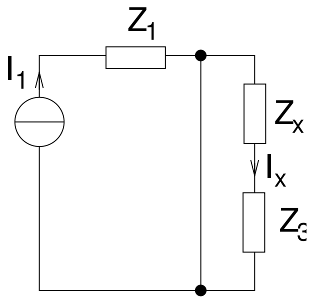
Lösungsvorschlag Klausur Elektronik II WS 09/10

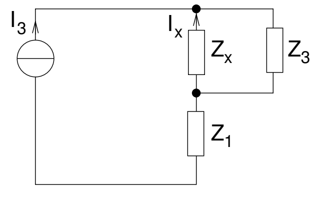
Aufgabe 1 (11 Punkte): Netzwerkberechnung

1. Überlagerungssatz:
   $I_x = \left.\frac{I_x}{I_1}\right|_{I_3=0,U_2=0} \cdot I_1 + \left.\frac{I_x}{I_3}\right|_{I_1=0,U_2=0} \cdot I_3 + \left.\frac{I_x}{U_2}\right|_{I_1=0,I_2=0} \cdot I_1$

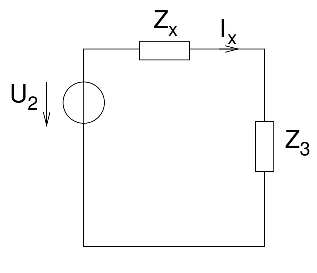
   Bezüglich I_1:
   Kurzschluss durch Spannungsquelle U_2 (s. Abbildung):
   $\left.\frac{I_x}{I_1}\right|_{I_3=0,U_2=0} = 0$



   Bezüglich I_3:
   Stromteiler durch Z_x und Z_3, Strom durch Z_x entspricht $-I_x$:
   $\left.\frac{I_x}{I_3}\right|_{I_1=0,U_2=0} = \frac{-Z_3}{Z_x+Z_3}$



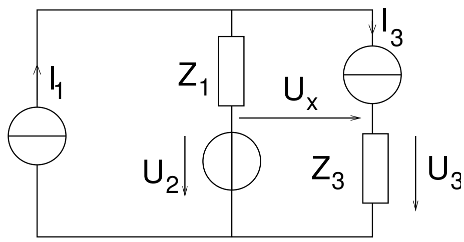
   Bezüglich U_2:
   $\left.\frac{I_x}{U_2}\right|_{I_1=I_2=0} = \frac{1}{Z_x+Z_3}$



Überlagerung der Anteile führt zur Gesamtlösung:
$I_x = \frac{-Z_3}{Z_x+Z_3} \cdot I_3 + \frac{1}{Z_x+Z_3} \cdot U_2$

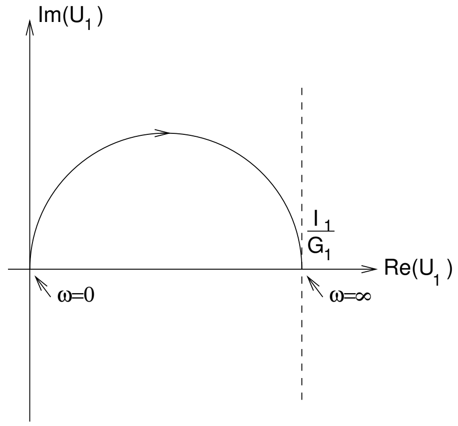
2. $Z_x \to \infty$
$U_x = Z_x \cdot I_x = \frac{Z_x Z_3}{Z_x+Z_3} \cdot I_3 + \frac{Z_x}{Z_x+Z_3} \cdot U_2$
$\lim_{Z_x \to \infty} U_x = -Z_3 I_3 + U_2$

alternativ: Masche auswerten
$U_x = U_2 - U_3 = U_2 - Z_3 I_3$

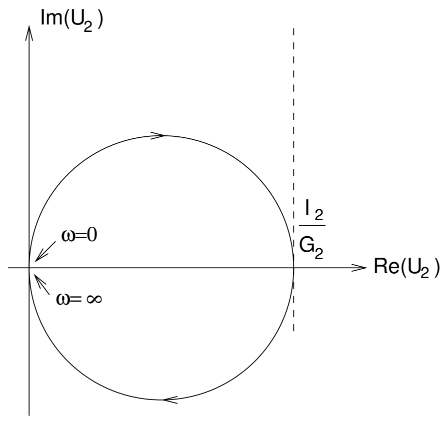


Aufgabe 2 (13 Punkte): Komplexe Rechnung, Ortskurve

1. a) $U_1 = \left(G_1 + \frac{1}{j\omega L_1}\right)^{-1} I_1 = \frac{j\omega L1}{1+j\omega L_1 G_1} I_1$
   $max|U_1| = \frac{I_1}{G_1}$

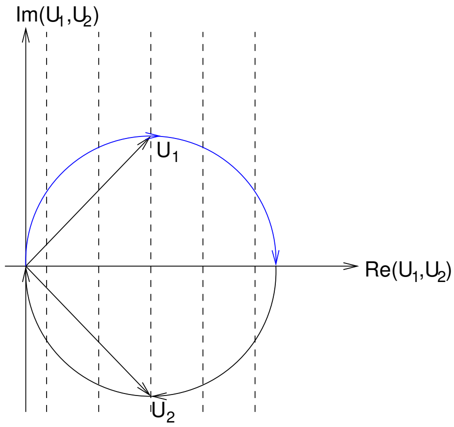


$U_2 = \frac{1}{G_2 + j\omega C_2 + \frac{1}{j\omega L_2}} I_2 = \frac{1}{G_2 + j\left(\omega C_2 - \frac{1}{\omega L_2}\right)} I_2$
max$|U_2| = \frac{I_2}{G_2}$



b) max$|U_2| = 2$max$|U_1|$
$\frac{I_1}{G_1} = 2\frac{I_2}{G_2}$
$G_2 I_1 = 2 I_2 G_1$

2. $max|U_1| = max|U_2|$
   kein Realteil $\Rightarrow U_1 - U_2$ senkrecht
   Betrag maximal $\Rightarrow$ Mitte des Kreis
   $U_1 = \frac{1}{2}(1+j)\max|U_1|$



$U_2 = \frac{1}{2}(1-j)\max|U_2|$
Betrachte U_1:

$$\phi(U_1) = 45^\circ$$

$$\begin{aligned} \phi = U_1 &= \frac{\phi(j\omega L)}{\phi(1 + j\omega L_1 G_1)} \stackrel{!}{=} \frac{90^\circ}{45^\circ} \\ \phi(j\omega L) &= 90^\circ \\ \phi(1 + j\omega L_1 G_1) &\stackrel{!}{=} 45^\circ \\ 1 &= \omega L_1 G_1 \\ \Rightarrow \omega_x &= \frac{1}{L_1 G_1} \end{aligned}$$

Betrachte U_2:

$$\phi(U_2) = -45^\circ$$

$$\begin{aligned} G_2 &= \omega_x C_2 - \frac{1}{\omega_x L_2} \\ C_2 &= \frac{1}{\omega_x^2 L_2} + \frac{G_2}{\omega_x} \\ &= \frac{L_1^2 G_1^2}{L_2} + G_2 L_1 G_1 \\ &= L_1 G_1^2 + L_1 G_1^2 = 2 L_1 G_1^2 \end{aligned}$$

Aufgabe 3 (14 Punkte): Schaltungsdimensionierung und -berechnung

1.

$$\frac{1}{2}U_0 = U_{BE0} + (R_T + R_E)I_E \qquad I_E \approx I_C \text{ da } \beta \gg 1$$
$$= U_{BE0} + (R_t + R_E)I_C$$
$$I_C = \frac{\frac{U_0}{2} - U_{BE0}}{R_T + R_E}$$
$$g_m = \frac{I_c}{U_T} = \frac{\frac{U_0}{2} - U_{BE0}}{U_T\,(R_T + R_E)} \qquad g_m = \left.\frac{\partial I_c}{\partial U_{be}}\right|_{v_{ce0}} = \frac{I_{c0}}{U_T}$$

2.

normal aktiv: $U_{BE} > 0$ $\quad U_{BC} < 0$

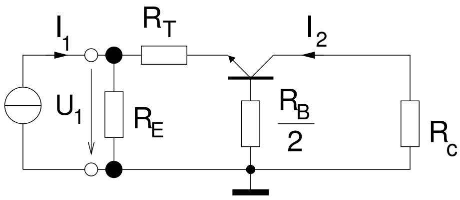
⇒minimales Potenzial: $U_C > \frac{1}{2}U_0$

maximales Potenzial: $U_C = U_0$

⇒AP: $U_C = \frac{3}{4}U_0 \quad \Rightarrow U_{RC} = \frac{1}{4}U_0 = R_C I_C$

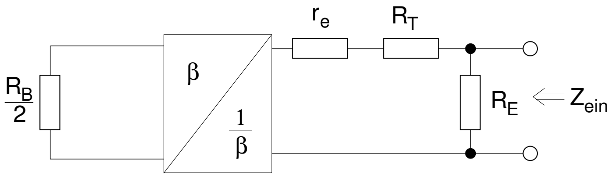
$$R_C = \frac{U_0}{4I_C}$$
$$R_C = \frac{U_0}{4\frac{\frac{U_0}{2} - U_{BE}}{R_T + R_E}} = \frac{U_0(R_T + R_E)}{4\left(\frac{U_0}{2} - U_{BE}\right)}$$

3. Basisgrundschaltung:



4.

$$Z_{ein} = R_E || \left(R_T + r_e + \frac{R_b}{2\beta}\right)$$



5.

$$1,2\,(R_E||(R_T + r_e)) = R_e||\left(R_T + r_e + \frac{R_b}{2\beta}\right)$$

6.

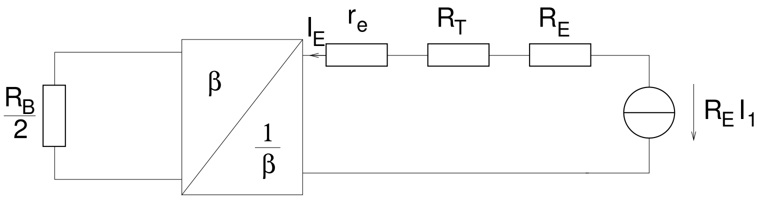
$$u_1 = R_E I_1$$

$$I_2 = i_E = i_C = \frac{R_E I_1}{R_T + r_e + R_E}$$

$$\frac{I_2}{I_1} = \frac{R_E}{R_T + r_e + R_E}$$

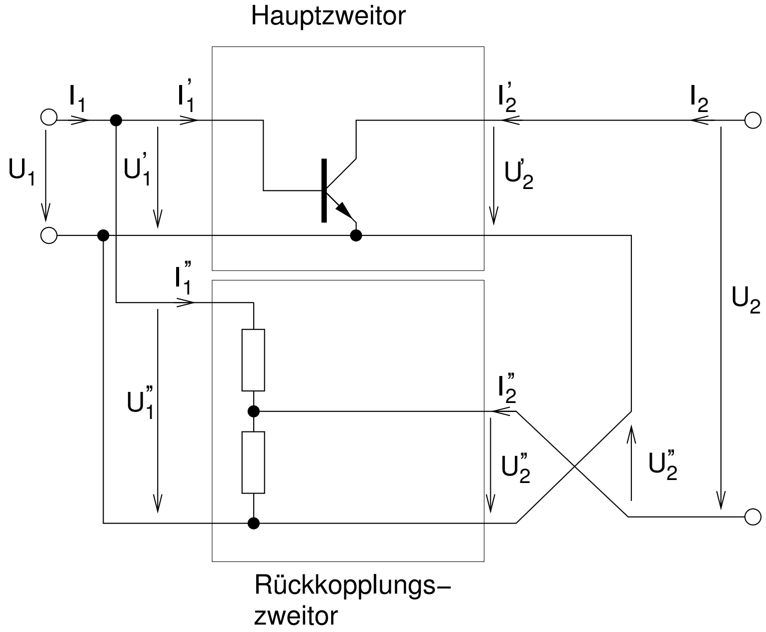
$$\Leftrightarrow I_2 = \frac{R_E I_1}{R_T + r_e + R_E}$$

$$\Rightarrow r_e \downarrow \Rightarrow I_C I_{AP} \uparrow$$



Aufgabe 4 (15 Punkte): Rückkopplung, Zweitor

1. Umzeichnen in Haupt- und Rückkopplungszweitor



2. Parallel-Serien Kopplung (PSK):

$$U_1 = U_1' = U_1'' \qquad U_2 = U_2' + U_2''$$

$$I_1 = I_1' + I_1'' \qquad I_2 = I_2' = I_2''$$

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = [G] \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$$U_2 = G_{21} U_1 + G_{22} I_2$$

$$I_1 = G_{11} U_1 + G_{12} I_2$$

3. Hauptzweitor:

$$G_{22}' = \left.\frac{U_2'}{I_2'}\right|_{U_1'=0} = \frac{1}{g_o} \qquad G_{21}' = \left.\frac{U_2'}{U_1'}\right|_{I_2'=0} = \frac{-g_m}{g_o}$$

$$G_{11}' = \left.\frac{I_1'}{U_1'}\right|_{I_2'=0} = 0 \qquad G_{12}' = \left.\frac{I_1'}{I_2'}\right|_{U_1'=0} = 0$$

$$G' = \begin{bmatrix} 0 & 0 \\ -\frac{g_m}{g_0} & \frac{1}{g_0} \end{bmatrix}$$

Rückkopplungszweitor:

$$G''_{11} = \left.\frac{I''_1}{U''_1}\right|_{I''_2=0} = \frac{1}{R_1+R_2} \qquad G''_{12} = \left.\frac{I''_1}{I''_2}\right|_{U''_1=0} = -\frac{R_2}{R_1+R_2}$$

$$G''_{21} = \left.\frac{U''_2}{U''_1}\right|_{I''_2=0} = \frac{R_2}{R_1+R_2} \qquad G''_{22} = \left.\frac{U''_2}{I''_2}\right|_{U''_1=0} = R_1||R_2$$

$$G'' = \begin{bmatrix} \frac{1}{R_1+R_2} & -\frac{R_2}{R_1+R_2} \\ \frac{R_2}{R_1+R_2} & R_1||R_2 \end{bmatrix}$$

$$G''* = \begin{bmatrix} \frac{1}{R_1+R_2} & \frac{R_2}{R_1+R_2} \\ -\frac{R_2}{R_1+R_2} & R_1||R_2 \end{bmatrix}$$

Gesamtschaltung:

$$G = \begin{bmatrix} \frac{1}{R_1+R_2} & \frac{R_2}{R_1+R_2} \\ -\frac{g_m}{g_0} - \frac{R_2}{R_1+R_2} & \frac{1}{g_0} + R_1||R_2 \end{bmatrix}$$

4. Ansteuerung mit Spannungsquelle ⇒ Rückkopplung wirkungslos
   ⇒ mit Stromquelle ansteuern (hochohmig) ⇒ optimale Rückwirkung

5. Ausgangsimpedanz:

$$Z_{aus} = \left.\frac{U_2}{I_2}\right|_{I_1=0}$$

$$I_1 = 0$$

$$\Rightarrow 0 = G_{11}U_1 + G_{12}U_2$$

$$U_1 = -\frac{G_{12}}{G_{11}} I_2$$

$$U_2 = -\frac{G_{12}G_{21}}{G_{11}} I_2 + G_{22} I_2$$

$$Z_{aus} = -\frac{G_{12}G_{21}}{G_1 1} + G_{22}$$

$$= R_2\left(\frac{g_0}{g_m} + \frac{R_2}{R_1+R_2}\right) + \frac{1}{g_0} + R_1||R_2$$

Aufgabe 5 (11 Punkte): Stabilität, Netzwerktheorie

1.

$$
\begin{aligned}
Z_{ein} &= \frac{U}{I} \\
Z_2 I_L &= j\omega L I_1 \\
U &= Z_1 I_L = Z_1 Z_2 \frac{1}{j\omega L} I \\
Z_{ein} &= \frac{Z_1 Z_2}{j\omega L} && \Rightarrow C^* = \frac{L}{Z_1 Z_2}
\end{aligned}
$$

2. Betrachte beliebige Wirkungsfunktion des Netzwerks:

$$
\begin{aligned}
\frac{I_1}{I_0} &= \frac{\frac{1}{G}}{\frac{1}{G} + Z_{ein}} = \frac{\frac{1}{G}}{\frac{1}{G} + \frac{Z_1 Z_2}{j\omega L}} \\
&= \frac{j\omega L \left(1 + \frac{j\omega}{\omega_2}\right)}{j\omega L \left(1 + \frac{j\omega}{\omega_2}\right) + \underbrace{Z_{10} Z_{20} G}_{\gamma}}
\end{aligned}
$$

Nullstellen des Nenners (Polstellen) bestimmen: $j\omega \rightarrow \infty$

$$
\begin{aligned}
\gamma &= Z_{10} Z_{20} G \\
0 &= \gamma + sL + \frac{s^2 L}{\omega_2} \\
0 &= \gamma \frac{\omega_2}{L} + s\omega_2 + s^2 \\
s_{1,2} &= -\frac{\omega_2}{2} \pm \sqrt{\left(\frac{\omega_2}{2}\right)^2 - \gamma \frac{\omega_2}{L}} \\
\text{instabil } &\Rightarrow \Re\left\{-\frac{\omega_2}{2} \pm \sqrt{\left(\frac{\omega_2}{2}\right)^2 - \gamma \frac{\omega_2}{L}}\right\} \geq 0 \\
&\Rightarrow \gamma \frac{\omega_2}{L} < 0 \text{ mit } \omega_2 > 0, L > 0 \Rightarrow Z_{10} Z_{20} < 0
\end{aligned}
$$

3.

$$\begin{aligned}
i_o(t) &= \delta(t) \\
\frac{I(s)}{I_0(s)} &= \frac{sL}{sL + s^2 \frac{L}{\omega_2} + g} \\
I(s) &= \frac{sL}{g + sL + s^2 \frac{L}{\omega_2}} I_0(s) \qquad \text{mit } I_0(s) = \mathcal{L}\left\{\delta(t)\right\} = 1 \\
\frac{I(s)}{I_0(s)} &= \frac{sL}{\frac{L}{\omega_2}(s - s_1)(s - s_2)} = \frac{s\omega_2}{(s - s_1)(s - s_2)} \\
i(t) &= \mathcal{L}^{-1}\left\{I(s)\right\} \\
&= \mathcal{L}^{-1}\left\{\frac{s\omega_2}{(s - s_1)(s - s_2)}\right\} \\
&= \sum_{i=1}^{2} \frac{Z(s)}{N'(s)} e^{st}\bigg|_{s=s_i} \\
&= \frac{s\omega_2}{s - s_2} e^{st}\bigg|_{s=s_1} + \frac{s\omega_2}{s - s_1} e^{st}\bigg|_{s=s_2} \\
&= \omega_2 \left(\frac{s_1}{s_1 - s_2} e^{s_1 t} + \frac{s_2}{s_2 - s_1} e^{s_2 t}\right)
\end{aligned}$$

Aufgabe 6 (16 Punkte): Gleichtakt-, Gegentaktzerlegung

1.

$$U^+ = \frac{U_1 + U_2}{2} = \frac{U_1 - U_1(1-\alpha)}{2} = \frac{\alpha}{2}U_1$$
$$U^- = \frac{U_1 - U_2}{2} = \frac{U_1 + U_1(1-\alpha)}{2} = (1-\frac{\alpha}{2})U_1$$

2.

$$R_1 = R_2$$
$$C_1 = C_2$$
$$Z_3 + j\omega L_1 = Z_4 + j\omega L_2$$

3. Zerlegung in Gleich- und Gegentakt:

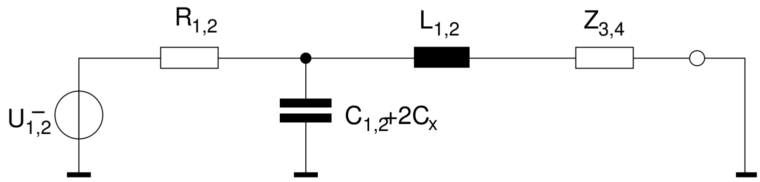


Abbildung 1: Gegentakt

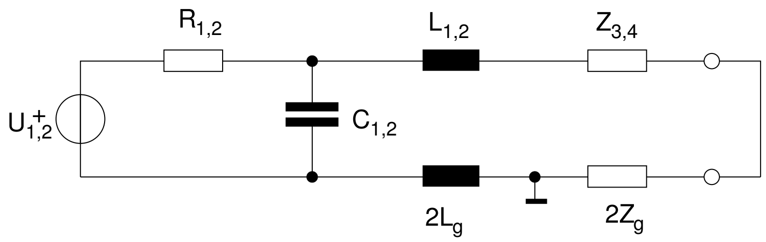


Abbildung 2: Gleichtakt

4. Ersetzen der Spannungsquellen durch Stromquellen:

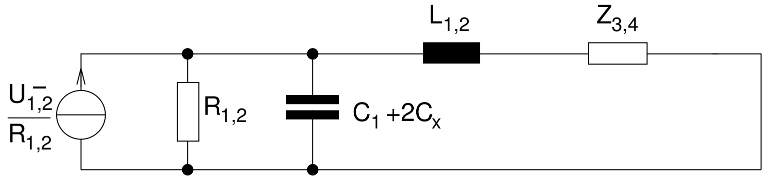


Abbildung 3: Gegentakt

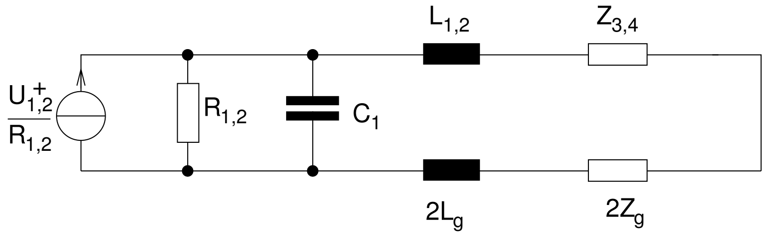


Abbildung 4: Gleichtakt

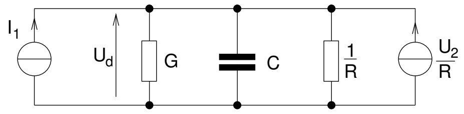
$$Z^{*-} = Z_3 + j\omega L_1$$
$$Z^{*+} = Z_3 + j\omega L_1 + 2Z_g + 2j\omega L_g$$
$$I_{3,4}^{-} = \pm \frac{\left(1-\frac{\alpha}{2}\right)U_1}{R_1} \frac{R_1 \frac{1}{j\omega(C_1+2C_x)}}{R_1 \frac{1}{j\omega(C_1+2C_x)} + Z^{*-}R_1 + Z^{*-}\frac{1}{j\omega(C_1+2C_x)}}$$
$$I_{3,4}^{+} = \frac{\frac{\alpha}{2}U_1}{R_1} \frac{R_1 \frac{1}{j\omega C_1}}{R_1 \frac{1}{j\omega C_1} + Z^{*+}R_1 + Z^{*+}\frac{1}{j\omega C_1}}$$
$$I_3 = I_3^- + I_3^+$$
$$I_4 = I_4^- + I_4^+$$

5.

$$I_3 - I_4 = 2I_3^-$$
$$= 2\frac{\left(1-\frac{\alpha}{2}\right)U_1}{R_1} \frac{R_1 \frac{1}{j\omega(C_1+2C_x)}}{R_1 \frac{1}{j\omega(C_1+2C_x)} + Z^{*-}R_1 + Z^{*-}\frac{1}{j\omega(C_1+2C_x)}}$$
$$\Rightarrow Z^{*-} \text{ ist keine Funktion von } L_g$$

Aufgabe 7 (15 Punkte): Operationsverstärker, Bode-Diagramm.

1. Ersatzschaltbild:



a)

$$U_d = \left(G + j\omega C + \frac{1}{R}\right)^{-1}\left(I_1 + \frac{U_2}{R}\right) \qquad \text{mit } v_u U_d = -U_2$$

$$I_1 + \frac{1}{R}U_2 = \left(G + j\omega C + \frac{1}{R}\right)\frac{-U_2}{v_u}$$

$$I_1 = U_2\left[-\frac{G + j\omega C + \frac{1}{R}}{v_u} - \frac{1}{R}\right]$$

$$F(j\omega) = \frac{U_2}{I_1} = \frac{1}{-\frac{G+j\omega C+\frac{1}{R}}{v_u} - \frac{1}{R}}$$

b)

$$lim_{v_u\to\infty}F(j\omega) = \frac{1}{-\frac{1}{R}} = -R = \frac{1}{F_2}$$

c)

$$F_2 = -\frac{1}{R}$$

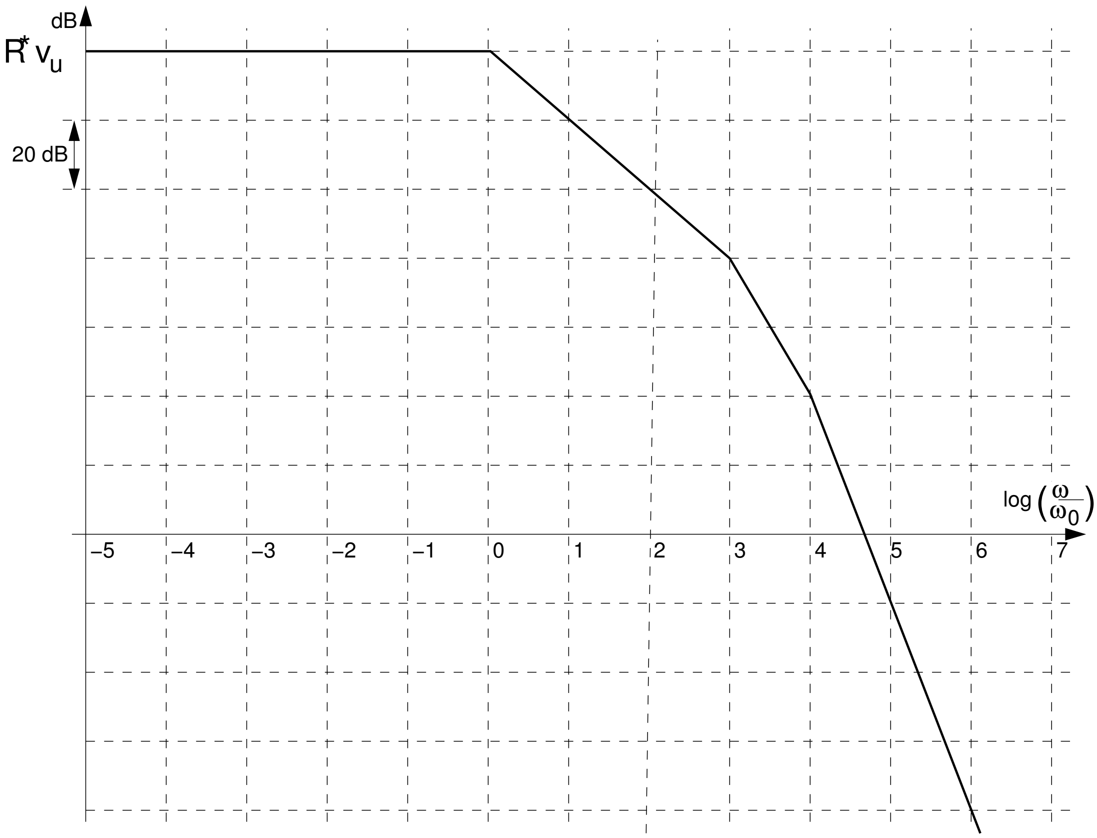
$$F(j\omega) = \frac{1}{-\frac{G+j\omega C+\frac{1}{R}}{V_u} - \frac{1}{R}}$$

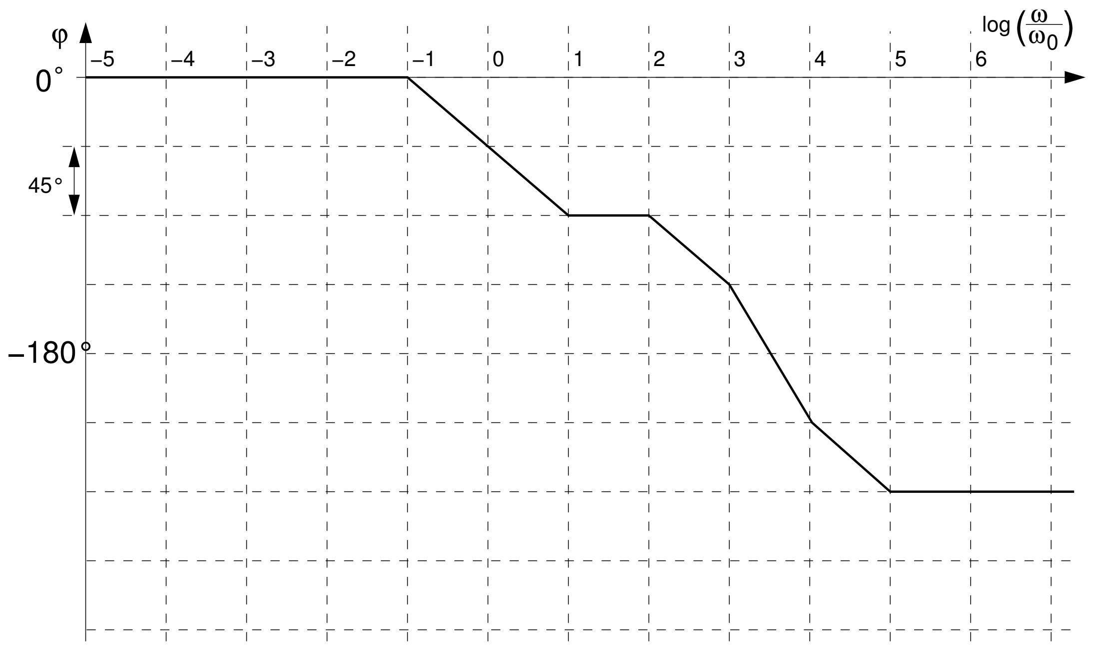
$$= \frac{-\frac{v_u}{G+j\omega C+\frac{1}{R}}}{1 - \frac{1}{R}\frac{-v_u}{G+j\omega C+\frac{1}{R}}}$$

$$F_a(j\omega) = \frac{-v_u}{\underbrace{G + \frac{1}{R}}_{\frac{1}{R^*}} + j\omega C}$$

$$= \frac{-R^* v_u}{1 + j\omega \underbrace{R^* C}_{\frac{1}{\omega_x}}}$$

2. a)


dB
R* v_u
20 dB
log (ω/ω0)
–5
–4
–3
–2
–1
0
1
2
3
4
5
6
7


φ
0°
45°
–180°
log (ω/ω0)
–5
–4
–3
–2
–1
0
1
2
3
4
5
6

b)

$$\left|\frac{1}{F_2}\right| = F_a(\omega_{45^\circ})$$

$$F_a(\omega_{45^\circ}) = \frac{F_a(\omega \to 0)}{1000} = \frac{-R^* v_u}{1000}$$

$$\Rightarrow |F_2| = \frac{1000}{R^* v_u}$$

3.

$$1 = F_2 \cdot F_a = \left|\frac{-R^* v_u}{1\Omega\omega_0 R^* C}\right| \qquad \omega_0 \gg \omega_x$$

$$1\Omega\omega_0 R^* C = R^* v_u$$

$$C = \frac{v_u}{\omega_0 1\Omega}$$