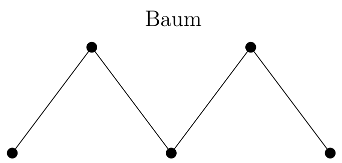
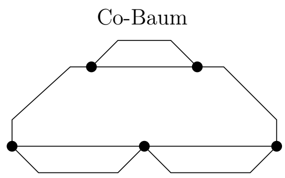
Aufgabe 1

a)



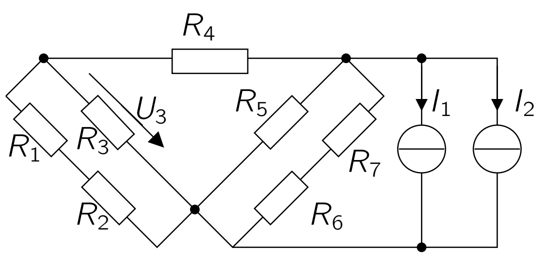


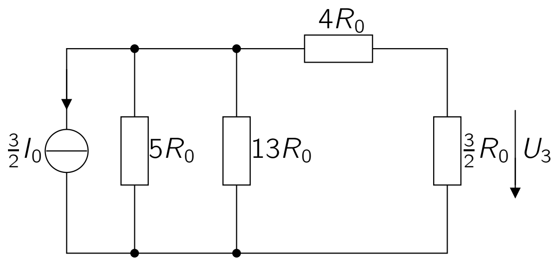
b)

Kobaum: 8 Zweige; 5 Ströme bekannt (Stromquellen) $\Rightarrow$ 3 unabhängige Ströme

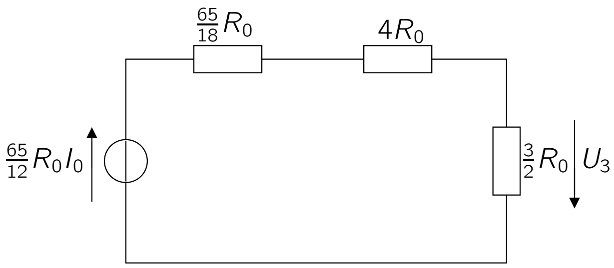
c)

umzeichnen:





$$\frac{5R_0 13R_0}{5R_0+13R_0} = \frac{65}{18}R_0$$



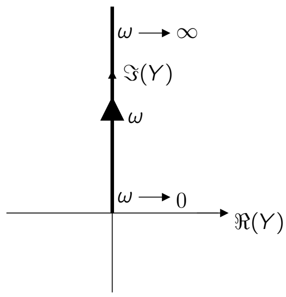
$$\Rightarrow -U_3 = \frac{65}{12}R_0 I_0 \cdot \frac{\frac{3}{2}\cancel{R_0}}{(\frac{65}{18}+4+\frac{3}{2})\cdot\cancel{R_0}} = \frac{65}{12}R_0 I_0 \cdot \frac{27}{65+72+27} = \frac{65}{{}_4\cancel{12}}R_0 I_0 \cdot \frac{\cancel{27}^{\,9}}{164} = \frac{585}{656}R_0 I_0 \;\Rightarrow\; a = \frac{-585}{656}$$

Aufgabe 2

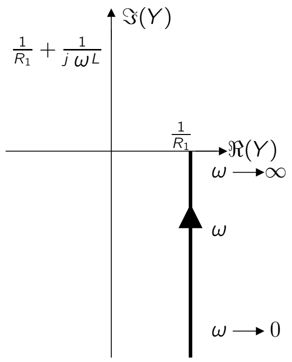
a)

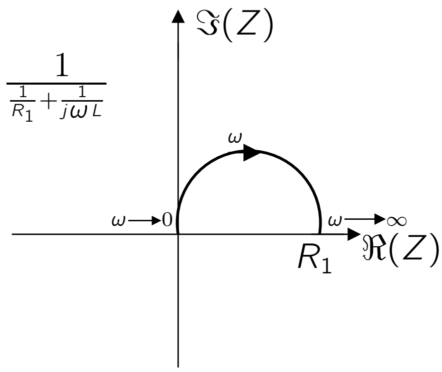
$$\underline{Y}_x = j\omega C_k \qquad \underline{Y}_y = \frac{1}{R_2 + (j\omega L // R1)} = \frac{1}{R_2 + \frac{j\omega L R1}{j\omega L + R1}}$$

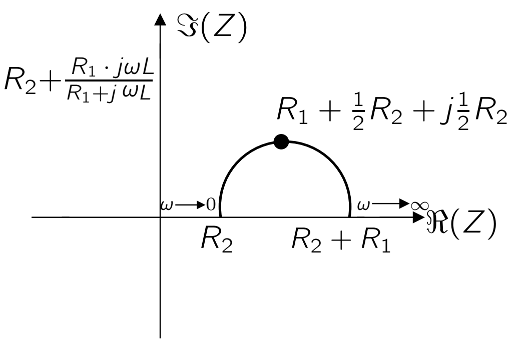
b)

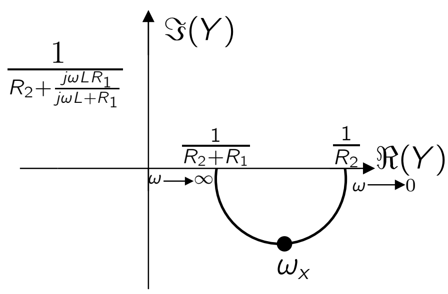


$$\underline{Y}_x(\omega = 0) = 0$$
$$\underline{Y}_x(\omega \to \infty) = j\infty$$









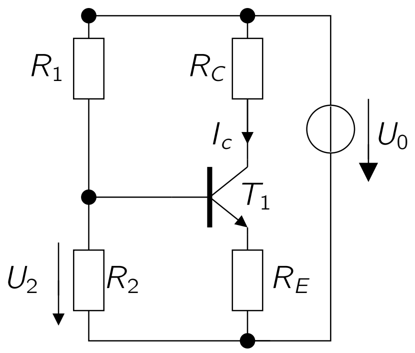
$$\underline{Y}_y(\omega \to 0) = \frac{1}{R_2}$$
$$\underline{Y}_y(\omega \to \infty) = \frac{1}{R_2 + R_1}$$
$$bei \;\; \omega_x \;\; gilt: \qquad \Re(Y_y) = \frac{1}{2}\left(\frac{1}{R_1 + R_2} + \frac{1}{R_2}\right)$$
$$\Im(Y_y) = -\frac{1}{2}\left(\frac{1}{R_2} - \frac{1}{R_1 + R_2}\right)$$

c)

$$\omega C_k = -\Im(Y_y) = \frac{1}{2}\left(\frac{1}{R_2} - \frac{1}{R_1 + R_1}\right)$$
$$C_k = \frac{1}{\omega_k}\left(\frac{R_1 + R_2 - R_2}{R_2(R_1 + R_2)}\right) = \frac{1}{\omega_x} \cdot \frac{R_1}{R_2(R_1 + R_2)}$$

Aufgabe 3

a)

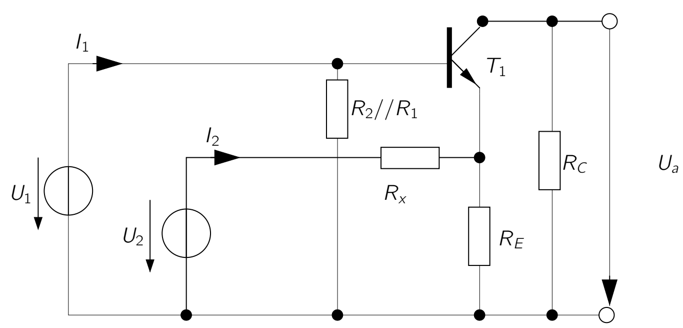


b)

$$R_C I_C = \frac{U_0}{2} = 10 U_{BE} \;\Rightarrow\; I_C = \frac{10 U_{BE}}{R_C}$$

$$U_2 = R_E I_E + U_{BE} \approx R_E I_C + U_{BE} =$$
$$\frac{\cancel{R_C}}{\cancel{10}} \frac{\cancel{10} U_{BE}}{\cancel{R_C}} + U_{BE} = 2U_{BE}$$
$$\Rightarrow \frac{R_2}{R_1 + R_2} = \frac{U_2}{U_0} = \frac{2U_{BE}}{20U_{BE}} = \frac{1}{10}$$
$$\Leftrightarrow 10R_2 = R_1 + R_2 \;\Leftrightarrow\; 9R_2 = R_1 \Leftrightarrow\; R_2 = R_1 \frac{1}{9}$$

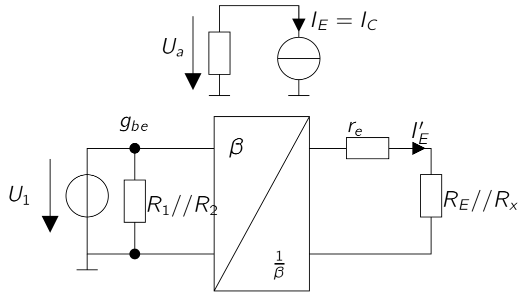
c)



d)

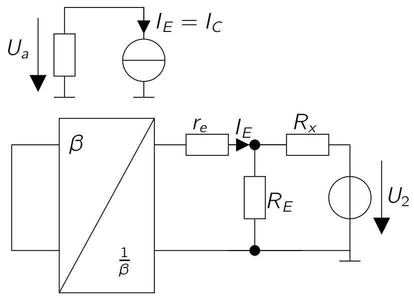
$U_2 = 0;$

$$I_E = \frac{U_1}{r_e + R_E//R_x}$$
$$U_a = -I_C R_C = -I_E R_C = -\frac{R_C}{r_e + R_E//R_x} U_1$$
$$\left.\frac{U_a}{U_1}\right|_{U_2=0} = -\frac{R_c}{r_e + R_E//R_x}$$



$U_1 = 0$

$$I_E = \frac{-\frac{1}{r_e}}{\frac{1}{r_e} + \frac{1}{R_E} + \frac{1}{R_x}} \cdot \frac{U_2}{R_x} = \frac{-1}{1 + \frac{r_e}{R_E} + \frac{r_e}{R_x}} \cdot \frac{U_2}{R_x}$$
$$= \frac{-1}{R_x(1 + \frac{r_e}{R_E}) + r_e} \cdot U_2$$
$$U_a = -I_C R_C = \frac{R_C}{R_x(1 + \frac{r_e}{R_E}) + r_e} \cdot U_2$$
$$\Rightarrow \left.\frac{U_a}{U_2}\right|_{U_1=0} = \frac{R_C}{R_x(1 + \frac{r_e}{R_E}) + r_e}$$
$$\Rightarrow U_a = -\frac{R_c}{r_e + R_E//R_x} U_1 + \frac{R_C}{R_x(1 + \frac{r_e}{R_E}) + r_e} U_2$$



e)

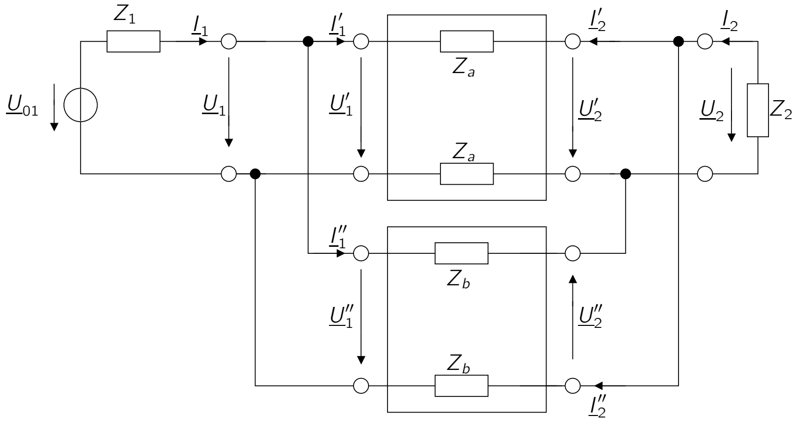
$$\frac{R_C}{r_e + R_E//R_x} \stackrel{!}{=} 1 \qquad \frac{R_C}{R_x(1 + \frac{r_e}{R_E}) + r_e} \stackrel{!}{=} 1$$
$$\Rightarrow R_E//R_x = R_x(1 + \frac{r_e}{R_E}) \quad \Rightarrow \quad \frac{R_E}{R_x + R_E} = 1 + \frac{r_e}{R_E}$$

$$R_E >> R_x, r_e \ \Rightarrow 1 = 1 \ \checkmark$$

$$\Rightarrow R_C = r_e + R_x$$

Aufgabe 4

a)



b)

i)PPK
ii)$\underline{Y}$-Parameter , weil

$$\underline{U}'_1 = \underline{U}''_1 = \underline{U}_1; \quad \underline{U}'_2 = \underline{U}''_2 = \underline{U}_2; \quad \underline{I}_1 = \underline{I}'_1 + \underline{I}''_1; \quad \underline{I}_2 = \underline{I}'_2 + \underline{I}''_2;$$

$$\begin{aligned}\underline{I}_1 = \underline{Y}'_{11}\underline{U}'_1 + \underline{Y}''_{11}\underline{U}''_1 + \underline{Y}'_{12}\underline{U}'_2 + \underline{Y}''_{12}\underline{U}''_2 &= \underline{Y}'_{11}\underline{U}_1 + \underline{Y}''_{11}\underline{U}_1 + \underline{Y}'_{12}\underline{U}_2 + \underline{Y}''_{12}\underline{U}_2 \\ &= (\underline{Y}'_{11} + \underline{Y}''_{12})\underline{U}_1 + (\underline{Y}'_{12} + \underline{Y}''_{12})\underline{U}_2\end{aligned}$$

$$\begin{aligned}\underline{I}_2 = \underline{Y}'_{21}\underline{U}'_1 + \underline{Y}''_{21}\underline{U}''_1 + \underline{Y}'_{22}\underline{U}'_2 + \underline{Y}''_{22}\underline{U}''_2 &= \underline{Y}'_{21}\underline{U}_1 + \underline{Y}''_{21}\underline{U}_1 + \underline{Y}'_{22}\underline{U}_2 + \underline{Y}''_{22}\underline{U}_2 \\ &= (\underline{Y}'_{21} + \underline{Y}''_{22})\underline{U}_1 + (\underline{Y}'_{22} + \underline{Y}''_{22})\underline{U}_2\end{aligned}$$

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}''$$

c)

$$\underline{Y}'_{11} = \left.\frac{\underline{I}'_1}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = \frac{1}{2\underline{Z}_a} \qquad \underline{Y}'_{12} = \left.\frac{\underline{I}'_1}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = -\frac{1}{2\underline{Z}_a}$$

$$\underline{Y}'_{21} = \left.\frac{\underline{I}'_2}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = -\frac{1}{2\underline{Z}_a} \qquad \underline{Y}'_{22} = \left.\frac{\underline{I}'_2}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = \frac{1}{2\underline{Z}_a}$$

$$\Rightarrow \underline{Y}' = \frac{1}{\underline{Z}_a}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\underline{Y}''_{11} = \left.\frac{\underline{I}''_1}{\underline{U}''_1}\right|_{\underline{U}''_2=0} = \frac{1}{2\underline{Z}_b} \qquad \underline{Y}''_{12} = \left.\frac{\underline{I}''_1}{\underline{U}''_2}\right|_{\underline{U}''_1=0} = \frac{1}{2\underline{Z}_b}$$

$$\underline{Y}''_{21} = \left.\frac{\underline{I}''_2}{\underline{U}''_1}\right|_{\underline{U}''_2=0} = \frac{1}{2\underline{Z}_b} \qquad \underline{Y}''_{22} = \left.\frac{\underline{I}''_2}{\underline{U}''_2}\right|_{\underline{U}''_1=0} = \frac{1}{2\underline{Z}_b}$$

$$\Rightarrow \underline{Y}' = \tfrac{1}{2\underline{Z}_b}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}'' = \begin{pmatrix} \frac{1}{2\underline{Z}_a} + \frac{1}{2\underline{Z}_b} & \frac{1}{2\underline{Z}_a} - \frac{1}{2\underline{Z}_b} \\ \frac{1}{2\underline{Z}_a} - \frac{1}{2\underline{Z}_b} & \frac{1}{2\underline{Z}_a} + \frac{1}{2\underline{Z}_b} \end{pmatrix}$$

d)

$$\underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 = \underline{Y}_{11}\underline{U}_1 - \underline{Y}_{12}\underline{Z}_2\underline{I}_2$$

$$\underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 = \underline{Y}_{21}\underline{U}_1 - \underline{Y}_{22}\underline{Z}_2\underline{I}_2 \;\Rightarrow\; I_2(1+\underline{Y}_{22}\underline{Z}_2) = \underline{Y}_{21}\underline{U}_1$$

$$\Rightarrow \frac{I_1}{I_2} = \underline{Y}_{11}\frac{(1+\underline{Y}_{22}\underline{Z}_2)}{\underline{Y}_{21}} - \underline{Y}_{12}Z_2$$

$$\Rightarrow \underline{F}_I = \frac{\underline{I}_2}{\underline{I}_1} = \frac{1}{\underline{Y}_{11}\frac{(1+\underline{Y}_{22}\underline{Z}_2)}{\underline{Y}_{21}} - \underline{Y}_{12}Z_2} = \frac{\underline{Y}_{21}}{\underline{Y}_{11}(1+\underline{Y}_{22}\underline{Z}_2) - \underline{Y}_{21}\underline{Y}_{12}Z_2}$$

$$\underline{F}_I = 0 \text{ für } \underline{Y}_{21} = 0 \Longleftrightarrow \tfrac{1}{2\underline{Z}_b} = \tfrac{1}{2\underline{Z}_a} \;\Longleftrightarrow\; \underline{Z}_a = \underline{Z}_b$$

Aufgabe 5

a)

$$z(s) = \frac{1}{sC + \frac{1}{R+sL} + \frac{1}{r_x}} = \frac{(R+sL)r_x}{sC(R+sL)r_x + r_x + R + sL}$$

b)

Spannung soll oszillieren $\Rightarrow$ schließe hochomige Last an Oszillator an

$\Rightarrow \underline{I} \mapsto 0 \quad \underline{U} = \underline{I} \cdot \underline{Z} \neq 0$ für Pole von $\underline{Z}$

c)

$$Z(s) = \frac{(R+sL)r_x}{sC(R+sL)r_x + r_x + R + sL}$$

$$\Rightarrow sCRr_x + s^2CLr_x + r_x + R + sL \stackrel{!}{=} 0$$

$$\Rightarrow s^2CLr_x + s \cdot (CRr_x + L) + r_x + R = 0$$

$$\Rightarrow s^2 + s\frac{CRr_x + L}{CLr_x} + \frac{r_x + R}{CLr_x} = 0$$

$$s_{1/2} = -\frac{1}{2} \cdot \frac{CRr_x + L}{CLr_x} \stackrel{+}{-} \sqrt{\left(\frac{CRr_x + L}{2CLr_x}\right)^2 - \frac{r_x + R}{CLr_x}}$$

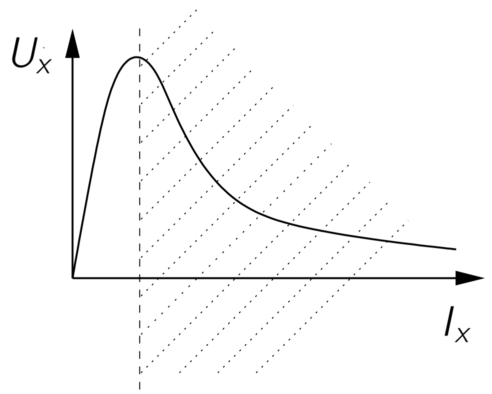
d)

konstante Amplitude wenn $\Re(s) = 0 = \sigma = \frac{R}{L} + \frac{1}{r_x C} \Rightarrow \frac{1}{r_x} = -\frac{RC}{L} \Rightarrow r_x = -\frac{L}{RC}$

e)

Schwingfrequenz: $s = \sigma + j\omega = 0 + j\sqrt{\frac{r_x+R}{CLr_x}}$ mit $\frac{r_x+R}{CLr_x} > 0 \rightarrow \omega = \sqrt{\frac{1}{CL}(1 + \frac{R}{r_x}}$

f)



g)

$$r_x = -\frac{L}{CR} = \frac{dU}{dI}\Big|_{AP} = \frac{(1+bI_x^2)a - 2bI_x aI_x}{(1+bI_x^2)^2}$$

$$\Rightarrow (1 + bI_x^2)a - 2abI_x^2 = r_x(1 + 2bI_x^2 + b^2I_x^4)$$

$$\Rightarrow I_x^4 + \left(\frac{2}{b} + \frac{a}{br_x}I_x^2\right) - \frac{a}{b^2r_x}$$

$$\Rightarrow I_x^2 = -\left(\frac{1}{b} + \frac{a}{2br_x}\right) \stackrel{+}{-} \sqrt{\left(\frac{1}{b} + \frac{a}{2br_x}\right)^2 + \frac{a}{b^2r_x}}$$

$$\Rightarrow I_x = +\sqrt{-\left(\frac{1}{b} + \frac{a}{2br_x}\right) + \sqrt{\left(\frac{1}{b} + \frac{a}{2br_x}\right)^2 + \frac{a}{b^2r_x}}}$$

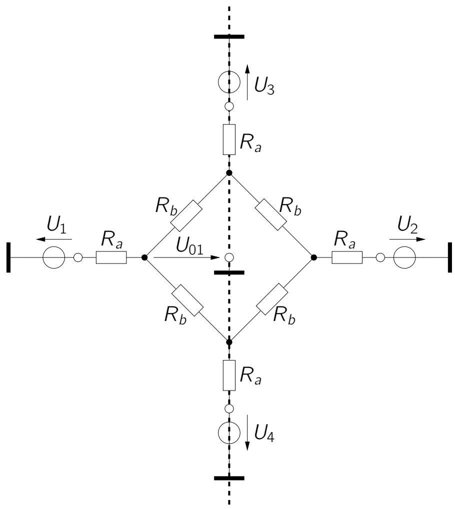
Aufgabe 6

a)

Torpaar 1 : $U_3 = U_4 = 0$

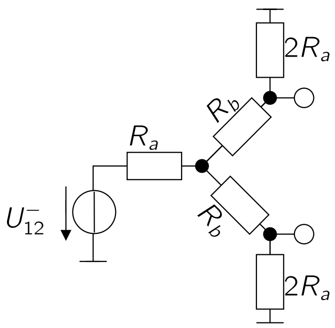
Torpaar 2 : $U_1 = U_2 = 0$

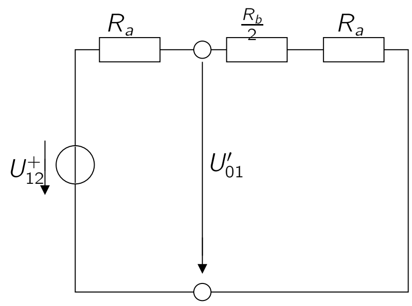
$U_{12}^{+} = \frac{U_1+U_2}{2}$ $U_{12}^{-} \frac{U_1-U_2}{2}$
$U_{34}^{+} = \frac{U_4+U_3}{2}$ $U_{43}^{-} \frac{U_4-U_3}{2}$



b)

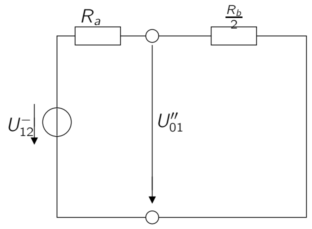
betrachte Torpaar 1 ,
Gleichtakt:





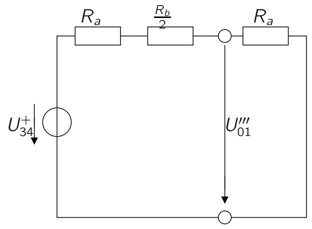
$$\Rightarrow \; U'_{01} = \underbrace{\frac{\frac{R_b}{2} + R_a}{\frac{R_b}{2} + 2R_a}}_{H_{12}^{+}} U_{12}^{+}$$

Gegentakt:



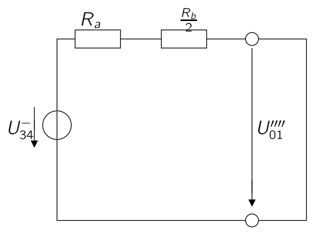
$$\Rightarrow U''_{01} = \underbrace{\frac{R_b}{2R_a + R_b}}_{H_{12}^-} U_{12}^-$$

betrachte Torpaar 2 ,
Gleichtakt:



$$\Rightarrow U'''_{01} = \underbrace{\frac{R_a}{\frac{R_b}{2} + 2R_a}}_{H^+34} U_{34}^+$$

Gegentakt:



$$\Rightarrow U''''_{01} = \underbrace{0}_{H_{34}^-}$$

$$\Rightarrow U_{01} = H_{12}^+ U_{12}^+ + H_{12}^- U_{12}^- +_{12} + H_{34}^+ U_{34}^+ + 0$$

$$= \frac{H_{12}^+ + H_{12}^-}{2} U_1 + \frac{H_{12}^+ - H_{12}^-}{2} U_2 + \frac{H_{34}^+ U_3}{2} + \frac{H_{34}^+ U_4}{2}$$

$$= \left(\frac{\frac{\frac{R_b}{2} + R_a}{\frac{R_b}{2} + 2R_a} + \frac{R_b}{2R_a + R_b}}{2} \right) U_1 + \left(\frac{\frac{\frac{R_b}{2} + R_a}{\frac{R_b}{2} + 2R_a} - \frac{R_b}{2R_a + R_b}}{2} \right) U_2 + \frac{\frac{R_a}{\frac{R_b}{2} + 2R_a}}{2} U_3 + \frac{\frac{R_a}{\frac{R_b}{2} + 2R_a}}{2} U_4$$

Aufgabe 7

a)

$$\frac{\underline{U}_p}{\underline{U}_1} = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}$$

$$\underline{U}_p = \underline{U}_d + \underline{U}_e \Leftrightarrow \; \underline{U}_d = \underline{U}_p - \underline{U}_e$$

$$\underline{U}_2 = v_u \underline{U}_d \frac{\underline{Z}_e}{R_a+\underline{Z}_e} \Leftrightarrow \; \underline{U}_2 = v_u(\underline{U}_p - \underline{U}_2)\frac{\underline{Z}_e}{R_a+\underline{Z}_e}$$

$$\Rightarrow \underline{U}_2(1 + v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}) = \underline{U}_p v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}$$

$$\frac{\underline{U}_2}{\underline{U}_p} = \frac{v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}{1+v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}$$

$$\Rightarrow \underline{F} = \frac{\underline{U}_p}{\underline{U}_1}\frac{\underline{U}_2}{\underline{U}_p} = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \cdot \frac{v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}{1+v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}$$

b)

$$\underline{F}_k = \frac{\underline{Z}_2'}{\underline{Z}_1'+\underline{Z}_2'} \frac{\underline{v}_u' \frac{\underline{Z}_e'}{Ra+\underline{Z}_e'}}{1+\underline{v}_u' \frac{\underline{Z}_e'}{Ra+\underline{Z}_e'}} \cdot \frac{\underline{Z}_2''}{\underline{Z}_1''+\underline{Z}_2''} \frac{\underline{v}_u'' \frac{\underline{Z}_e''}{Ra+\underline{Z}_e''}}{1+ \frac{\underline{Z}_e''}{Ra+\underline{Z}_e''}}$$

$$\underline{Z}_1' = \frac{1}{j\omega C_1}; \underline{Z}_2' = R_2;\; \underline{Z}_1'' = R_1;\; \underline{Z}_2'' = \frac{1}{j\omega C_2};\; |\underline{Z}_e| \mapsto \infty;$$

$$\underline{Z}_e' = \underline{Z}_1'' + \underline{Z}_2'' = R_1 + \frac{1}{j\omega C_2};$$

$$\underline{v}_u' = \underline{v}_u'' = \alpha \in \mathbb{R},\; \alpha >> 1$$

$$\underline{F}_k \overset{|\underline{Z}_e''|\mapsto\infty}{\approx} \frac{R_2}{R_2+\frac{1}{j\omega C_1}} \frac{\alpha \frac{R_1+\frac{1}{j\omega C_2}}{R_a+R_1+\frac{1}{j\omega C_2}}}{1+\alpha \frac{R_1+\frac{1}{j\omega C_2}}{R_a+R_1+\frac{1}{j\omega C_2}}} \cdot \frac{\frac{1}{j\omega C_2}}{R_1+\frac{1}{j\omega C_2}} \frac{\alpha}{1+\alpha}$$

$$= \frac{j\omega R_2C_1}{1+j\omega R_2C_1} \frac{\alpha(R_1+\frac{1}{j\omega C_2})}{R_a+(1+\alpha)(R_1+\frac{1}{j\omega C_2})} \frac{1}{1+j\omega R_1C_2} \frac{\alpha}{1+\alpha}$$

$$= \frac{j\omega R_2C_1}{1+j\omega R_2C_1} \frac{\alpha(R_1+\frac{1}{j\omega C_2})j\omega C_2}{(1+\alpha)[(R_1+\frac{R_a}{1+\alpha})j\omega C_2+1]} \frac{1}{1+j\omega R_1C_2} \frac{\alpha}{1+\alpha}$$

$$= \frac{\alpha^2}{(1+\alpha)^2} \frac{j\omega R_2C_1}{1+j\omega R_2C_1} \frac{\cancel{1+j\omega R_1C_2}}{[j\omega(R_1+\frac{R_a}{1+\alpha})C_2+1]} \frac{1}{\cancel{1+j\omega R_1C_2}}$$

$$= \frac{\alpha^2}{(1+\alpha)^2} \frac{j\omega R_2C_1}{(1+j\omega R_2C_1)[j\omega(R_1+\frac{R_a}{1+\alpha})C_2+1]}$$

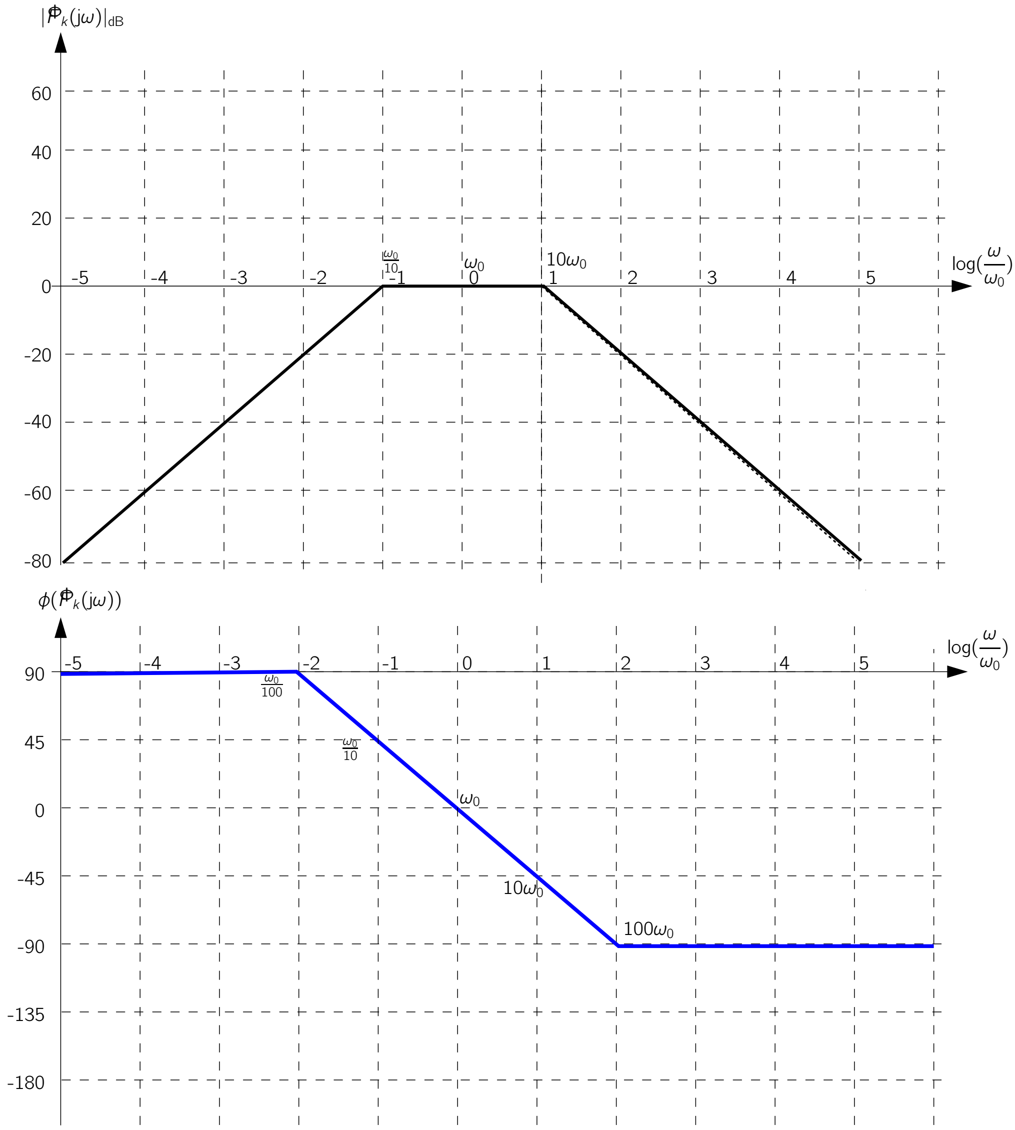
c)

$$\alpha R_1 >> R_a$$

$$\Rightarrow \underline{F}_k = \frac{\alpha^2}{(1+\alpha)^2} \frac{j\omega R_2C_1}{(1+j\omega R_2C_1)(j\omega R_1C_2+1)}$$

$$\Rightarrow R_2C_1 = \frac{10}{\omega_0} \qquad R_2C_1 = \frac{1}{10\omega_0}$$

d)


$|\Phi_k(j\omega)|_{dB}$
60
40
20
0
-20
-40
-60
-80
-5 -4 -3 -2 -1 0 1 2 3 4 5
$\frac{\omega_0}{10}$
ω_0
$10\omega_0$
$\log(\frac{\omega}{\omega_0})$
$\phi(\Phi_k(j\omega))$
90
45
0
-45
-90
-135
-180
$\frac{\omega_0}{100}$
$\frac{\omega_0}{10}$
ω_0
$10\omega_0$
$100\omega_0$