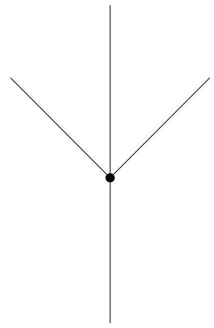
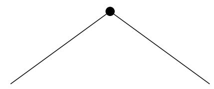
Aufgabe 1

a)

Baum:

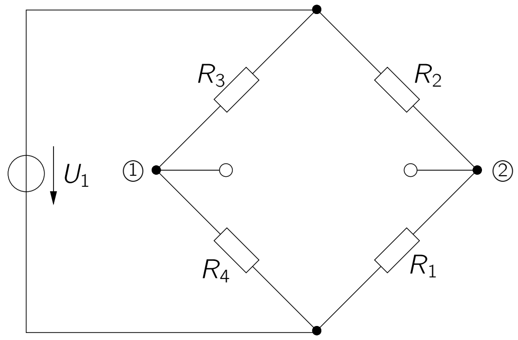


Co-Baum:



b)

Vereinfachtes Netzwerk:



Berechnung der Spannung U_{12}:

$$U_{12} = U_1 \cdot \left(\frac{R_4}{R_3 + R_4} - \frac{R_1}{R_1 + R_2} \right)$$

c)

Berechung der Verlustleistung:

$$P_{\Sigma} = U_1^2 \cdot \left(\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \right)$$

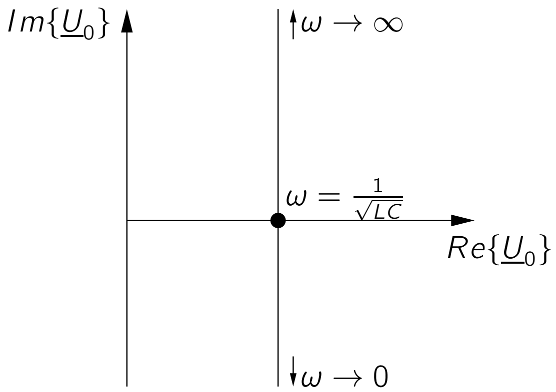
Aufgabe 2

a)

$$\frac{\underline{U}_1}{\underline{U}_0} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$

$$\frac{\underline{U}_2}{\underline{U}_0} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{C_1}{C_1 + C_2}$$

b)



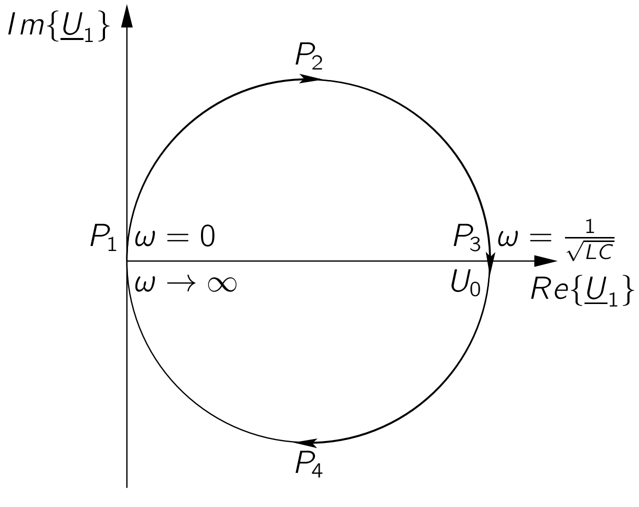
$$\frac{\underline{U}_0}{\underline{U}_1} = 1 + j\omega\frac{L}{R} + \frac{1}{j\omega RC}$$

$$\omega\frac{L}{R} = \frac{1}{\omega RC}$$

$$\Leftrightarrow \omega^2 = \frac{1}{LC}$$

$$\Leftrightarrow \omega = \sqrt{\frac{1}{LC}}$$

→ Kehrwertbildung: $\frac{\underline{U}_1}{\underline{U}_0} = \frac{R}{R+j\omega L+\frac{1}{j\omega C}}$



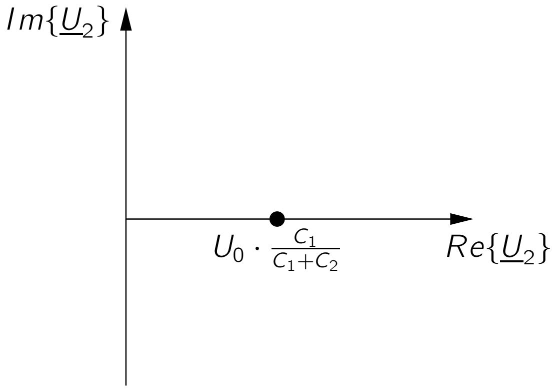
$$P_1 : U_1 = 0 \qquad \omega = 0, \omega = \infty$$

$$P_2 : U_1 = \frac{U_0}{2} + j\frac{U_0}{2}$$

$$P_3 : U_1 = U_0 \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$P_4 : U_1 = \frac{U_0}{2} - j\frac{U_0}{2}$$

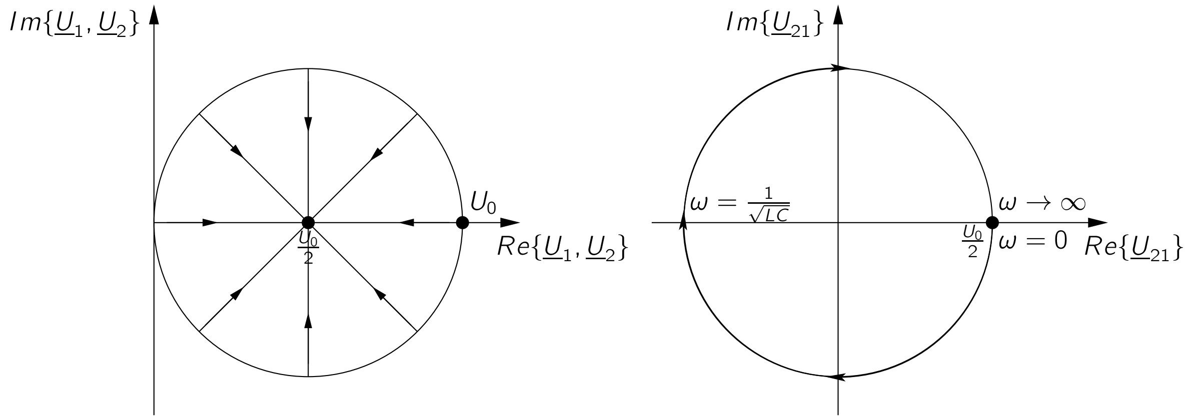
c)



d)

$$U_0 \cdot \frac{C_1}{C_1 + C_2} = \frac{U_0}{2} \Rightarrow C_2 = C_1$$

e)



Aufgabe 3

a)

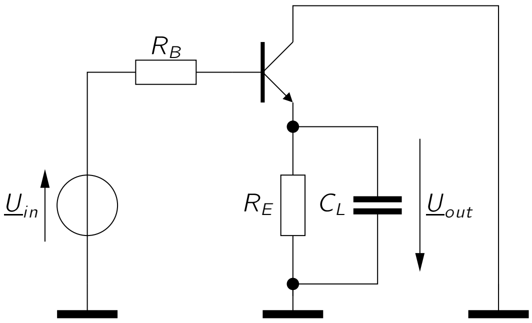
$$I_C = \frac{U_0 - U_{BE,0} - U_{in}}{R_E}$$
$$g_m = \frac{I_C}{U_T} = \frac{U_0 - U_{BE,0} - U_{in}}{U_T R_E}$$

b)

Mit: $U_{BE} > 0$ $\quad U_{CE} > U_{BE}$ $\quad (U_{CB} > 0)$

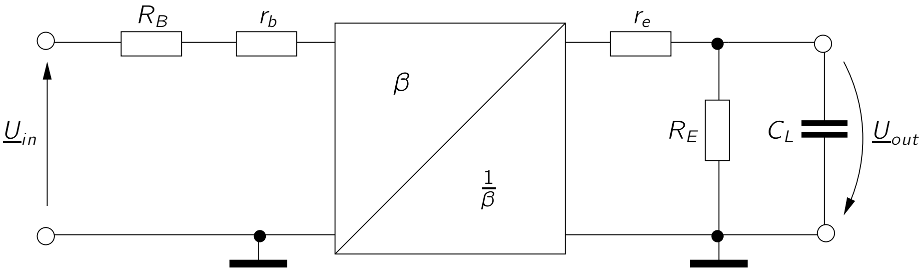
minimales $U_{in} = 0\,\mathrm{V}$
maximales $U_{in} = U_0 - U_{BE}$, für $U_{R_E} = 0\,\mathrm{V}$

c)



$\Rightarrow$ Kollektorgrundschaltung

d)



$$\frac{\underline{U}_{out}}{\underline{U}_{in}} = -\frac{R_E \Big|\Big| \frac{1}{j\omega C_L}}{R_E \Big|\Big| \frac{1}{j\omega C_L} + \frac{1}{\beta}\left(\frac{1}{g_{be}} + r_b + R_B\right)}$$

$$= -\frac{R_E}{R_E + j\omega C_L \left(R_E + \frac{1}{j\omega C_L}\right) \cdot \frac{1}{\beta}\left(\frac{1}{g_{be}} + r_b + R_B\right)}$$

e)

$$\left.\frac{\underline{U}_{out}}{\underline{U}_{in}}\right|_{\omega=0} = -\frac{R_E}{R_E + \underbrace{\frac{1}{\beta}\left(\frac{1}{g_{be}} + r_b + R_B\right)}_{R_E \text{ lt. Text}}} = -\frac{R_E}{R_E + R_E} = -\frac{1}{2}$$

$$\left|\left.\frac{\underline{U}_{out}}{\underline{U}_{in}}\right|_{\omega=0}\right| = \frac{R_E}{R_E + (j\omega_0 C_L R_E + 1)\cdot R_E}$$

$$= \frac{R_E}{2R_E + j\omega_0 C_L R_E^2}$$

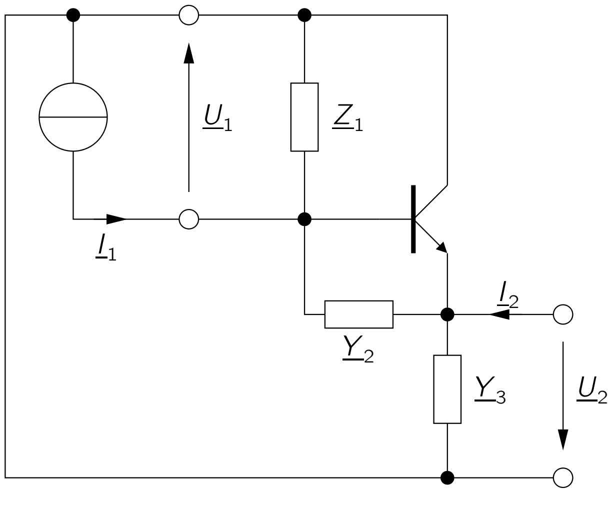
$$= \frac{1}{2 + j\omega_0 C_L R_E} = \frac{1}{2\cdot\sqrt{2}}$$

$$\Rightarrow \omega_0 C_L R_E = 2$$

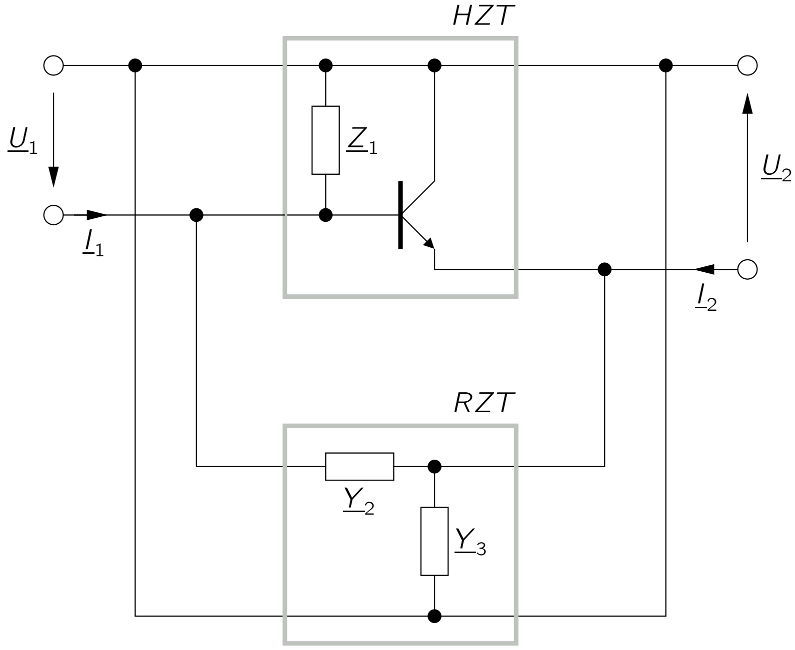
$$\Leftrightarrow \omega_0 = \frac{2}{C_L R_E}$$

Aufgabe 4

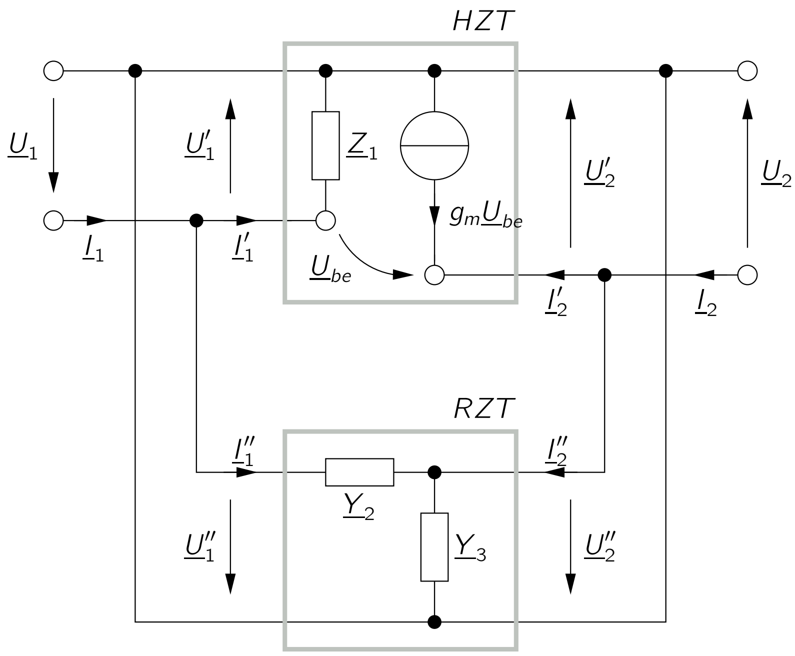
a)



b)



c)



d)

i) Parallel-Parallel-Kopplung

ii) Y-Matrizen

$$\begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} \underline{I}'_1 \\ \underline{I}'_2 \end{pmatrix} + \begin{pmatrix} \underline{I}''_1 \\ \underline{I}''_2 \end{pmatrix}$$

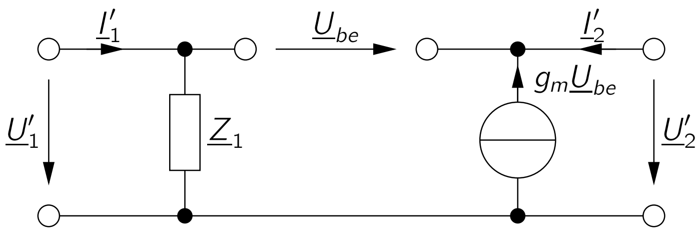
$$= \underline{Y}' \begin{pmatrix} \underline{U}'_1 \\ \underline{U}'_2 \end{pmatrix} + \underline{Y}'' \begin{pmatrix} \underline{U}''_1 \\ \underline{U}''_2 \end{pmatrix}$$

$$= \underline{Y}' \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} + \underline{Y}'' \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix}$$

$$= \left[\underline{Y}' + \underline{Y}''\right] \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix}$$

e)

HZT:

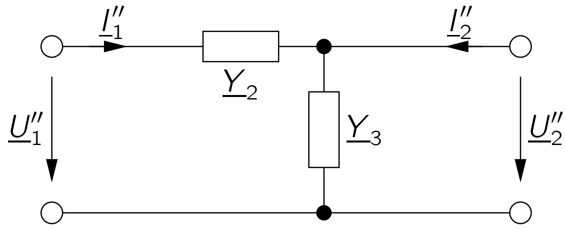


$$\underline{Y}'_{11} = \left.\frac{\underline{I}'_1}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = \frac{1}{\underline{Z}_1} \qquad \underline{Y}'_{12} = \left.\frac{\underline{I}'_1}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = 0$$

$$\underline{Y}'_{21} = \left.\frac{\underline{I}'_2}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = -g_m \qquad \underline{Y}'_{22} = \left.\frac{\underline{I}'_2}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = g_m$$

$$\Rightarrow \underline{Y}' = \begin{pmatrix} \frac{1}{\underline{Z}_1} & 0 \\ -g_m & g_m \end{pmatrix}$$

RZT:



$\underline{Y}_{11}'' = \underline{Y}_2$ $\underline{Y}_{12}'' = -\underline{Y}_2$

$\underline{Y}_{21}'' = -\underline{Y}_2$ $\underline{Y}_{22}'' = \underline{Y}_2 + \underline{Y}_3$

$$\Rightarrow \underline{Y}'' = \begin{pmatrix} \underline{Y}_2 & -\underline{Y}_2 \\ -\underline{Y}_2 & \underline{Y}_2 + \underline{Y}_3 \end{pmatrix}$$

Gesamtmatrix:

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}'' = \begin{pmatrix} \frac{1}{\underline{Z}_1} + \underline{Y}_2 & -\underline{Y}_2 \\ -\underline{Y}_2 - g_m & \underline{Y}_2 + \underline{Y}_3 + g_m \end{pmatrix}$$

f)

$$\underline{Z}_{11} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{I}_2=0}$$

$$\overset{\underline{I}_2=0}{\Rightarrow} \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 = 0$$

$$\Leftrightarrow \quad \underline{U}_2 = -\frac{\underline{Y}_{21}}{\underline{Y}_{22}}\underline{U}_1$$

liefert eingesetzt:

$$\Rightarrow \underline{I}_1 = \underline{Y}_{11}\underline{U}_1 - \frac{\underline{Y}_{21}\underline{Y}_{12}}{\underline{Y}_{22}}\underline{U}_1$$

$$\Leftrightarrow \frac{\underline{U}_1}{\underline{I}_1} = \frac{1}{\underline{Y}_{11} - \frac{\underline{Y}_{21}\underline{Y}_{12}}{\underline{Y}_{22}}}$$

$$= \frac{1}{\frac{1}{\underline{Z}_1} + \underline{Y}_2 - \frac{\underline{Y}_2(\underline{Y}_2 - g_m)}{\underline{Y}_2 + \underline{Y}_3 + g_m}}$$

$$= \frac{\underline{Y}_2 + \underline{Y}_3 + g_m}{\left(\frac{1}{\underline{Z}_1} + \underline{Y}_2\right)(\underline{Y}_2 + \underline{Y}_3 + g_m) - \underline{Y}_2\left(\underline{Y}_2 + g_m\right)}$$

alternativ $\underline{Z}_{11}$ aus Z-Matrix:

$$\begin{aligned}\underline{Z} &= \underline{Y}^{-1}\\ &= \frac{1}{\underline{Y}_{11}\underline{Y}_{22} - \underline{Y}_{21}\underline{Y}_{12}} \begin{pmatrix} \underline{Y}_{22} & -\underline{Y}_{12} \\ -\underline{Y}_{21} & \underline{Y}_{11} \end{pmatrix}\\ \Rightarrow \underline{Z}_{11} &= \frac{\underline{Y}_{22}}{\underline{Y}_{11}\underline{Y}_{22} - \underline{Y}_{21}\underline{Y}_{12}}\\ &= \frac{\underline{Y}_2 + \underline{Y}_3}{\left(\frac{1}{\underline{Z}_1} + \underline{Y}_2\right)(\underline{Y}_2 + \underline{Y}_3) - \underline{Y}_2\left(\underline{Y}_2 + g_m\right)}\end{aligned}$$

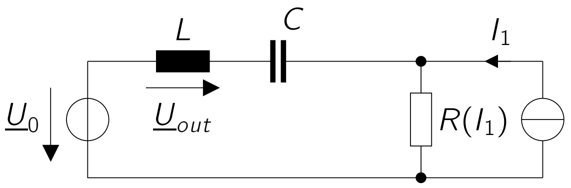
Aufgabe 5

a)

$$\underline{Y}_{in} = \frac{\underline{I}_0}{\underline{U}_0} = \frac{1}{sL + \frac{1}{sC} + R(I_1)}$$

$$= \frac{sC}{s^2LC + sRC + 1}$$

b)



$H = \frac{\underline{U}_{out}}{\underline{U}_0}$, da wenn $\underline{U}_0 \to 0$ sich Netzwerktopologie nicht verändert

c)

Polstellen von $\underline{Y}_{in}$:

$$s^2LC + sRC + 1 = 0$$

$$\Leftrightarrow \quad \left(s + \frac{R}{2L}\right)^2 = \left(\frac{R}{2L}\right)^2 - \frac{1}{2L}$$

$$\Leftrightarrow \quad s = \pm\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} - \frac{R}{2L}$$

aufklingend:

$$\mathrm{Re}\{s\} > 0$$

$$-\frac{R}{2L} > 0$$

$$R < 0$$

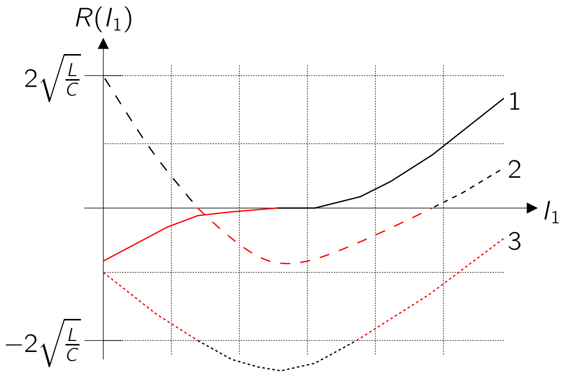
sinusförmig:

$$\mathrm{Im}\{s\} \neq 0$$

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{2L} < 0$$

$$|R| < \sqrt{2\frac{L}{C}}$$

d)



e)

$$i(t) = \sum_k \frac{Z(s_k)}{N'(s_k)} e^{s_k t} \qquad , s_k \text{ Polstellen}$$

$$Z(s_k) = sC$$

$$N'(s_k) = 2sLC + RC$$

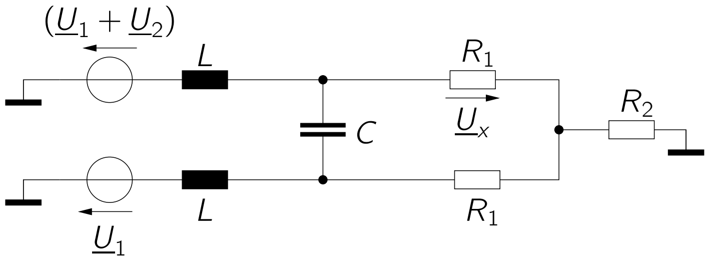
$$\Rightarrow i(t) = c_1 e^{st} + c_2 e^{s^* t}$$

$$= \underbrace{e^{\sigma t}}_{\text{aufklingend}} \left(c_1 \underbrace{e^{j\omega t} + c_2 e^{-j\omega t}}_{\text{sinusförmig}}\right)$$

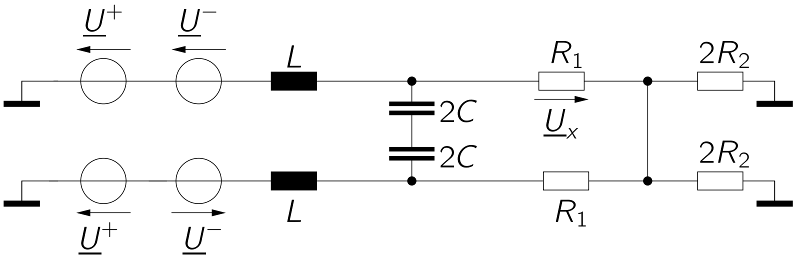
Aufgabe 6

a)

Quellen anders anordnen (äquivalente Umformung):



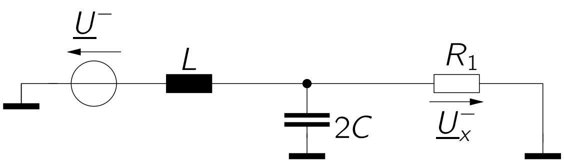
Darstellen durch Gleich- und Gegentaktquellen:



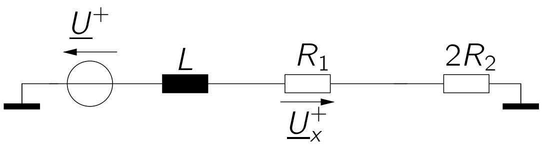
$$\underline{U}_1 + \underline{U}_2 = \underline{U}^+ + \underline{U}^-$$
$$\underline{U}_1 = \underline{U}^+ - \underline{U}^-$$
$$\Leftrightarrow \qquad \underline{U}^+ = \underline{U}_1 + \frac{1}{2}\underline{U}_2$$
$$\underline{U}^- = \frac{1}{2}\underline{U}_2$$

b)

Einphasiges Gegentakt-Ersatzschaltbild:



Einphasiges Gleichtakt-Ersatzschaltbild:



c)

$$\underline{U}_x^- = \frac{\frac{1}{\frac{1}{R_1}+j\omega 2C}}{j\omega L + \frac{1}{\frac{1}{R_1}+j\omega 2C}}\underline{U}^- = \frac{1}{1-2\omega^2 LC + j\omega\frac{L}{R_1}}\underline{U}^-$$

$$\underline{U}_x^+ = \frac{R_1}{j\omega L + R_1 + 2R_2}\underline{U}^+$$

$$\Rightarrow \qquad \underline{U}_x = \underline{U}_x^- + \underline{U}_x^+ = \frac{1}{1-2\omega^2 LC + j\omega\frac{L}{R_1}}\underline{U}^- + \frac{R_1}{j\omega L + R_1 + 2R_2}\underline{U}^+$$

$$= \frac{1}{1-2\omega^2 LC + j\omega\frac{L}{R_1}}\frac{1}{2}\underline{U}_2 + \frac{R_1}{j\omega L + R_1 + 2R_2}\left(\underline{U}_1 + \frac{1}{2}\underline{U}_2\right)$$

d)

R_2 weglassen $\Rightarrow R_2 \to \infty$.

$$\underline{U}_x = \frac{1}{1-2\omega^2 LC + j\omega\frac{L}{R_1}}\frac{1}{2}\underline{U}_2 + \frac{R_1}{j\omega L + R_1 + 2R_2}\left(\underline{U}_1 + \frac{1}{2}\underline{U}_2\right) \to \frac{1}{1-2\omega^2 LC + j\omega\frac{L}{R_1}}\frac{1}{2}\underline{U}_2$$

Nur noch der Gegentaktpfad trägt zum Ergebnis bei. Insbesondere leistet die Quelle $\underline{U}_1$ keinen Beitrag mehr. Dies ist auch aus dem Schaltplan ersichtlich, da der Masseknoten nur noch mit der Quelle $\underline{U}_1$ verbunden ist und an kein weiteres Bauelement angeschlossen ist. Die Quelle $\underline{U}_1$ ist damit wirkungslos.

Aufgabe 7

a)

$$\underline{U}_+ = \frac{R_1}{R_1 + \frac{1}{j\omega C}}\underline{U}_1 = \frac{j\omega R_1 C}{1 + j\omega R_1 C}\underline{U}_1$$

$$\underline{U}_- = \frac{R_1}{R_1 + R_2}\underline{U}_2$$

$$\underline{U}_2 = \underline{v}_u(j\omega)\underline{U}_d = \underline{v}_u(j\omega)(\underline{U}_+ - \underline{U}_-) = \underline{v}_u(j\omega)\left(\frac{j\omega R_1 C}{1 + j\omega R_1 C}\underline{U}_1 - \frac{R_1}{R_1 + R_2}\underline{U}_2\right)$$

$$\left(1 + \frac{\underline{v}_u(j\omega)R_1}{R_1 + R_2}\right)\underline{U}_2 = \underline{v}_u(j\omega)\frac{j\omega R_1 C}{1 + j\omega R_1 C}\underline{U}_1$$

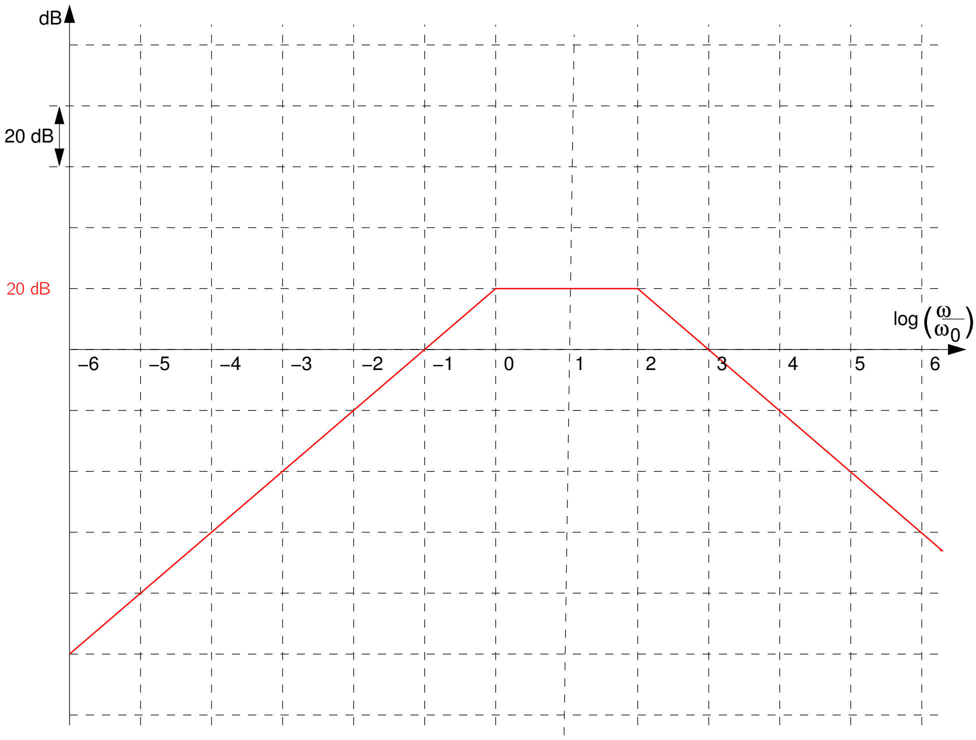
$$\underline{F}(j\omega)\frac{\underline{U}_2}{\underline{U}_1} = \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{\underline{v}_u(j\omega)}{\left(1 + \frac{\underline{v}_u(j\omega)R_1}{R_1+R_2}\right)} = \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{1}{\left(\frac{1}{\underline{v}_u(j\omega)} + \frac{R_1}{R_1+R_2}\right)}$$

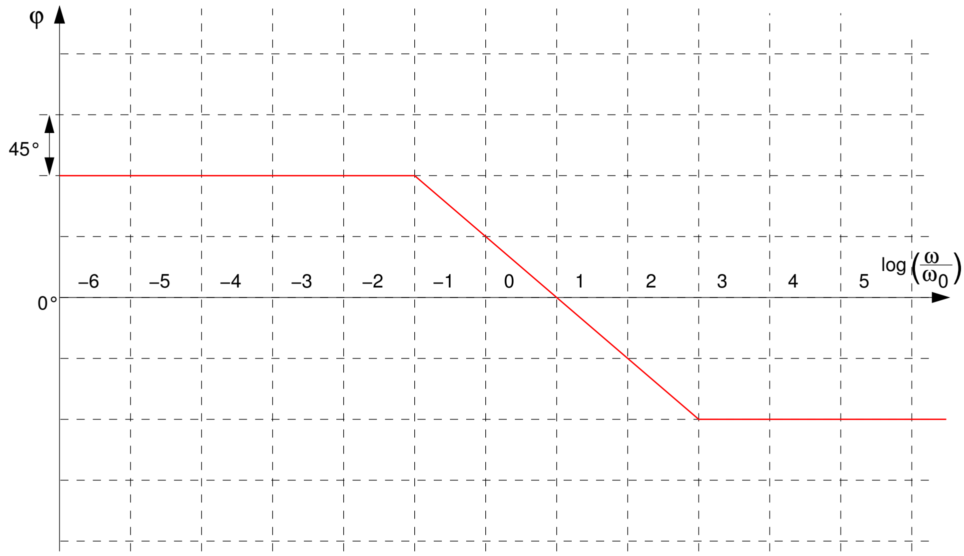
b)

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{1}{\left(\frac{1}{\underline{v}_u(j\omega)} + \frac{R_1}{R_1+R_2}\right)} \xrightarrow{|\underline{v}_u(j\omega)|\to\infty} \frac{R_1 + R_2}{R_1}\frac{j\omega R_1 C}{1 + j\omega R_1 C}$$

c)

$$\begin{aligned}
\underline{F}(j\omega) &= \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{1}{\left(\frac{1}{\underline{v}_u(j\omega)} + \frac{R_1}{R_1+R_2}\right)} = \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{1}{\left(\frac{1+\frac{j\omega}{\omega_0}}{v_0} + \frac{R_1}{R_1+R_2}\right)} \\
&= \frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{v_0}{\left(1 + \frac{j\omega}{\omega_0} + \frac{v_0 R_1}{R_1+R_2}\right)} = \frac{v_0}{1 + \frac{v_0 R_1}{R_1+R_2}}\frac{j\omega R_1 C}{1 + j\omega R_1 C}\frac{1}{1 + \frac{j\omega}{\omega_0\left(1+\frac{v_0 R_1}{R_1+R_2}\right)}} \\
&= 10\frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}}\frac{1}{1 + \frac{j\omega}{100\omega_0}}
\end{aligned}$$





d)

Der Betragsgang schneidet die 0-dB-Achse bei $\frac{1}{10}\omega_0$ und bei $1000\omega_0$.

e)

Betrachten des Frequenzgangs zeigt, dass der Vorfaktor gleichermaßen von R_2 abhängt wie die zweite Knickposition. Die Position des zweiten Nulldurchgangs bleibt daher unverändert.

$$\underline{F}(j\omega) = \frac{v_0}{1+\frac{v_0 R_1}{R_1+R_2}} \frac{j\omega R_1 C}{1+j\omega R_1 C} \frac{1}{1+\frac{j\omega}{\omega_0\left(1+\frac{v_0 R_1}{R_1+R_2}\right)}} := \frac{v_0}{f(R_2)} \frac{j\omega R_1 C}{1+j\omega R_1 C} \frac{1}{1+\frac{j\omega}{\omega_0 f(R_2)}}$$

Die erste Knickfrequenz hängt nicht von R_2 ab. Es ändert sich jedoch der maximale Betrag:

$$\begin{aligned}\underline{F}(j\omega) &= \frac{v_0}{1+\frac{v_0 R_1}{R_1+R_2}} \frac{j\omega R_1 C}{1+j\omega R_1 C} \frac{1}{1+\frac{j\omega}{\omega_0\left(1+\frac{v_0 R_1}{R_1+R_2}\right)}} \\ &\approx 2\frac{\frac{j\omega}{\omega_0}}{1+\frac{j\omega}{\omega_0}} \frac{1}{1+\frac{j\omega}{500\omega_0}}\end{aligned}$$

Der gesamte Betragsgang wird um den Faktor 5 abgesenkt (von 20 dB auf 6 dB). Entsprechend steigt die erste Nullstelle um den Faktor 5 auf $\frac{1}{2}\omega_0$ an.