\Rightarrow Z_{ein} = \frac{U_1}{I_1} = \frac{1}{Y_{11} + \frac{Y_{12}Y_{21}}{\frac{1}{R_L} - Y_{22}}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{\frac{1}{R_1}\left(g_m + \frac{1}{R_1}\right)}{\frac{1}{R_L} - \left(g_m + \frac{1}{R_1}\right)}}$$
$$\Rightarrow Z_{ein}|_{R_L \to 0} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$
$$Z_{ein}|_{R_L \to \infty} = \frac{1}{\frac{1}{R_2}} = R_2$$

$\Rightarrow$ Ein sinkender Lastwiderstand verschiebt die Eingangsimpedanz von R_2 nach $R_1||R_2$. Die Eingangsimpedanz sinkt also ebenfalls.

Aufgabe 5

a)

$$\begin{aligned}
\underline{Y}_1(s) &= \frac{1}{R_2 + \frac{1}{j\omega C}} + \frac{1}{R + j\omega L} \\
&= \frac{j\omega C}{1 + j\omega R_2 C} + \frac{1}{R_1 + j\omega L} \\
&= \frac{j\omega C\left(R_1 + j\omega L\right) + 1 + j\omega R_2 C}{\left(1 + j\omega R_2 C\right)\left(R_2 + j\omega L\right)} \\
\text{Pole: } s_1 &= -\frac{1}{R_2 C} \\
s_2 &= -\frac{R_1}{L} \\
\underline{Z}_1 &= \frac{\underline{U}_1}{\underline{I}_1}
\end{aligned}$$

für $\underline{I}_1 \rightarrow$ (Ursache verschwindet): andere Netzwerktopologie wie in Aufgabenstellung

b)

Stabil: Pole in linker, offener Halbebene (LHP)
$Re\left\{\underline{s}_1\right\} < 0$ da $R_2 > 0 \rightarrow$ immer erfüllt
$Re\left\{\underline{s}_2\right\} < 0$ wenn $R_1 > 0$

c)

$$\begin{aligned}
\underline{Y}_1(s) &= \frac{1}{R - j\frac{1}{sC}} + \frac{1}{R + sL} \\
&= \frac{sC\left(R + sL\right) + 1 + sRC}{\left(1 + sRC\right)\left(R + sL\right)} \\
&= \frac{2sRC + s^2 LC + 1}{sR^2 C + SL + s^2 RLC + R} \\
&= \frac{„R + sL + \frac{1}{sC}}{R^2 + \frac{L}{C} + sRL + \frac{R}{sC}} \\
&= \frac{2R + sL + \frac{1}{sC}}{R^2 + \frac{L}{C} + R\left(sL + \frac{1}{sC}\right)} \\
&= \frac{2R + \left(sL + \frac{1}{sC}\right)}{R\left(R + \frac{L}{RC} + \left(sL + \frac{1}{sC}\right)\right)}
\end{aligned}$$

$$\text{konstanter reeller Leitwert gefordert} \Rightarrow R + \frac{L}{RC} = 2R \Rightarrow \frac{L}{C} = R^2$$

d)

$$\text{Sei } \underline{Y}_1(j\omega) = \frac{1}{R}$$

$$\underline{P}_L = \underline{U}_L \cdot \underline{I}_L^*$$

$$\underline{U}_L = \underline{U}_1 \cdot \frac{j\omega L}{R + j\omega L}$$

$$\underline{I}_L^* = \left(\underline{U}_1 \cdot \frac{1}{R + j\omega L}\right)^*$$

$$\underline{P}_C = \underline{U}_C \cdot \underline{I}_C^*$$

$$\underline{U}_C = \underline{U}_1 \cdot \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}$$

$$\underline{I}_C^* = \left(\underline{U}_1 \cdot \frac{1}{\frac{1}{j\omega C} + R}\right)^*$$

$$\frac{\underline{P}_L}{\underline{P}_C} = \frac{\frac{j\omega L}{R+j\omega L} \cdot \frac{1}{R-j\omega L}}{\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} \cdot \frac{1}{R-\frac{1}{j\omega C}}}$$

$$= \frac{j\omega L}{\frac{1}{j\omega C}} \cdot \frac{R^2 + \left(\frac{1}{\omega C}\right)^2}{R^2 + (\omega L)^2}$$

$$= -1 \cdot \omega^2 LC \frac{R^2 + \left(\frac{1}{\omega C}\right)^2}{R^2 + (\omega L)^2}$$

$$R = \alpha L = \frac{1}{\alpha C}$$

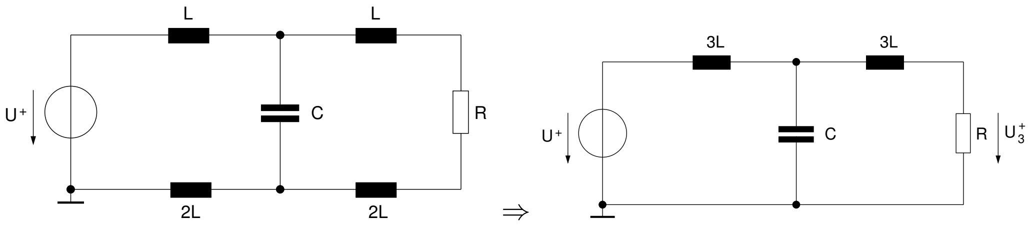
Aufgabe 6

a)

$$\underline{U}^+ = \frac{1}{2}(\underline{U}_1 + \underline{U}_2) \qquad \underline{U}^- = \frac{1}{2}(\underline{U}_1 - \underline{U}_2)$$

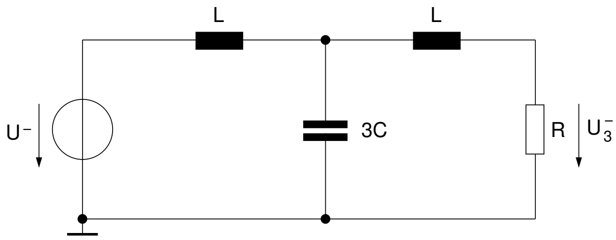
$$\underline{U}_1 = \underline{U}^+ + \underline{U}^- \qquad \underline{U}_2 = \underline{U}^+ - \underline{U}^-$$

Gleichtakt:



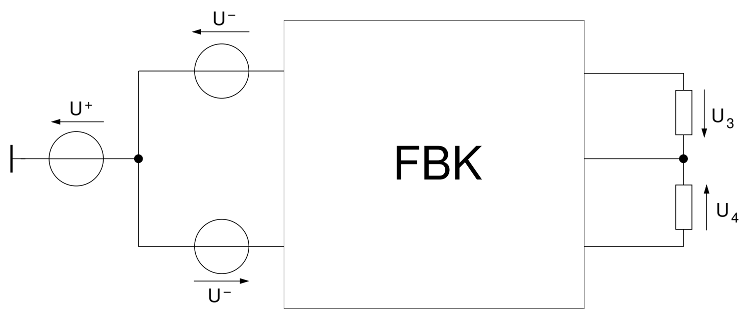
$$\frac{\underline{U}_3^+}{\underline{U}^+} = H^+$$

Gegentakt:



$$\frac{\underline{U}_3^-}{\underline{U}^-} = H^-$$

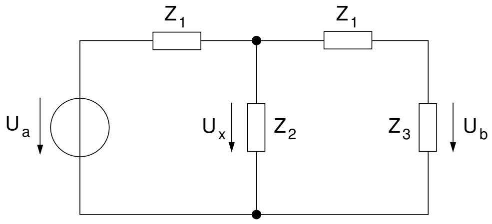
b)



$$U_1 = U^+ + U^- \qquad U_3 = H^+U^+ + H^-U^-(= U_3^+ + U_3^-)$$

$$U_3^+ = U_4^+, U_3^- = -U_4^- \qquad U_4 = H^+U^+ - H^-U^-(= U_3^+ - U_3^- = U_4^+ U_4^-)$$

Allgemein:



$$U_x \frac{Z_3}{Z_1 + Z_3} = U_b$$

$$Z_x = Z_2 || (Z_1 + Z_3) = \frac{Z_2 (Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}$$

$$U_a \frac{Z_x}{Z_1 + Z_x} = U_x = \frac{Z_2 (Z_1 + Z_3) U_a}{(Z_1 + Z_2 + Z_3)(Z_1 + Z_x)} = \frac{Z_2 (Z_1 + Z_3) U_a}{Z_1 (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$$

$$U_b = U_a \frac{Z_2 (Z_1 + Z_3)}{Z_1 (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)} \frac{Z_3}{(Z_1 + Z_3)}$$

$$\frac{U_b}{U_a} = \frac{Z_2 Z_3}{Z_1 (Z_1 + Z_2 + Z_3) + Z_2 (Z_1 + Z_3)}$$

$$\Rightarrow H^+ : Z_1 = j\omega 3L, Z_2 = \frac{1}{j\omega C}, Z_3 = R$$

$$H^+ = \frac{\frac{R}{j\omega C}}{j\omega 3L \left(j\omega 3L + \frac{1}{j\omega C} + R\right) + \frac{1}{j\omega C}(j\omega 3L + R)}$$

$$= \frac{R}{j\omega 3L (2 + j\omega RC - \omega^2 3LC) + R}$$

$$H^- : H^+ (3L \to L, C \to 3C)$$

$$H^- = \frac{R}{j\omega L (2 + j\omega 3RC - \omega^2 3LC) + R}$$

$$\left.\frac{U_3}{U_1}\right|_{\underbrace{U_2 = 0}_{U^+ = \frac{1}{2}U_1, U^- = \frac{1}{2}U_1}} = \frac{H^+ U^+ + H^- U^-}{U^+ + U^-} = \frac{H^+ \frac{1}{2} U_1 + H^- \frac{1}{2} U_1}{U_1} = \frac{H^+}{2} + \frac{H^-}{2}$$

$$\left.\frac{U_4}{U_1}\right|_{U_2=0} = \frac{H^+ U^+ - H^- U^-}{U^+ + U^-} = \frac{H^+ \frac{1}{2} U_1 - H^- \frac{1}{2} U_1}{U_1} = \frac{H^+}{2} - \frac{H^-}{2} = \frac{1}{2}\left(H^+ - H^-\right)$$

$$= \frac{R}{j\omega 3L (2 + j\omega RC - \omega^2 3LC) + R} - \frac{R}{j\omega L (2 + j\omega 3RC - \omega^2 3LC) + R}$$

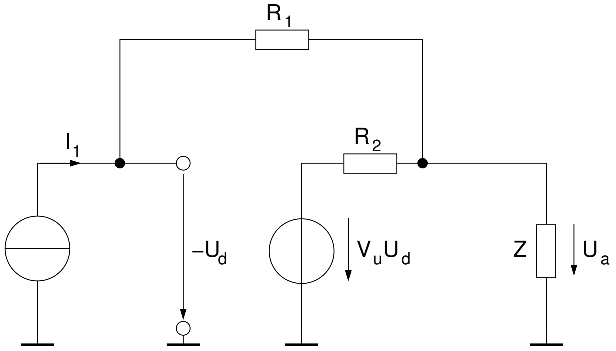
$$L \to 0:$$
$$\left.\frac{U_4}{U_1}\right|_{L\to 0} = \frac{R}{R} - \frac{R}{R} = 0$$

$$R \to \infty:$$
$$\left.\frac{U_4}{U_1}\right|_{R\to \infty} = \frac{1}{j\omega 3L\left(j\omega C\right)+1} - \frac{1}{j\omega L\left(j\omega 3C\right)+1} = 0$$

$\Rightarrow$beide Fälle vermeiden Übersprechen

Aufgabe 7

a)



$$
\begin{aligned}
U_a + U_d &= -I_1 R_1 \\
-\frac{U_a}{Z} + I_1 &= \frac{U_a - v_u U_d}{R_2} \\
-\frac{U_a}{Z} + I_1 &= \frac{U_a - v_u\left(-U_a - I_1 R_1\right)}{R_2} = \frac{U_a + v_u\left(U_a + I_1 R_1\right)}{R_2} \\
-\frac{U_a}{Z} - \frac{U_a}{R_2} - \frac{v_u U_a}{R_2} &= -I_1 + \frac{v_u I_1 R_1}{R_2} \\
U_a\left(\frac{1}{Z} + \frac{1}{R_2} + \frac{v_u}{R_2}\right) &= I_1\left(1 - \frac{v_u R_1}{R_2}\right) \\
\frac{U_a}{I_1} &= \frac{1 - \frac{v_u R_1}{R_2}}{\frac{1}{Z} + \frac{1}{R_2}\left(1 + v_u\right)} \\
&= \frac{R_2 - v_u R_1}{1 + \frac{R_2}{Z} + v_u}
\end{aligned}
$$

b)

Standardform:

$$
\begin{aligned}
F &= \frac{F_a}{1 + F_a F_2} \\
\rightarrow F_a &= R_2 - v_u R_1 \\
F_2 F_a &= \frac{R_2}{Z} + v_u \\
\Rightarrow F_2 &= \frac{\frac{R_2}{Z} + v_u}{F_a} \\
&= \frac{\frac{R_2}{Z} + v_u}{R_2 - v_u R_1}
\end{aligned}
$$

c)

$$
\begin{aligned}
F_2F_a &= \frac{R_2}{Z} + v_u = \frac{R_2}{j\omega L} + \frac{v_0}{1+\frac{j\omega}{\omega_0}} \\
&= \frac{R_2\left(1+\frac{j\omega}{\omega_0}\right) + j\omega L v_o}{j\omega L\left(1+\frac{j\omega}{\omega_0}\right)} \\
&= \frac{R_2 + j\omega\left(\frac{R_2}{\omega_0} + L v_0\right)}{j\omega L\left(1+\frac{j\omega}{\omega_0}\right)} \\
&= \frac{R_2}{L}\frac{1 + j\omega\left(\frac{1}{\omega_0} + v_0\frac{L}{R_2}\right)}{j\omega\left(1+\frac{j\omega}{\omega_0}\right)}
\end{aligned}
$$

mit den angegebenen Werten:

$$
= \frac{1 + j\omega\left(\frac{1}{\omega_0} + \frac{10^4}{\omega_0}\right)}{j\frac{\omega}{\omega_0}\left(1+\frac{j\omega}{\omega_0}\right)}
$$

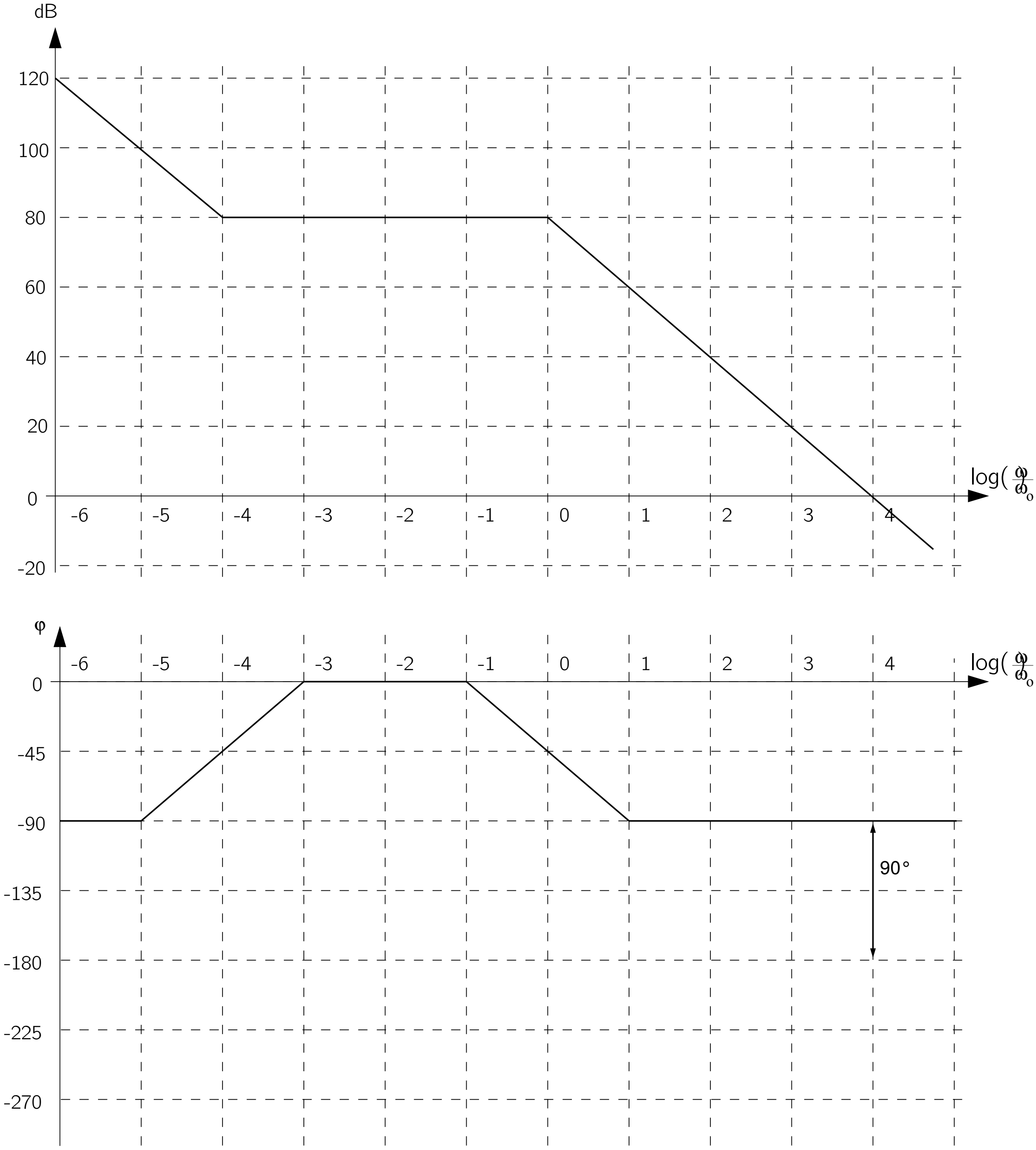
Näherung (Fehler im Bode-Diagramm nicht erkennbar):

$$
\frac{1}{\omega_0} + \frac{10^4}{\omega_0} \approx \frac{10^4}{\omega_0}
$$

$$
F_2F_a = \frac{1+\frac{j\omega}{10^{-4}\omega_0}}{j\frac{\omega}{\omega_0}\left(1+\frac{j\omega}{\omega_0}\right)}
$$

$$
\omega = 10^{-4}\omega_0 \Rightarrow F_2F_a \approx \frac{1+j}{j10^{-4}\,(1)}
$$

d)


dB
120
100
80
60
40
20
0
-20
-6 -5 -4 -3 -2 -1 0 1 2 3 4
log(ω/ω₀
φ
0
-45
-90
-135
-180
-225
-270
90°