

### Aufgabe 1

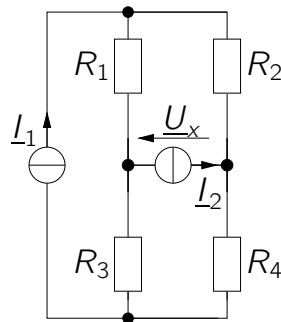
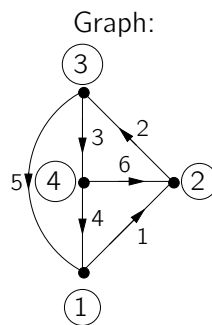
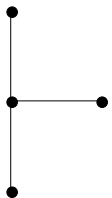


Abb. 1: Gegebenes Netzwerk.

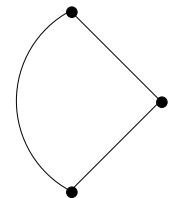
a)



Baum:



Co-Baum:



b)

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix} \end{matrix}$$

Bezugsknoten 1  $\Rightarrow$  streiche 1. Zeile

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix}$$

c)

$$Y = \begin{pmatrix} \frac{1}{R_4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Y_n = AYA^T$$

$$= \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{R_4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{R_4} & 0 & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ 0 & \frac{1}{R_1} & -\frac{1}{R_1} \\ 0 & 0 & \frac{1}{R_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{R_4} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_1} & -\frac{1}{R_1} \\ 0 & -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} \end{pmatrix}$$

$$\begin{aligned}
 I_{qn} &= A(I_g - YU_g) \\
 &= \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_1 \\ I_2 \end{pmatrix} \\
 &= \begin{pmatrix} -I_2 \\ -I_1 \\ I_2 \end{pmatrix}
 \end{aligned}$$

d)

$$\begin{pmatrix} \frac{1}{R_4} + \frac{1}{R_2} & -\frac{1}{R_2} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_1} & -\frac{1}{R_1} \\ 0 & -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} \end{pmatrix} \begin{pmatrix} U_{n2} \\ U_{n3} \\ U_{n4} \end{pmatrix} = \begin{pmatrix} -I_2 \\ -I_1 \\ I_2 \end{pmatrix}$$

Einsetzen:

$$R_1 = R_2 = R_x$$

$$R_3 = R_4 = R_y$$

$$\begin{pmatrix} \frac{1}{R_x} + \frac{1}{R_y} & -\frac{1}{R_x} & 0 \\ -\frac{1}{R_x} & \frac{2}{R_x} & -\frac{1}{R_x} \\ 0 & -\frac{1}{R_x} & \frac{1}{R_x} + \frac{1}{R_y} \end{pmatrix} \begin{pmatrix} U_{n2} \\ U_{n3} \\ U_{n4} \end{pmatrix} = \begin{pmatrix} -I_2 \\ -I_1 \\ I_2 \end{pmatrix}$$

$$(I) \quad \left( \frac{1}{R_x} + \frac{1}{R_y} \right) U_{n2} - \frac{1}{R_x} U_{n3} = -I_2$$

$$(III) \quad -\frac{1}{R_x} U_{n3} + \left( \frac{1}{R_x} + \frac{1}{R_y} \right) U_{n4} = I_2$$

(I) - (III):

$$\left( \frac{1}{R_x} + \frac{1}{R_y} \right) U_{n2} - \left( \frac{1}{R_x} + \frac{1}{R_y} \right) U_{n4} = -2I_2$$

$$U_{24} = U_{n2} - U_{n4} = \frac{-2I_2}{\left( \frac{1}{R_x} + \frac{1}{R_y} \right)}$$

## Aufgabe 2

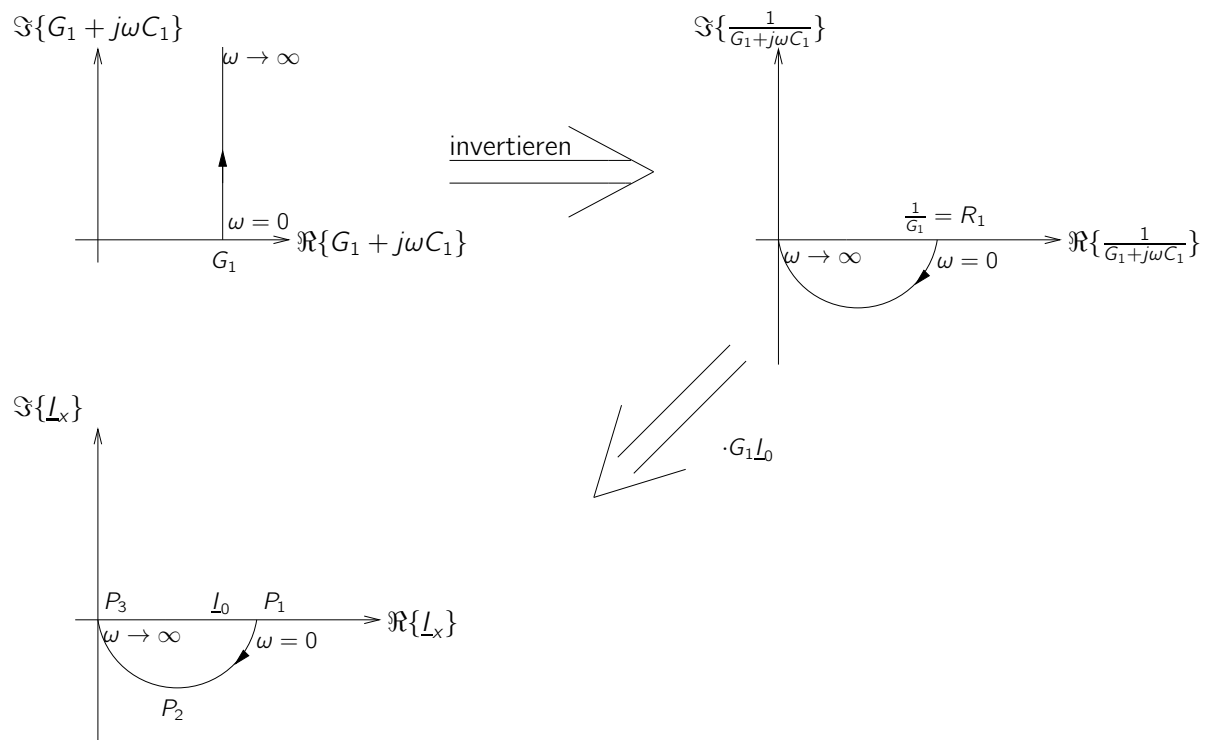
a)

$$\frac{I_x}{I_0} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + j\omega C_1} = \frac{G_1}{G_1 + j\omega C_1} \quad G_1 = \frac{1}{R_1}$$

$$\frac{I_y}{I_0} = \frac{\frac{j\omega C_2}{R_2}}{\frac{1}{R_2} + j\omega C_2} = \frac{j\omega C_2}{G_2 + j\omega C_2} \quad G_2 = \frac{1}{R_2}$$

b)

$I_x$ :



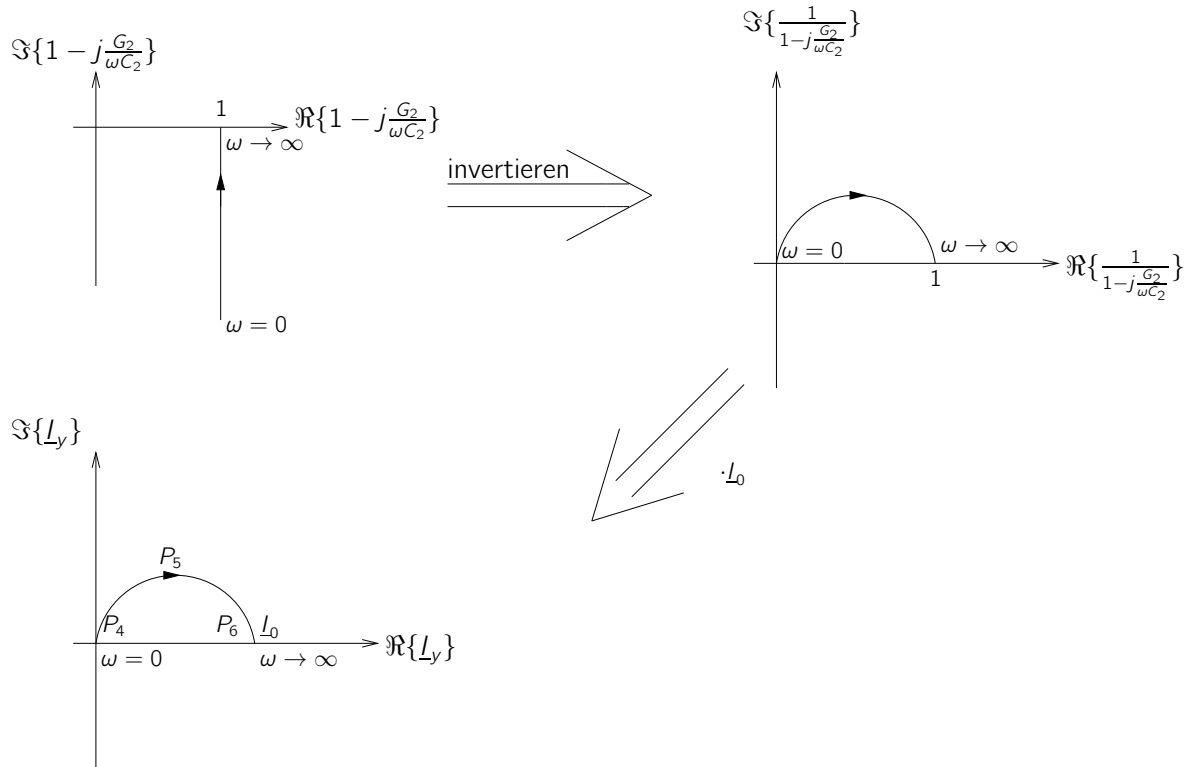
$$P_1 : I_x(\omega = 0) = I_0$$

$$P_2 : I_x\left(\omega = \frac{G_1}{C_1}\right) = \frac{I_0}{2}(1 - j)$$

$$P_3 : I_x(\omega \rightarrow \infty) = 0$$

$$I_y = \frac{1}{1 - j\frac{G_2}{\omega C_2}} I_0$$

$I_y$ :



$$P_4 : I_y(\omega = 0) = 0$$

$$P_5 : I_y\left(\omega = \frac{G_2}{C_2}\right) = \frac{I_0}{2}(1 + j)$$

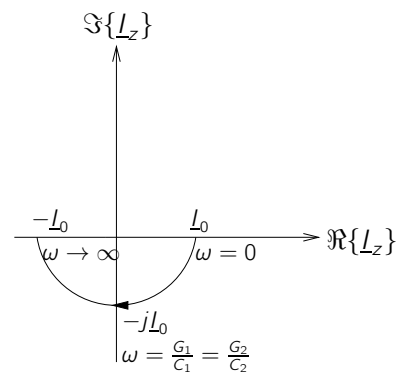
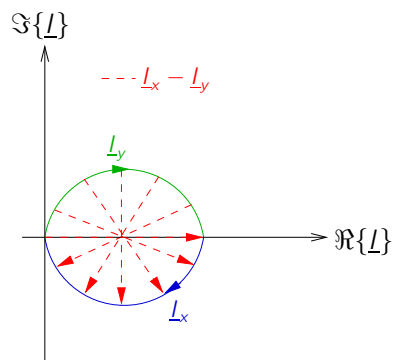
$$P_6 : I_y(\omega \rightarrow \infty) = L_0$$

c)

i)

$$R_1 C_1 = R_2 C_2 \Rightarrow \frac{G_1}{C_1} = \frac{G_2}{C_2}$$

$$L_z = L_x - L_y$$

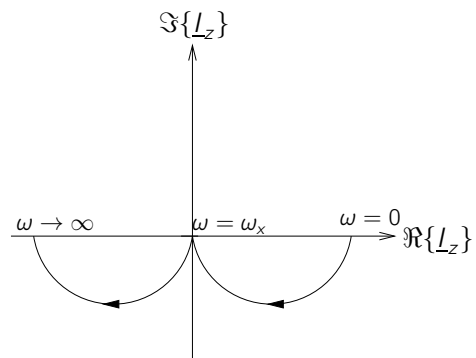
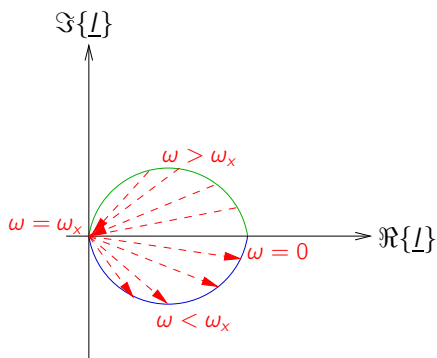


$$\begin{aligned} \underline{I}_z(\omega = 0) &= \underline{I}_0 \\ \underline{I}_z\left(\omega = \frac{\omega_x}{C_1}\right) &= -j\underline{I}_0 \\ \underline{I}_z(\omega \rightarrow \infty) &= -\underline{I}_0 \end{aligned}$$

ii)

$$\begin{aligned} \underline{I}_x(\omega_x) &\approx 0 \\ \underline{I}_y(\omega_x) &\approx 0 \\ \underline{I}_z(\omega_x) &\approx 0 \end{aligned}$$

Bei der Frequenz  $\omega_x$  hat  $\underline{I}_y$  den Ursprung noch nicht verlassen,  $\underline{I}_x$  ist aber bereits dort angelangt.



### Aufgabe 3

a)

$$I_E \cdot R_E + U_{BE,0} = \frac{U_0}{2}$$

$$I_E = \frac{\frac{U_0}{2} - U_{BE,0}}{R_E} \approx I_C$$

$$g_m = \frac{I_C}{U_T} = \frac{\frac{U_0}{2} - U_{BE,0}}{R_E \cdot U_T}$$

b)

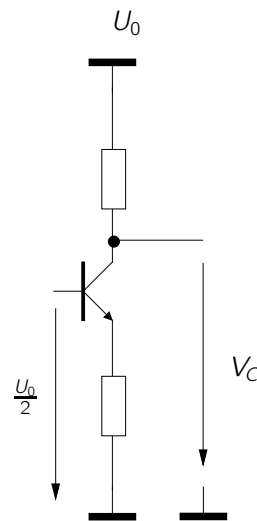
$$V_{C,min} = \frac{U_0}{2}$$

$$V_{C,max} = U_0$$

$$I_C \cdot R_C \stackrel{!}{=} U_0 - \frac{V_{C,min} + V_{C,max}}{2}$$

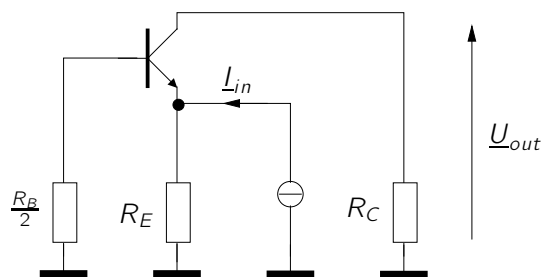
$$R_C = \frac{1}{I_C} \cdot \left( U_0 - \frac{\frac{U_0}{2} + U_0}{2} \right)$$

$$= \frac{1}{I_C} \cdot \frac{U_0}{4}$$

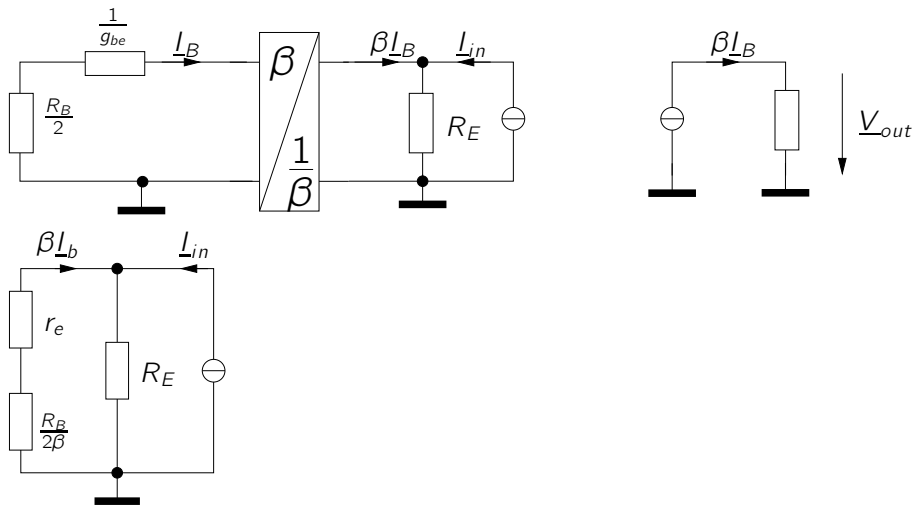


c)

Basis-Grundschtung (BGS)



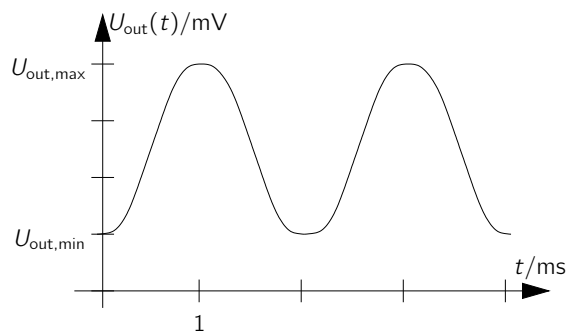
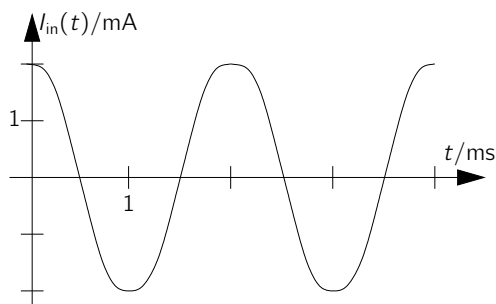
d)



$$\begin{aligned}
 V_{out} &= +R_C \cdot \beta I_b \\
 &= -R_C \cdot I_{in} \cdot \frac{R_E}{r_e + \frac{R_B}{2\beta} + R_E} \\
 Z_{trans} &= -R_C \cdot \frac{R_E}{r_e + \frac{R_B}{2\beta} + R_E}
 \end{aligned}$$

e)

$$\begin{aligned}
 Z_{trans} &= -R_C = -100 \Omega \\
 U_0 &= 4 \text{ V} \\
 U_{out}(t) &= \left( U_0 - \frac{V_{C,min} + V_{C,max}}{2} \right) - 100 \Omega \cdot I_{in}(t)
 \end{aligned}$$



$$\begin{aligned}
 U_{out,max} &= 1,2 \text{ V} \\
 U_{out,min} &= 0,8 \text{ V}
 \end{aligned}$$



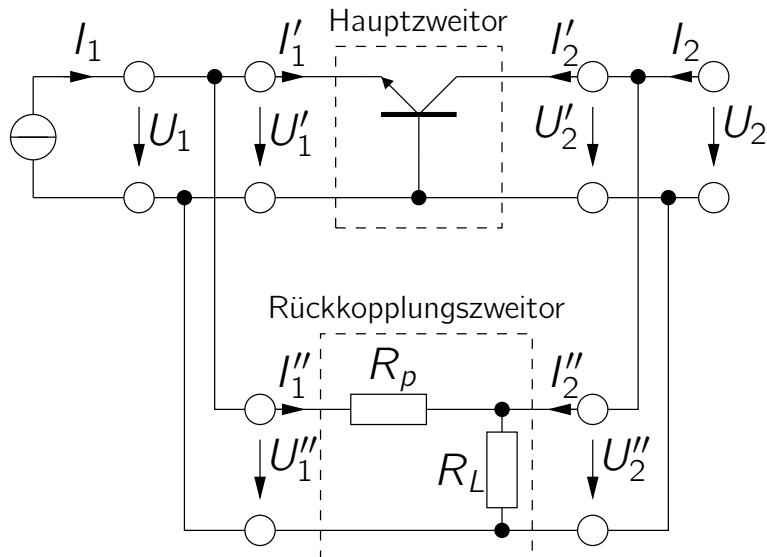
**f)**

$$3\text{V} - I_{\max} \cdot 100\ \Omega = \frac{U_0}{2} = 2\text{V}$$

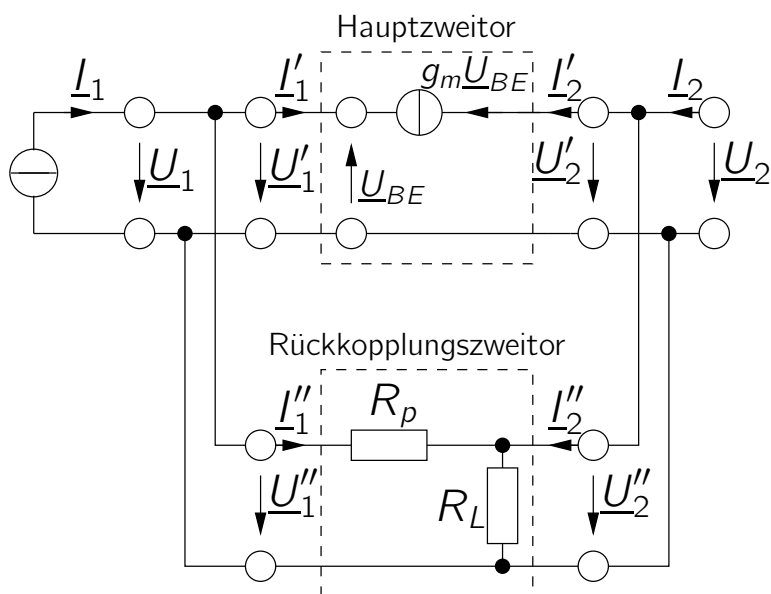
$$I_{\max} < 10\text{ mA}$$

### Aufgabe 4

a)



b)



c)

i)

Parallel-Parallel-Kopplung (PPK)

ii)

Y-(Admittanz-)Matrix

$$\begin{aligned}
 \underline{U}_1 &= \underline{U}'_1 = \underline{U}''_1 \\
 \underline{I}_1 &= \underline{I}'_1 + \underline{I}''_1 \\
 &= \underline{Y}'_{11}\underline{U}_1 + \underline{Y}''_{11}\underline{U}_1 + \underline{Y}'_{12}\underline{U}_2 + \underline{Y}''_{12}\underline{U}_2 \\
 &= (\underline{Y}'_{11} + \underline{Y}''_{11})\underline{U}_1 + (\underline{Y}'_{12} + \underline{Y}''_{12})\underline{U}_2
 \end{aligned}$$

$$\begin{aligned}
 \underline{U}_2 &= \underline{U}'_2 = \underline{U}''_2 \\
 \underline{I}_2 &= \underline{I}'_2 + \underline{I}''_2 \\
 &= \underline{Y}'_{21}\underline{U}_1 + \underline{Y}''_{21}\underline{U}_1 + \underline{Y}'_{22}\underline{U}_2 + \underline{Y}''_{22}\underline{U}_2 \\
 &= (\underline{Y}'_{21} + \underline{Y}''_{21})\underline{U}_1 + (\underline{Y}'_{22} + \underline{Y}''_{22})\underline{U}_2
 \end{aligned}$$

d)

$$\underline{Y}'_{11} = \left. \frac{\underline{I}'_1}{\underline{U}'_1} \right|_{\underline{U}'_2=0} = g_m$$

$$\underline{Y}'_{21} = \left. \frac{\underline{I}'_2}{\underline{U}'_1} \right|_{\underline{U}'_2=0} = -g_m$$

$$\underline{Y}'_{12} = \left. \frac{\underline{I}'_1}{\underline{U}'_2} \right|_{\underline{U}'_1=0} = 0$$

$$\underline{Y}'_{22} = \left. \frac{\underline{I}'_2}{\underline{U}'_2} \right|_{\underline{U}'_1=0} = 0$$

$$\underline{Y}''_{11} = \left. \frac{\underline{I}''_1}{\underline{U}''_1} \right|_{\underline{U}''_2=0} = \frac{1}{R_p}$$

$$\underline{Y}''_{21} = \left. \frac{\underline{I}''_2}{\underline{U}''_1} \right|_{\underline{U}''_2=0} = -\frac{1}{R_p}$$

$$\underline{Y}''_{12} = \left. \frac{\underline{I}''_1}{\underline{U}''_2} \right|_{\underline{U}''_1=0} = -\frac{1}{R_p}$$

$$\underline{Y}''_{22} = \left. \frac{\underline{I}''_2}{\underline{U}''_2} \right|_{\underline{U}''_1=0} = \frac{1}{R_p} + \frac{1}{R_L}$$

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}''$$

$$= \begin{pmatrix} g_m + \frac{1}{R_p} & -\frac{1}{R_p} \\ -g_m - \frac{1}{R_p} & \frac{1}{R_p} + \frac{1}{R_L} \end{pmatrix}$$

**e)**

$$I_2 = 0 = Y_{21}U_1 + Y_{22}U_2 \Leftrightarrow U_1 = -\frac{Y_{22}}{Y_{21}}U_2$$

$$I_1 = Y_{11}U_1 + Y_{12}U_2 = \left(Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right)U_2$$

$$\Leftrightarrow \frac{U_2}{I_1} = \frac{1}{Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}} = \frac{Y_{21}}{Y_{12}Y_{21} - Y_{11}Y_{22}} = Z_T$$

Einsetzen:

$$Z_T = \frac{-g_m - \frac{1}{R_p}}{-\frac{1}{R_p} \left(-g_m - \frac{1}{R_p}\right) - \left(g_m + \frac{1}{R_p}\right) \left(\frac{1}{R_p} + \frac{1}{R_L}\right)} = \frac{1}{-\frac{1}{R_p} + \frac{1}{R_p} + \frac{1}{R_L}} = R_L$$

 $\Rightarrow Z_T$  ist nur abhängig von  $R_L$ .

## Aufgabe 5

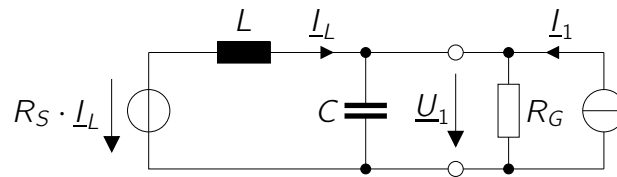


Abb. 5: Zu untersuchende Schaltung.

a)

$$\underline{U}_1 = \frac{I_L + I_1}{\frac{1}{R_G} + sC}$$

$$I_L = \frac{R_S \cdot I_L - \underline{U}_1}{sL}$$

$$I_L \left(1 - \frac{R_S}{sL}\right) = -\frac{\underline{U}_1}{sL}$$

$$\underline{U}_1 = -sL I_L \left(1 - \frac{R_S}{sL}\right) = R_S I_L - sL I_L$$

$$R_S I_L - sL I_L = \frac{I_L}{\frac{1}{R_G} + sC} + \frac{I_1}{\frac{1}{R_G} + sC}$$

$$I_L \left(\frac{R_S}{R_G} + sC R_S - \frac{sL}{R_G} - s^2 LC - 1\right) = I_1$$

$$\frac{I_L}{I_1} = \frac{1}{\frac{R_S}{R_G} + sC R_S - \frac{sL}{R_G} - s^2 LC - 1}$$

b)

Verantwortlich für Stabilität: Pole der Wirkungsfunktion

Alle Wirkungsfunktionen mit gleicher Ursache haben die gleichen Pole (Determinante der Knotenadmittanzmatrix).  $\Rightarrow$  Untersuchung der Stabilität mit jeder Wirkungsfunktion des Netzwerkes möglich.

c)

$$\frac{R_S}{R_G} - 1 + s \left( R_S C - \frac{L}{R_G} \right) - s^2 LC = 0$$

$$\underbrace{\frac{-\frac{R_S}{R_G} + 1}{LC}}_q + s \underbrace{\left( -\frac{R_S}{L} + \frac{1}{R_G C} \right)}_p + s^2 = 0$$

$$\Rightarrow s_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

mit

$$p = \frac{1}{R_G C} - \frac{R_S}{L} = \frac{L - R_G R_S C}{L R_G C}$$

$$q = \frac{1 - \frac{R_S}{R_G}}{L C} = \frac{R_G - R_S}{L R_G C}$$

**d)**

aufklingend, sinusförmig  $\Rightarrow s = \sigma + j\omega$ , mit  $\sigma > 0$

$$\Rightarrow -\frac{p}{2} > 0$$

$$\left(\frac{p}{2}\right)^2 - q < 0$$

$$-(L - R_G R_S C) > 0 \Leftrightarrow R_S > \frac{L}{R_G C}$$

$$\frac{L^2 - 2R_G R_S C L + R_G^2 R_S^2 C^2}{4L^2 R_G^2 C^2} - \frac{R_G - R_S}{L R_G C} < 0$$

$$L^2 - 2R_G R_S C L + R_G^2 R_S^2 C^2 - 4L R_G^2 C + 4L R_G R_S C < 0$$

$$L^2 + 2R_G R_S C L + R_G^2 R_S^2 C^2 - 4L R_G^2 C < 0$$

$$(L + R_G R_S C)^2 - 4L R_G^2 C < 0$$

$$L + R_G R_S C < \pm 2R_G \sqrt{L C}$$

$$R_S < \frac{\pm 2R_G \sqrt{L C} - L}{R_G C}$$

$$R_S > 0 \Rightarrow R_S < \frac{2R_G \sqrt{L C} - L}{R_G C}$$

$$R_S > \frac{L}{R_G C}$$

$$\frac{L}{R_G C} < R_S < \frac{2R_G \sqrt{L C} - L}{R_G C}$$

**e)**

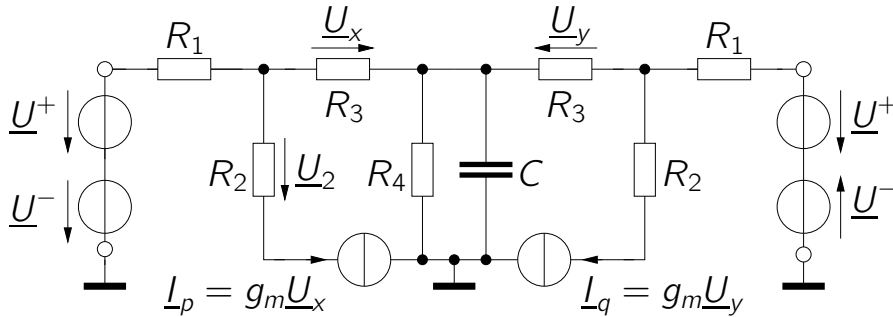
$$\frac{L}{R_G C} < \frac{2R_G \sqrt{L C} - L}{R_G C}$$

$$\cancel{L} < \cancel{2} R_G \sqrt{L C}$$

$$R_G > \sqrt{\frac{L}{C}}$$

### Aufgabe 6

a)



Phasoren:

$$\underline{U}_a = \underline{U}^+ + \underline{U}^-$$

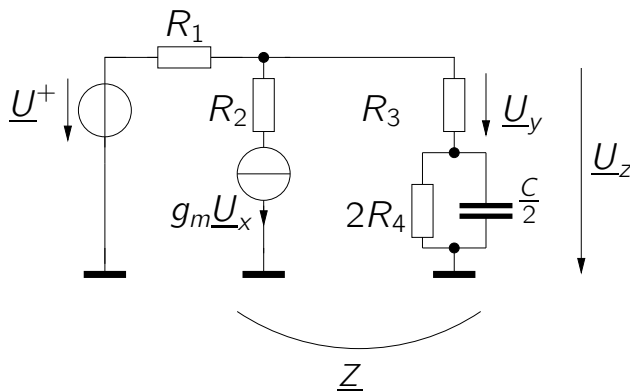
$$\underline{U}_b = \underline{U}^+ - \underline{U}^-$$

$$\underline{U}^+ = \frac{\underline{U}_a + \underline{U}_b}{2}$$

$$\underline{U}^- = \frac{\underline{U}_a - \underline{U}_b}{2}$$

b)

Gleichtakt:



$$\underline{Z} = R_3 + \left(2R_4 \parallel \frac{C}{2}\right)$$

$$\underline{U}_z = \frac{\underline{Z}}{R_3} \underline{U}_x$$

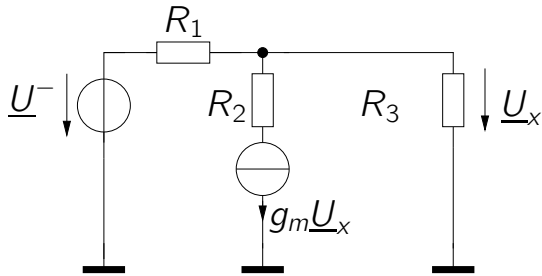
$$\frac{\underline{U}^+ - \underline{U}_z}{R_1} - g_m \underline{U}_x - \frac{\underline{U}_x}{R_3} = 0$$

$$\Leftrightarrow \frac{\underline{U}^+}{R_1} - \left(\frac{\underline{Z}}{R_3 R_1} + g_m + \frac{1}{R_3}\right) \underline{U}_x = 0$$

$$\underline{U}_x = \frac{1}{R_1 \left( \frac{Z}{R_3 R_1} + g_m + \frac{1}{R_3} \right)} \underline{U}^+$$

$$\underline{U}_2^+ = R_2 g_m \underline{X} \underline{U}^+$$

Gegentakt:



$$\frac{\underline{U}^- - \underline{U}_x}{R_1} - g_m \underline{U}_x - \frac{\underline{U}_x}{R_3} = 0$$

$$\frac{\underline{U}^-}{R_1} - \left( \frac{1}{R_1} + g_m + \frac{1}{R_3} \right) \underline{U}_x = 0$$

$$\underline{U}_x = \frac{1}{R_1 \left( \frac{1}{R_1} + g_m + \frac{1}{R_3} \right)} \underline{U}^-$$

$$\underline{U}_2^- = \frac{g_m R_2}{R_1 \left( \frac{1}{R_1} + g_m + \frac{1}{R_3} \right)} \underline{U}^- = R_2 g_m \underline{Y} \underline{U}^-$$

c)

$$\begin{aligned} \underline{U}_2 &= \underline{U}_2^+ + \underline{U}_2^- \\ &= R_2 g_m \underline{X} \underline{U}^+ + R_2 g_m \underline{Y} \underline{U}^- \\ &= R_2 g_m \underline{X} \frac{\underline{U}_a + \underline{U}_b}{2} + R_2 g_m \underline{Y} \frac{\underline{U}_a - \underline{U}_b}{2} \\ &= (\underline{X} + \underline{Y}) \frac{R_2 g_m}{2} \underline{U}_a + (\underline{X} - \underline{Y}) \frac{R_2 g_m}{2} \underline{U}_b \end{aligned}$$

d)

Vorzeichenänderung im Gegentakt bei  $g_m$ .

## Aufgabe 7

a)

$$\frac{\underline{U}_1 + \underline{U}_d}{R_1} = I_1$$



$$\frac{U_2 + U_d}{R_2} = I_2$$

$$U_d = \frac{U_2}{V_u}$$

$$I_1 + I_2 = -j\omega C U_d$$

$$\Rightarrow U_d \left( \frac{1}{R_1} + \frac{1}{R_2} + j\omega C \right) + \frac{U_1}{R_1} + \frac{U_2}{R_2} = 0$$

$$\Leftrightarrow \frac{U_2}{V_u} \left[ \frac{1}{V_u} \left( \frac{1}{R_1} + \frac{1}{R_2} + j\omega C \right) + \frac{1}{R_2} \right] = -\frac{1}{R_1} U_1$$

$$\Leftrightarrow \frac{U_2}{U_1} = -\frac{\frac{1}{R_2} + \frac{1}{V_u} \left( \frac{1}{R_1} + \frac{1}{R_2} + j\omega C \right)}{1 + \frac{1}{R_2} + j\omega C R_1} = \underline{F(j\omega)}$$

**b)**

$$|V_u| \rightarrow \infty$$

$$\Rightarrow \underline{F(j\omega)} = -\frac{R_2}{R_1} = \frac{1}{\underline{F}_2}$$

**c)**

$$\underline{F(j\omega)} = \frac{-\frac{V_u}{1 + \frac{R_1}{R_2} + j\omega R_1 C}}{1 + \underbrace{\left( -\frac{R_1}{R_2} \right) \left( -\frac{V_u}{1 + \frac{R_1}{R_2} + j\omega R_1 C} \right)}_{E_0 \text{ (Schleifenverstärkung)}}}$$

$$\underline{F}_2 = -\frac{R_1}{R_2}$$

$$\underline{F}_a = -\frac{V_u}{1 + \frac{R_1}{R_2} + j\omega R_1 C} = -\frac{V_u}{\left( 1 + \frac{R_1}{R_2} \right) \left( 1 + j\omega \frac{R_1 R_2}{R_1 + R_2} C \right)}$$

**d)**

$$V_u = \frac{V_0}{\left( 1 + \frac{j\omega}{10\omega_0} \right) \left( 1 + \frac{j\omega}{10000\omega_0} \right)}$$

$$C = \frac{R_1 + R_2}{\omega_0 R_1 R_2}$$

$$\Rightarrow \underline{F}_a = -\frac{\frac{V_0 R_2}{R_1 + R_2}}{\left( 1 + \frac{j\omega}{\omega_0} \right) \left( 1 + \frac{j\omega}{10\omega_0} \right) \left( 1 + \frac{j\omega}{10000\omega_0} \right)}$$

