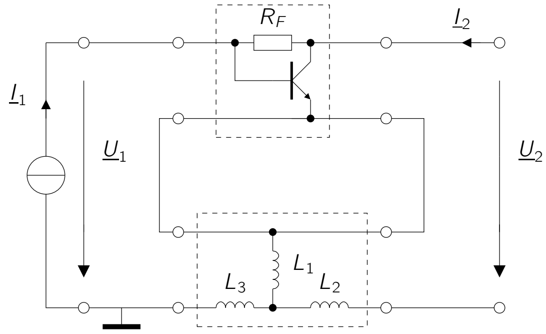
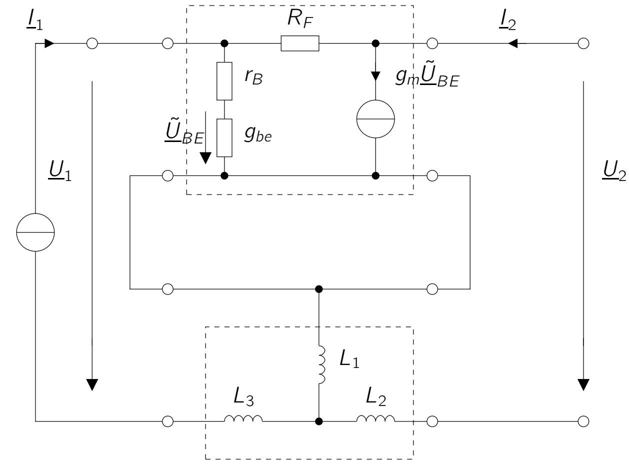
Aufgabe 1

a) Wechselstromersatzschaltbild



b) Kleinsignalersatzschaltbild



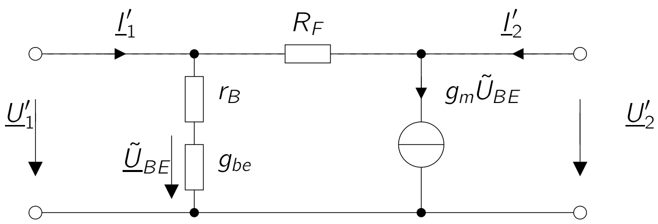
c) (i) seriell-seriell-Kopplung (SSK)

(ii) Z-Parameter , da

$$\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} = \begin{pmatrix} \underline{U}_1' \\ \underline{U}_2' \end{pmatrix} + \begin{pmatrix} \underline{U}_1'' \\ \underline{U}_2'' \end{pmatrix} = [\underline{Z}'] \begin{pmatrix} \underline{I}_1' \\ \underline{I}_2' \end{pmatrix} + [\underline{Z}''] \begin{pmatrix} \underline{I}_1'' \\ \underline{I}_2'' \end{pmatrix} = \underbrace{([\underline{Z}'] + [\underline{Z}''])}_{[\underline{Z}]} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix}$$

⇒Gesamtmatrix ist die Summe der Teilmatrizen

d) Hauptzweitor:



$$\underline{Z}'_{11} = \left.\frac{\underline{U}'_1}{\underline{I}'_1}\right|_{\underline{I}'_2=0} : \ \underline{I}'_2 = \frac{\underline{U}'_1}{r_B + \frac{1}{g_{be}}} + g_m \tilde{U}_{be} = \frac{\underline{U}'_1}{r_B + \frac{1}{g_{be}}} + g_m \frac{\frac{1}{g_{be}}}{r_B + \frac{1}{g_{be}}} \underline{U}'_1$$

$$= \frac{1+\beta}{r_b + \frac{1}{g_{be}}} \underline{U}'_1$$

$$\Rightarrow \underline{Z}'_{11} = \frac{r_B + \frac{1}{g_{be}}}{1+\beta_0} = \frac{1 + r_B g_{be}}{g_{be} + g_m}$$

$$\underline{Z}'_{12} = \left.\frac{\underline{U}'_1}{\underline{I}'_2}\right|_{\underline{I}'_1=0} : \ \underline{I}'_2 = g_m \tilde{U}_{BE} + g_{be} \underline{\tilde{U}}_{BE} \ ; \ \underline{\tilde{U}}_{BE} = \underline{U}'_1 \frac{\frac{1}{g_{be}}}{r_B + \frac{1}{g_{be}}} = \frac{1}{1 + g_{be} r_B} \underline{U}'_1$$

$$\Rightarrow \underline{I}'_2 = (g_m + g_{be}) \frac{1}{1 + g_{be} r_B} \underline{U}'_1$$

$$\Rightarrow \underline{Z}'_{12} = \frac{1 + g_{be} r_B}{g_m + g_{be}} = \frac{r_B + \frac{1}{g_{be}}}{1+\beta_0}$$

$$\underline{Z}'_{21} = \left.\frac{\underline{U}'_2}{\underline{I}'_1}\right|_{\underline{I}'_2=0} : \ \underline{I}'_1 = (g_{be} + g_m) \tilde{U}_{BE} \ ; \ \underline{U}'_2 = \underline{U}'_1 - R_F g_m \underline{\tilde{U}}_{BE} = \underline{\tilde{U}}_{BE} + r_B g_{be} \underline{\tilde{U}}_{BE} - R_F g_m \underline{\tilde{U}}_{BE}$$

$$\Rightarrow \underline{Z}'_{21} = \frac{1 - r_b g_{be} - R_F g_m}{g_{be} + g_m} = \frac{\frac{1}{g_{be}} + r_B - \beta_0 R_F}{1+\beta_0}$$

$$\underline{Z}'_{22} = \left.\frac{\underline{U}'_2}{\underline{I}'_2}\right|_{\underline{I}'_1=0} : \ \underline{\tilde{U}}_{BE} = \underline{U}'_1 \frac{1}{1 + g_{be} r_B}$$

$$\underline{U}'_1 = \underline{U}'_2 - R_F g_{be} \underline{\tilde{U}}_{BE}$$

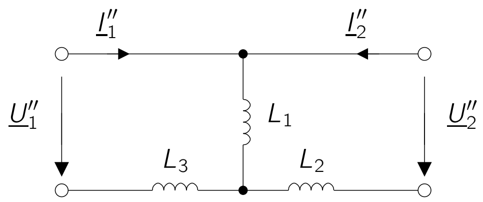
$$\Rightarrow \underline{\tilde{U}}_{BE} = \underline{U}'_2 \frac{1}{1 + g_{be} r_B} - \frac{R_F g_{be}}{1 + g_{be} r_B} \underline{\tilde{U}}_{BE} \Leftrightarrow (1 + g_{be} r_B + R_F g_{be}) \underline{\tilde{U}}_{BE} = \underline{U}'_2$$

$$\underline{I}'_2 = (g_m + g_{be}) \underline{\tilde{U}}_{BE}$$

$$\Rightarrow \underline{Z}'_{22} = \frac{1 + g_{be} r_B + R_F g_{be}}{g_m + g_{be}} = \frac{\frac{1}{g_{be}} + r_B + R_F}{1+\beta_0}$$

$$\Rightarrow [\underline{Z}'] = \begin{pmatrix} \frac{1 + r_B g_{be}}{g_{be} + g_m} & \frac{1 + r_B g_{be}}{g_{be} + g_m} \\ \frac{1 + r_B g_{be} - R_F g_m}{g_{be} + g_m} & \frac{1 + r_B g_{be} + R_F g_m}{g_{be} + g_m} \end{pmatrix}$$

Rückkoppelzweitor:



$$\left.\begin{array}{r}\underline{Z}_{11}'' = j\omega(L_1 + L_3)\\ \underline{Z}_{12}'' = \underline{Z}_{21}'' = j\omega L_1\\ \underline{Z}_{22}'' = j\omega(L_1 + L_2)\end{array}\right\} \Rightarrow [\underline{Z}''] = \begin{pmatrix} j\omega(L_1 + L_3) & j\omega L_1 \\ j\omega L_1 & j\omega(L_1 + L_2)\end{pmatrix}$$

$$[Z] = [\underline{Z}'] + [\underline{Z}'']$$

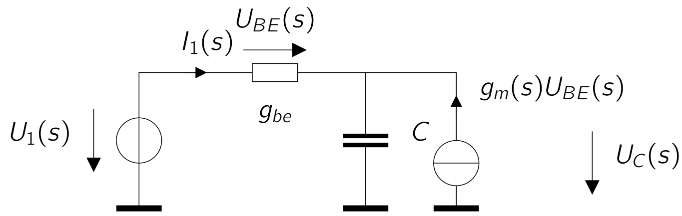
e) Verstärkung:

$$\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} = \begin{pmatrix} \underline{Z}_{11} & \underline{Z}_{22} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{pmatrix} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix}$$

$$\Rightarrow \underline{V} = \left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{\underline{I}_2=0} = \underline{Z}_{21}$$

Aufgabe 2

a) Wechselstrom-Kleinsignal-Ersatzschaltbild:



b)

$$
\begin{aligned}
I_1(s) &= g_{be}U_{BE}(s) \\
U_{BE}(s) &= U_1(s) - U_C(s) \\
U_C(s) &= \frac{1}{sC}(I_1(s) + g_m(s)U_{BE}(s)) = \frac{1}{sC}(g_{be} + g_m(s))U_{BE}(s) \\
\Leftrightarrow U_{BE}(s) &= U_1(s) - \frac{1}{sC}(g_{be} + g_m(s))U_{BE}(s) \Leftrightarrow U_{BE}(s) = \frac{1}{1 + \frac{1}{sC}(g_{be} + g_m(s))}U_1(s) \\
\Rightarrow Y_1(s) &= \frac{g_{be}}{1 + \frac{1}{sC}(g_{be} + g_m(s))} = \frac{sCg_{be}}{sC + g_{be} + g_m(s)}
\end{aligned}
$$

c) Nein, da andere Ursache und damit andere Netzwerktopologie

d) Polstelle, Wirkfunktion

$$
Y_1(s) = \frac{sCg_{be}}{sC + g_{be} + \frac{g_{m0}}{1 + \frac{s}{\omega 0}}} = \frac{sCg_{be}(1 + \frac{s}{\omega 0})}{sC(1 + \frac{s}{\omega 0}) + g_{be}(1 + \frac{s}{\omega 0}) + g_{m0}}
$$

$$
\begin{aligned}
& sC(1 + \frac{s}{\omega_0}) + g_{be}(1 + \frac{s}{\omega_0}) + g_{m0} = 0 \Leftrightarrow s^2\frac{C}{\omega 0} + sC + s\frac{g_{be}}{\omega_0} + g_{be} + g_{m0} = 0 \\
& \Leftrightarrow s^2 + s(\omega_0 + \frac{g_{be}}{C}) + (g_{be} + g_{m0})\frac{\omega_0}{C} = 0 \\
& \left(s + \frac{1}{2}(\omega_0 + \frac{g_{be}}{C})\right)^2 = -(g_{be} + g_{m0})\frac{\omega_0}{C} + \left(\frac{1}{2}(\omega_0 + \frac{g_{be}}{C})\right)^2 \\
& \Leftrightarrow s_{1,2} = -\frac{1}{2}(\omega_0 + \frac{g_{be}}{C}) \pm \sqrt{\frac{1}{4}(\omega_0 + \frac{g_{be}}{C})^2 - (g_{be} + g_{m0})\frac{\omega_0}{C}}
\end{aligned}
$$

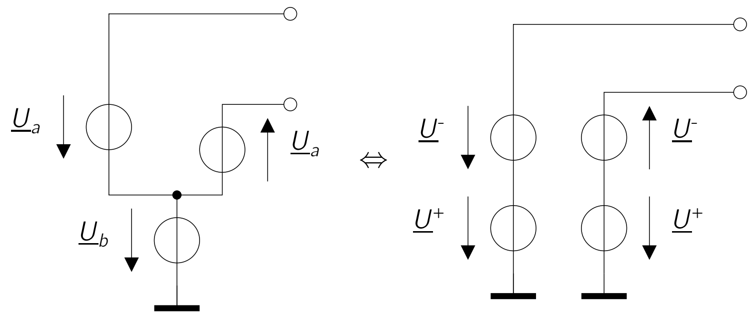
e) stabil $\Rightarrow Re\{S_{1,2}\}<0$

$$
\Rightarrow \text{entweder Wurzel imaginär oder Wurzel} < +\frac{1}{2}(\omega_0 + \frac{g_{be}}{C})
$$

$$
\Rightarrow (g_{be} + g_{m0})\frac{\omega_0}{C} > 0 \Rightarrow C < \infty \text{ und } C > 0
$$

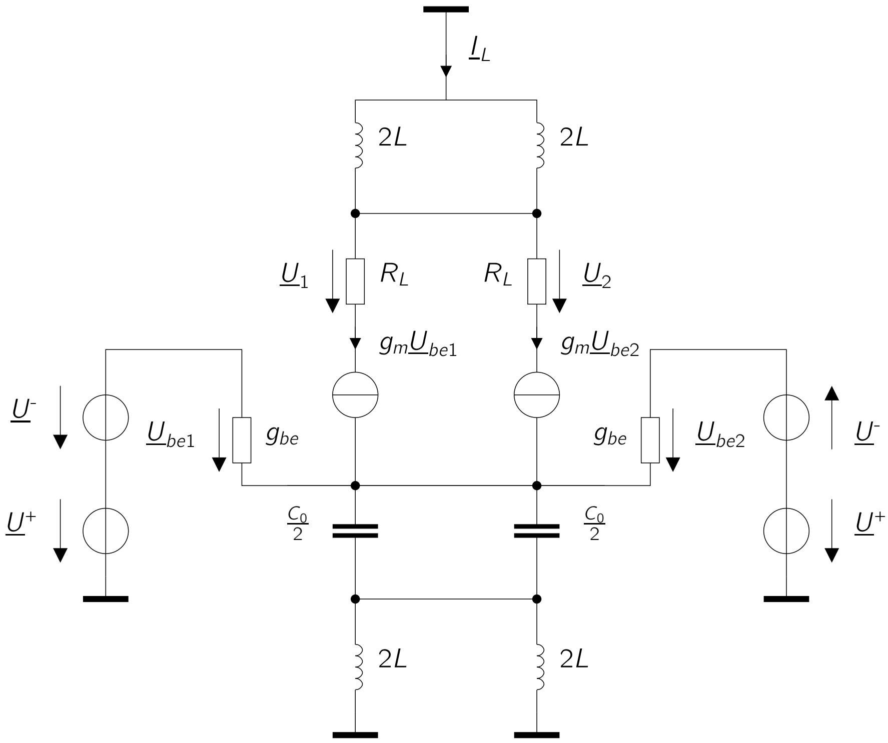
Aufgabe 3

a)

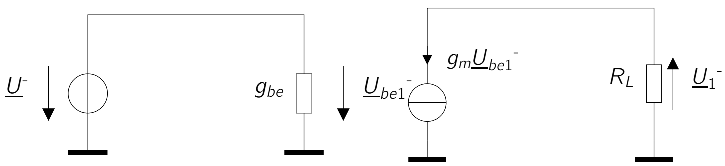


$$\underline{U}^{+} = \underline{U}_b \quad ; \quad \underline{U}^{-} = \underline{U}_a$$

b) Ersatzschaltbild

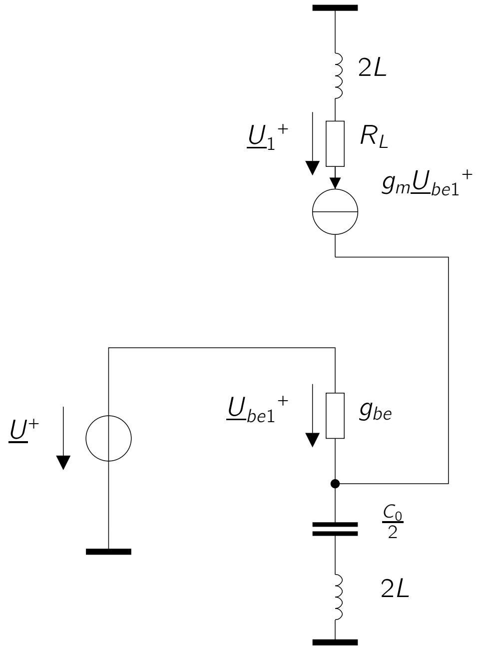


<u>Gegentakt</u>:



$$\Rightarrow \underline{U}_1^{-} = R_L g_m \underline{U}_{be1}^{-} = R_L g_m \underline{U}^{-}$$

<u>Gleichtakt</u>:



$$\underline{U}_1{}^+ = R_L g_m \underline{U}_{be1}{}^+$$

$$\underline{U}_{be1}{}^+ = \underline{U}^+ - \left(j\omega 2L + \frac{1}{j\omega\frac{C_0}{2}}\right)(g_m + g_{be})\underline{U}_{be1}{}^+$$

$$\Leftrightarrow \underline{U}_{be1}{}^+ = \frac{1}{1 + (g_{be} + g_m)\left(j\omega 2L + \frac{1}{j\omega\frac{C_0}{2}}\right)}\underline{U}^+$$

$$\Rightarrow \underline{U}_1{}^+ = \frac{g_m R_L}{1 + 2(g_{be} + g_m)(j\omega L + \frac{1}{j\omega C_0})}\underline{U}^+$$

c) Spannung:

$$\underline{U}_1 = \underline{U}_1{}^+ + \underline{U}_1{}^- = \frac{g_m R_L}{1 + 2(g_{be} + g_m)(j\omega L + \frac{1}{j\omega C_0})}\underline{U}_b + g_m R_L \underline{U}_a$$

d) siehe Originalschaltung : $\underline{I}_2 = \frac{\underline{U}_1}{R_L} + \frac{\underline{U}_2}{R_L}$

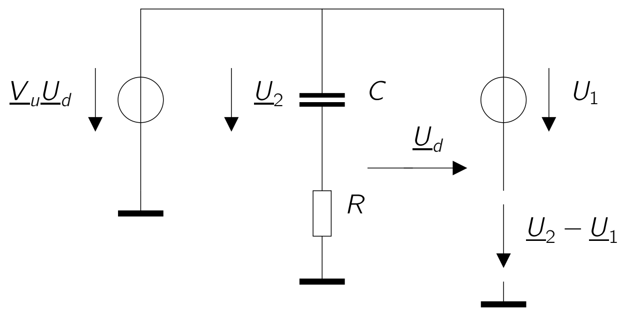
U_1 siehe (c)

$U_2 = \underline{U}_2{}^+ + \underline{U}_2{}^- = \underline{U}_1{}^+ - \underline{U}_1{}^-$ (Symmetrie)

$$\Rightarrow \underline{I}_2 = \frac{\underline{U}_1{}^+ + \underline{U}_1{}^-}{R_L} + \frac{\underline{U}_1{}^+ - \underline{U}_1{}^-}{R_L} = 2\frac{\underline{U}_1{}^+}{R_L} = \frac{2g_m}{1 + (g_{be} + g_m)(j\omega L + \frac{1}{j\omega C_0})}\underline{U}_b$$

Aufgabe 4

a) Frequenzgang



$$\underline{U}_d = \frac{R}{R + \frac{1}{j\omega C}}\underline{U}_2 - (\underline{U}_2 - \underline{U}_1) = \left(\frac{j\omega RC}{1 + j\omega RC} - 1\right)\underline{U}_2 + \underline{U}_1$$

$$= \underline{U}_1 - \frac{1}{1 + j\omega RC}\underline{U}_2$$

$$\underline{U}_2 = \underline{V}_u\underline{U}_D = \underline{V}_u\underline{U}_1 - \frac{\underline{V}_u}{1 + j\omega RC}\underline{U}_2 \quad \Leftrightarrow \left(1 + \frac{\underline{V}_u}{1 + j\omega RC}\right)\underline{U}_2 = \underline{V}_u\underline{U}_1$$

$$\Rightarrow \underline{F} = \frac{1}{\frac{1}{\underline{V}_u} + \frac{1}{1 + j\omega RC}}$$

b) Sonderfall:

$$\underline{F} \xrightarrow{V_u \to \infty} 1 + j\omega RC$$

c) $\underline{F}_2$:

$$\underline{F} = \frac{1}{\frac{1}{\underline{F}_a} + \underline{F}_2} \xrightarrow{F_a \to \infty} \frac{1}{\underline{F}_2} \stackrel{!}{=} 1 + j\omega RC \Leftrightarrow \underline{F}_2 = \frac{1}{1 + j\omega RC}$$

$$\frac{1}{\underline{F}_a} + \underline{F}_2 \stackrel{!}{=} \frac{1}{\underline{V}_u} + \frac{1}{1 + j\omega RC} \Leftrightarrow \underline{F}_a = \underline{V}_u$$

$$\underline{F}_0 = \underline{F}_2\underline{F}_a = \frac{\underline{V}_u}{1 + j\omega RC}$$

d) Betrag und Phase der Schleifenverstärkung:

$$\underline{F}_0 = \frac{1}{a}\frac{V_0}{(1 + \frac{j\omega}{\omega_0})(1 + \frac{j\omega}{10\omega_0})(1 + \frac{j\omega}{100\omega_0})} \quad \text{(mit a=1)}$$

e) $20\,\text{dB}\ log\left(\frac{V_0}{a}\right) = 80\,\text{dB} \Leftrightarrow \frac{V_0}{a} = 10^4$

f) Die Schaltung ist instabil, da an der Stelle wo ($\underline{F}_0$)die 0 dB-Achse schneidet, die Phase $\varphi(\underline{F}_0) < -180°$ ist.

g) Schnittpunkt mit 0 dB-Achse muss bei $w = 10\ \omega_0$ sein.

$$\Rightarrow |\underline{F}_0(\omega = 0)| = 20\,\text{dB} \Rightarrow\ 20\,\text{dB}\ log\left(\frac{v_0}{a}\right) = 20\,\text{dB} \Rightarrow \frac{v_0}{a} = 10$$
$$\Leftrightarrow v_0 = 10a = 10$$

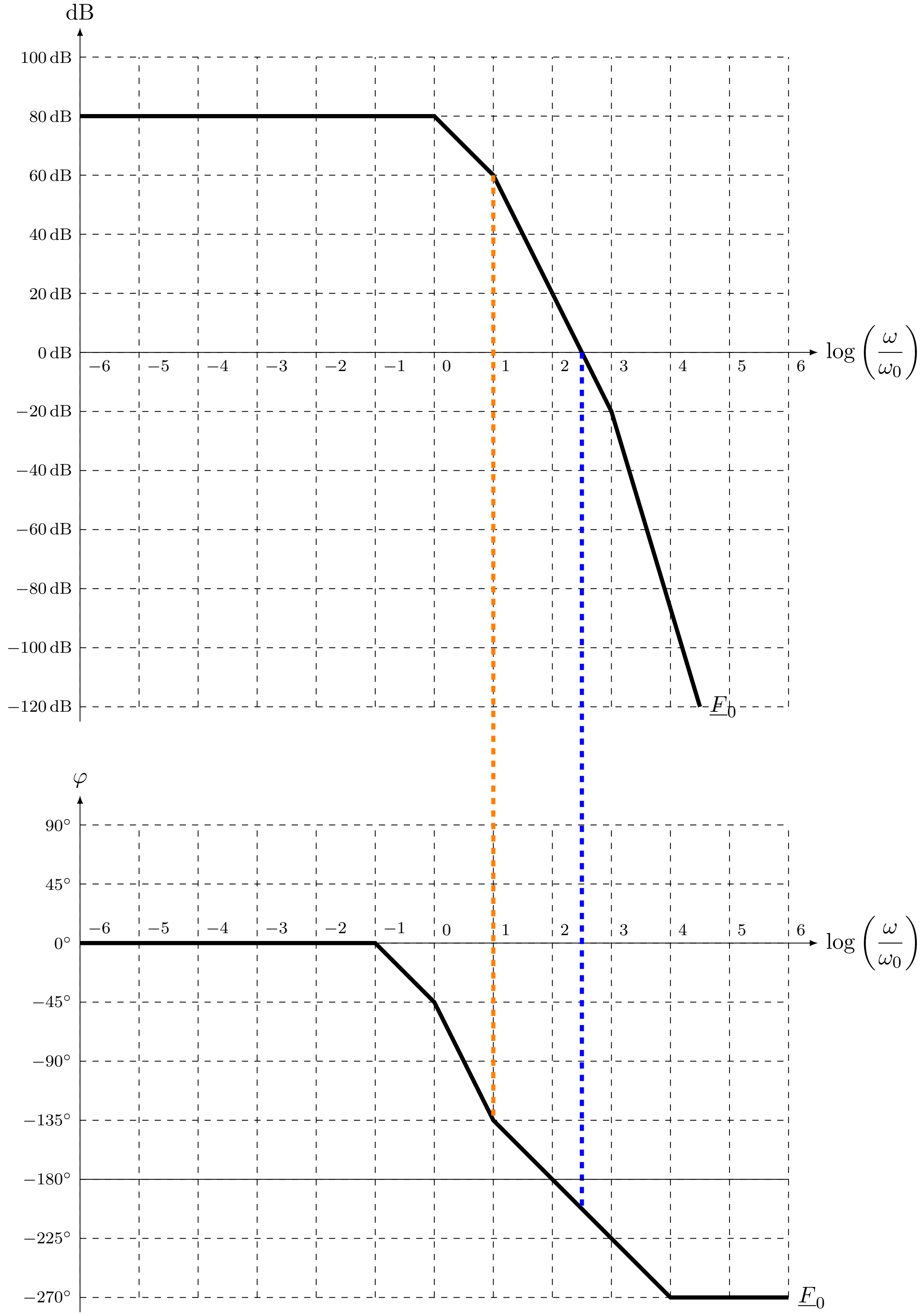


Abbildung 1: Lösung Bode-Diagramm.