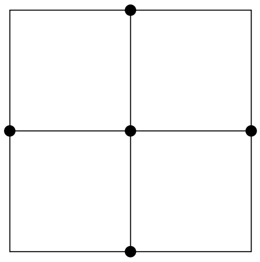
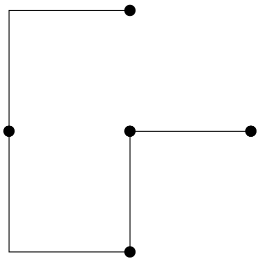
Aufgabe 1

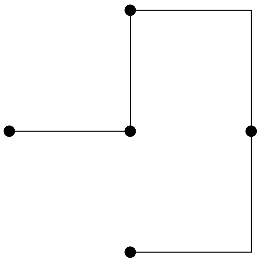
a)

Graph:



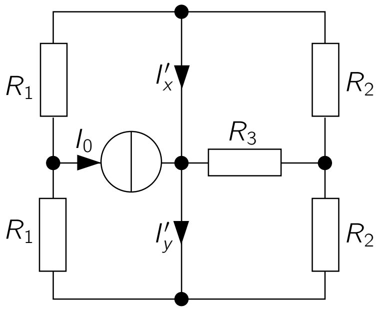
Baum: Co-Baum:



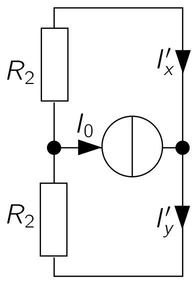


b)

lineare Überlagerung:
1) $U_1 = 0$

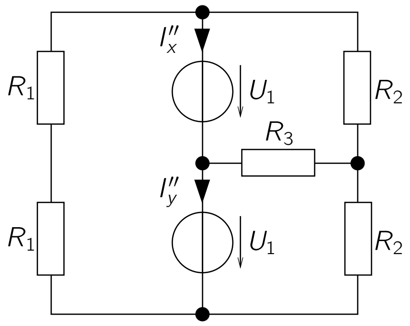


Daraus ergibt sich:

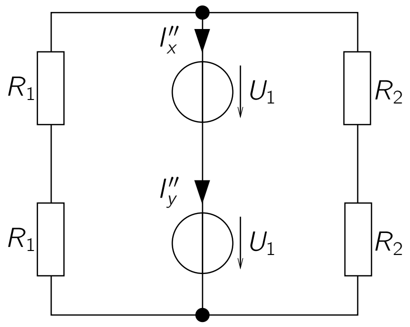


$$\Rightarrow I'_x = -\frac{I_0}{2}; I'_y = \frac{I_0}{2}$$

2) $I_0 = 0$



Daraus ergibt sich:



$$\Rightarrow I''_x = -\frac{U_1}{R_1} - \frac{U_1}{R_2} = -\frac{R_1 + R_2}{R_1 R_2} U_1$$

Die gesuchten Ströme ergeben sich somit zu:

$$\Rightarrow I_x = I'_x + I''_x = -\frac{I_0}{2} - \frac{R_1 + R_2}{R_1 R_2} U_1$$
$$I_y = I'_y + I''_y = \frac{I_0}{2} - \frac{R_1 + R_2}{R_1 R_2} U_1$$

c)

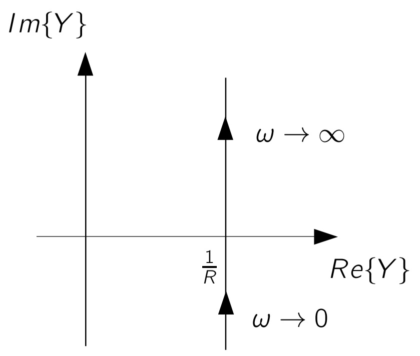
$$P_{\sum} = \frac{R_1}{2} I_0^2 + 2\,\frac{R_1 + R_2}{R_1 R_2} U_1^2$$

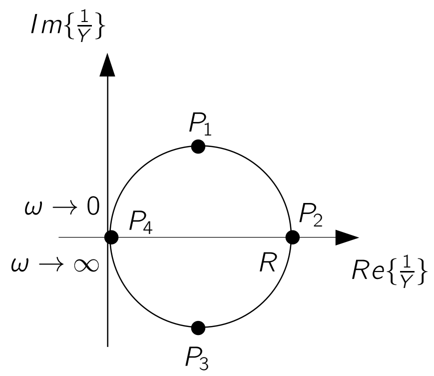
Aufgabe 2

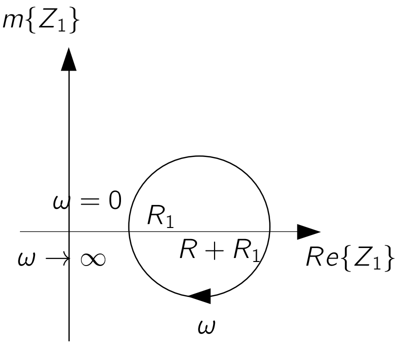
a)

$\underline{Z_1} = R_1 + \frac{1}{y}$; $\frac{1}{y} = \frac{1}{jwC + \frac{1}{R} + \frac{1}{jwL}}$

b)







Maximum des Imaginärteil:

$$P_1^* = \frac{R}{2} + j \cdot \frac{R}{2}$$

Maximum des Realteil:

$$P_2^* = R + j \cdot 0$$

Minimum des Imaginärteil:

$$P_3^* = \frac{R}{2} - j \cdot \frac{R}{2}$$

Minimum des Realteil:

$$P_4^* = 0 + j \cdot 0$$

$P_i = P_i^* + R_1$

1.

$Re\{\frac{1}{y}\} = Im\{\frac{1}{y}\}, f = \pm 45°$

$f\left(\frac{1}{y}\right) = 45° \rightarrow f(y) = -45°$

$P_1 \rightarrow \frac{1}{R} = -\omega \cdot C + \frac{1}{\omega \cdot L}$

$\omega^2 \cdot C + \omega \cdot \frac{1}{R} = \frac{1}{L}$

$\left(\omega + \frac{1}{2 \cdot R \cdot C}\right)^2 = \frac{1}{L} + \left(\frac{1}{2 \cdot R \cdot C}\right)^2$

$\omega_1 = \sqrt{\frac{1}{L} + \left(\frac{1}{2 \cdot R \cdot C}\right)} - \frac{1}{2 \cdot R \cdot C}$

$\omega_3 \rightarrow \frac{1}{R} = \omega \cdot C + \frac{1}{\omega \cdot L}$

$\omega^2 \cdot C - \omega \cdot \frac{1}{R} = -\frac{1}{\omega \cdot L}$

$\omega_3 = \sqrt{\left(\frac{1}{2 \cdot R \cdot C}\right)^2 - \frac{1}{\omega \cdot L}} + \frac{1}{2 \cdot R \cdot C}$

2.

$Im\{\frac{1}{y}\} = 0 \rightarrow \omega \cdot C - \frac{1}{\omega \cdot L} = 0$

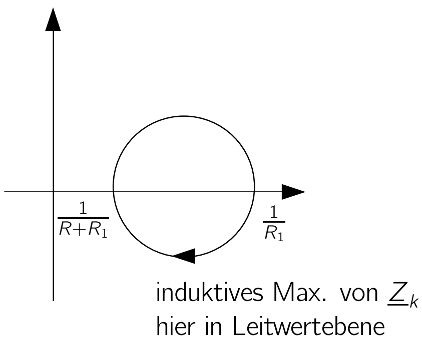
$\omega_2 = \sqrt{\frac{1}{L \cdot C}}$

3.

$\omega_4 = 0$

c)

$\underline{Z_k} || \underline{Z_1} \rightarrow$ Leitwerte $\underline{y_k} + \underline{y_1}$

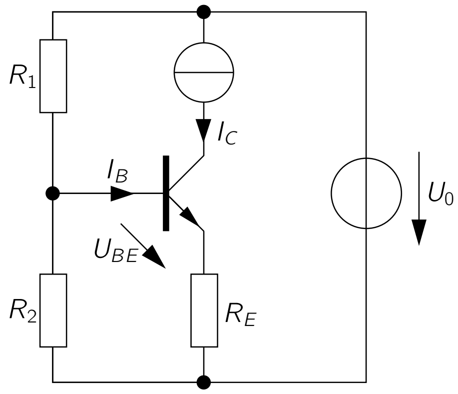


$\underline{y_k} = j \cdot \frac{\left(\frac{1}{R_1} - \frac{1}{R_1+R}\right)}{2} := w_1 \cdot C$

Aufgabe 3

Gleichstrom ESB

a)



$C \to$ Leerlauf
R_L stromlos $\to$ Leerlauf

$$I_C = B \cdot I_B$$
$$I_C = I_L$$

$$U_0 = U_{BE} + (B+1) \cdot I_B \cdot R_E$$

$$\frac{U_2}{R_2} = \frac{U_0 - U_2}{R_1} - I_B$$

$$\frac{U_{BE} + (B+1) \cdot I_B \cdot R_E}{R_2} = \frac{U_0 - U_{BE} - (B+1) \cdot I_B \cdot R_E}{R_1} - I_B$$

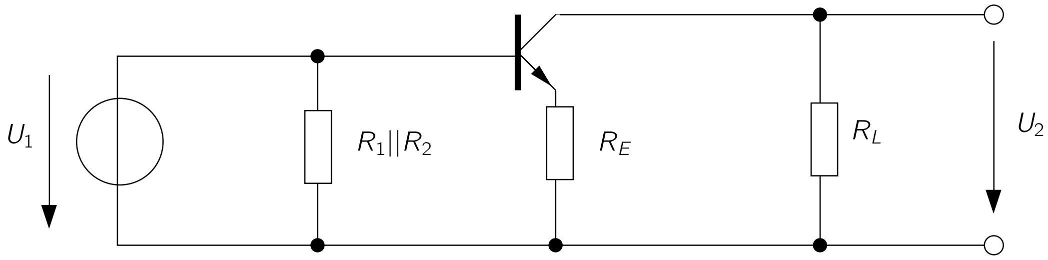
$$\begin{aligned} R_1 &= \frac{U_0 - U_{BE} - (B+1) \cdot I_B \cdot R_E}{\frac{U_{BE} + (B+1) \cdot I_B \cdot R_E}{R_2} + I_B} \qquad \text{mit } I_B = \frac{I_L}{B} \\ &= \frac{U_0 - U_{BE} - \frac{B+1}{B} \cdot I_L \cdot R_E}{\frac{U_{BE} + \frac{B+1}{B} \cdot I_L \cdot R_E}{R_2} + I_B} \\ &= \frac{U_0 - U_{BE} - \left(1 + \frac{1}{B}\right) \cdot I_L \cdot R_E}{U_{BE} + \left(1 + \frac{1}{B}\right) \cdot I_L \cdot R_E + I_B \cdot R_2} \cdot R_2 \end{aligned}$$

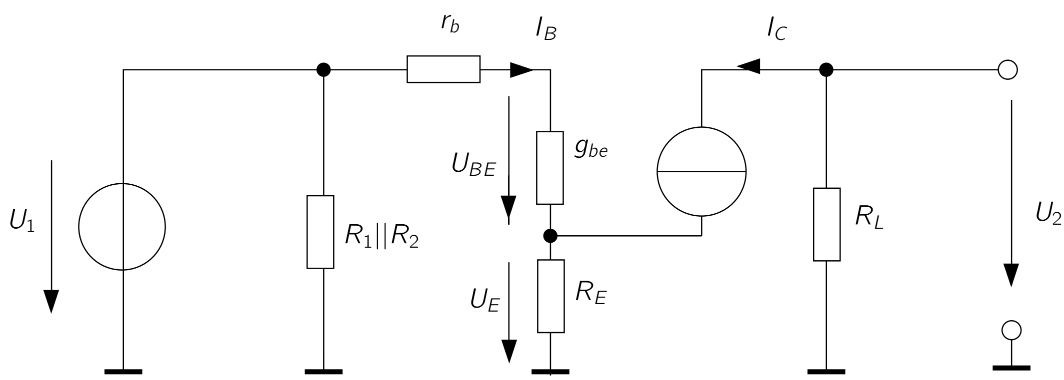
b)

Verlustleistung unabhängig von der Aussteuerung

$$
\begin{aligned}
P_V &= I_B \cdot U_{BE} + I_C \cdot U_{CE} \\
&= \frac{I_L}{B} \cdot U_{BE} + I_L \cdot U_{CE} \\
U_{CE} &= U_0 - I_L \cdot \left(1 + \frac{1}{B}\right) \cdot R_E - 0 \text{ , da } R_L \text{ stromlos} \\
P_V &= \frac{I_L}{B} \cdot U_{BE} + I_L \cdot \left(U_0 - I_L \cdot \left(1 + \frac{1}{B}\right) \cdot R_E\right) \\
&= I_L \cdot \left(\frac{U_{BE}}{B} + U_0 - I_L \cdot \left(1 + \frac{1}{B}\right) \cdot R_E\right)
\end{aligned}
$$

c)





$I_C = \beta \cdot I_B$, $U_E = R_E \cdot (I_C + I_B)$

$= gm \cdot U_{BE}$

$I_B = \frac{(U_1 - U_E)}{r_b + \frac{1}{g_{be}}}$

$= \frac{U_1 - R_E \cdot (\beta+1) \cdot I_B}{r_b + \frac{1}{g_{be}}}$

$I_B \cdot \left(1 + \frac{R_E \cdot (\beta+1)}{r_b + \frac{1}{g_{be}}}\right) = \frac{U_1}{r_b + \frac{1}{g_{be}}}$

$I_B \cdot \left(r_b + \frac{1}{g_{be}} + R_E \cdot (\beta + 1)\right) = U_1$

$I_C = \beta \cdot I_B$

$\frac{I_C}{\beta} \cdot \left(r_b + \frac{1}{g_{be}} + R_E \cdot (1 + \beta)\right) = U_1$

$I_C = \frac{\beta}{r_b + \frac{1}{g_{be}} + R_E \cdot (1+\beta)} \cdot U_1$

$U_2 = -I_C \cdot R_L$

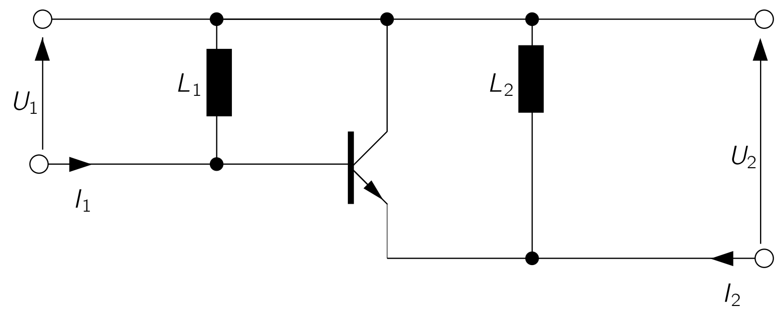
$-\frac{U_2}{R_L} = \cdots$

$\frac{U_2}{U_1} = -\frac{\beta \cdot R_L}{r_b + \frac{1}{g_{be}} + R_E \cdot (1+\beta)}$

Aufgabe 4

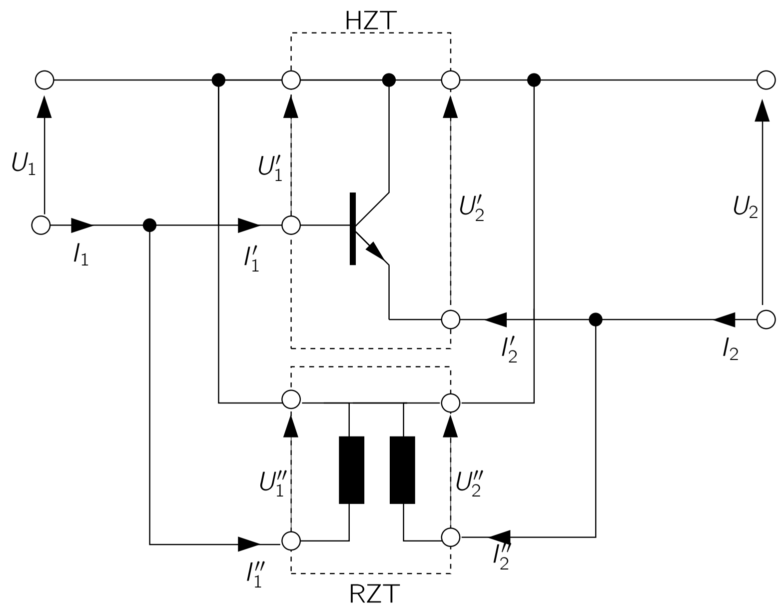
a)

Wechselstrom-Ersatzschaltbild



$\Rightarrow$ Kollektor-Grundschaltung

b)



c)

Parallel-Parallel-Kopplung $\Rightarrow$ Y-Matrix

$U_1 = U_1' = U_1''$, $U_2 = U_2' = U_2''$

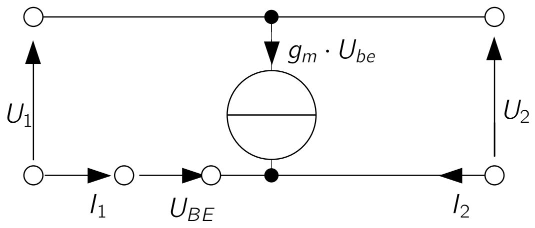
$I_1 = I_1' + I_1''$, $I_2 = I_2' + I_2''$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = (Y) \cdot \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$\underline{Y} = \underline{Y}' + \underline{Y}''$

d)

Hauptzweitor:



$Y'_{11} = \frac{I'_1}{U'_1}|_{U'_2=0} = 0$

$Y'_{12} = \frac{I'_1}{U'_2}|_{U'_1=0} = 0$

$Y'_{21} = \frac{I'_2}{U'_1}|_{U'_2=0} = -gm$

$Y'_{22} = \frac{I'_2}{U'_2}|_{U'_1=0} = gm$

$Y' = \begin{pmatrix} 0 & 0 \\ -gm & gm \end{pmatrix}$

Rückkopplungszweitor:

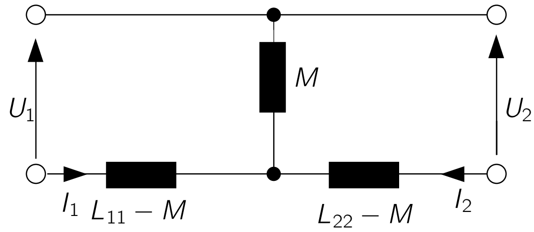
$I'_1 = I_x$, $I'_2 = I_y$

$\underline{Z}'' = \begin{pmatrix} j \cdot \omega \cdot L_{11} & j \cdot \omega \cdot M \\ j \cdot \omega \cdot M & j \cdot \omega \cdot L_{22} \end{pmatrix}$

$Y'' = Z''^{-1} =$

$\frac{1}{\omega^2 \cdot M^2 - L_{11} \cdot L_{22} \cdot \omega^2} \begin{pmatrix} j \cdot \omega \cdot L_{22} & -j \cdot \omega \cdot M \\ -j \cdot \omega \cdot M & j \cdot \omega \cdot L_{11} \end{pmatrix}$

alternative:



$Y''_{11} = \frac{I''_{11}}{\underline{U_1}''}|_{U''_2=0}$

$= \left(\left(\frac{1}{j \cdot \omega \cdot M} + \frac{1}{j \cdot \omega \cdot (L_{22} - M)} \right)^{-1} + j \cdot \omega \left(L_{11} - M \right)^{-1} \right)^{-1}$

$= \frac{1}{\frac{1}{\frac{1}{j \cdot \omega \cdot M} + \frac{1}{j \cdot \omega \cdot (L_{22} - M)}} + j \cdot \omega \cdot (L_{11} - M)}$

$= \left[\frac{-(L_{22} - M) \cdot \omega^2 \cdot M}{j \cdot \omega \cdot L_{22}} + j \cdot \omega \cdot (L_{11} - M) \right]$

$= \frac{j \cdot \omega \cdot L_{22}}{\omega^2 \cdot M^2 \cancel{- \omega^2 \cdot M \cdot L_{22}} - \omega^2 \cdot L_{11} \cdot L_{22} \cancel{+ \omega^2 \cdot M \cdot L_{22}}}$

$= \frac{j \cdot \omega \cdot L_{22}}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})}$

analog für Y''_{12} , Y''_{21} , Y''_{22}

$$Y = Y' + Y'' = \begin{pmatrix} \frac{j \cdot \omega \cdot L_{22}}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})} & -\frac{j \cdot \omega \cdot M}{\omega^2 \cdot (M - L_{11} \cdot L_{22})} \\ -gm - \frac{j \cdot \omega \cdot M}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})} & gm + \frac{j \cdot \omega \cdot L_{11}}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})} \end{pmatrix}$$

e)

$Y_{22} = gm + \frac{j \cdot \omega \cdot L_{11}}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})}$

$= gm + \frac{L_{11}}{L_{22}} \cdot \frac{j \cdot \omega \cdot L_{22}}{\omega^2 \cdot (M^2 - L_{11} \cdot L_{22})}$

$= gm + \frac{L_{11}}{L_{22}} \cdot Y_{11}$

$\Rightarrow b = gm$

$a = \frac{L_{11}}{L_{22}}$

Aufgabe 5

a)

$$\underline{Y} = Ge^{-j\omega T}$$

$\underline{Y}$, da $\underline{U}$ gemessen und $\underline{I}$ eingestellt wird (Stromquelle).

b)

$$\underline{I} = Ge^{-j\omega T}\underline{U}$$

$$\underline{U} = \underline{U}_q - R_q\underline{I}$$

$$\Leftrightarrow \frac{\underline{I}}{Ge^{-sT}} = \underline{U} - R_q\underline{I}$$

$$\Leftrightarrow \underline{I}(\frac{1}{Ge^{-sT}} + R_q) = \underline{U}$$

$$\Leftrightarrow \frac{\underline{I}}{\underline{U}_q} = \frac{1}{\frac{1}{Ge^{-sT}} + R_q} = \frac{Ge^{-sT}}{1 + R_qGe^{-sT}} = \underline{H}$$

$$\Rightarrow 1 + R_qGe^{-sT} = 0$$

mit $s = \sigma + j\omega$ folgt:

$$\Leftrightarrow \frac{R_q}{R}e^{-(\sigma+j\omega)T} = -1$$

$$\Leftrightarrow \frac{R_q}{R}e^{-\sigma T}e^{-j\omega T} = -1$$

$$\Leftrightarrow \frac{R_q}{R}e^{-\sigma T} = 1 \wedge e^{-j\omega T} = -1$$

$$\Rightarrow \omega T = \pm(2n+1)\pi \text{ mit } n = 0, 1, 2, ...$$

$$\Leftrightarrow e^{-\sigma T} = \frac{R}{R_q}$$

$$\Leftrightarrow -\sigma T = ln\left(\frac{R}{R_q}\right)$$

$$\Leftrightarrow \sigma = \frac{1}{T}ln\left(\frac{R_q}{R}\right)$$

c)

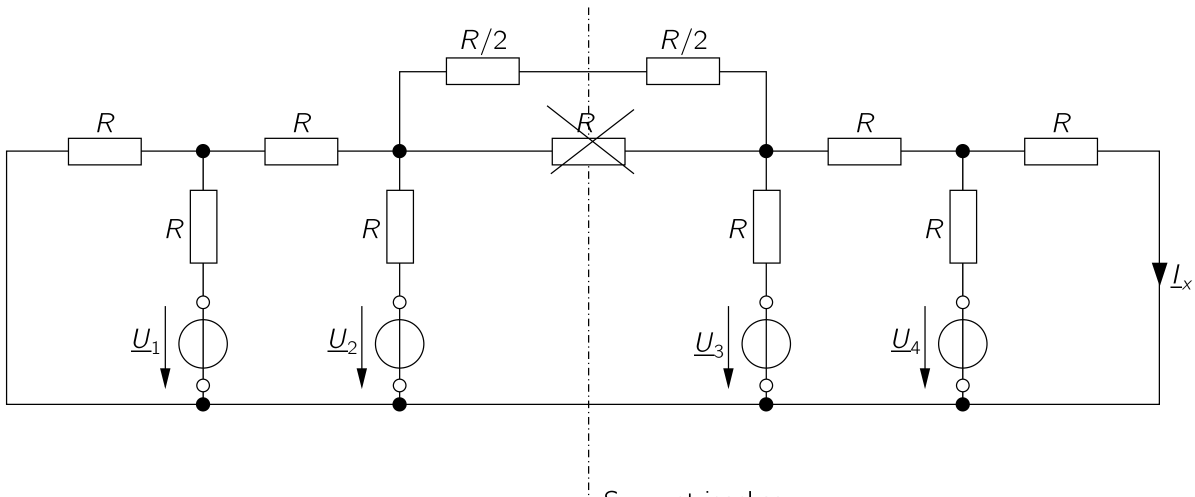
$$\sigma = \frac{1}{T}ln\left(\frac{R_q}{R}\right) \wedge \omega T = \pm(2n+1)\pi \text{ mit } n = 0, 1, 2, ...$$

d)

$$\omega = \frac{2n+1}{T}\pi \text{ mit } n = 0, 1, 2, ...$$

Aufgabe 6

a)



Überlagerungssatz:
Fall 1:

$$\underline{U}_1, \underline{U}_4 \neq 0; \qquad \underline{U}_2 = \underline{U}_3 = 0$$

Fall 2:

$$\underline{U}_2, \underline{U}_3 \neq 0; \qquad \underline{U}_1 = \underline{U}_4 = 0$$

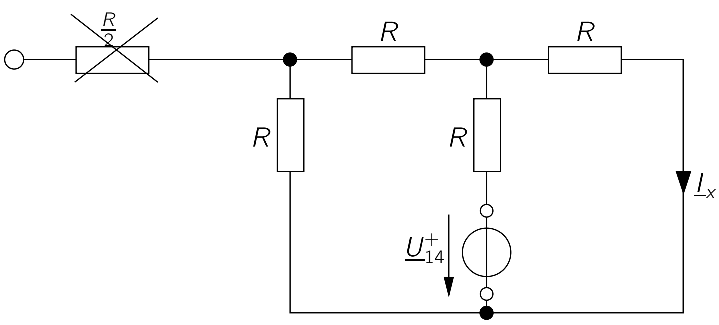
Fall 1:

$$\underline{U}_{14}^{+} = \frac{\underline{U}_1 + \underline{U}_4}{2}$$

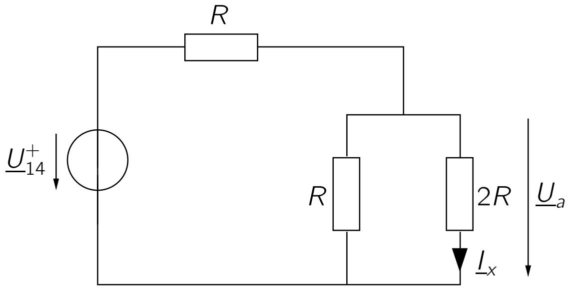
Fall 2:

$$\underline{U}_{14}^{-} = \frac{\underline{U}_1 - \underline{U}_4}{2}$$

Gleichtakt-ESB:



$$\Downarrow$$



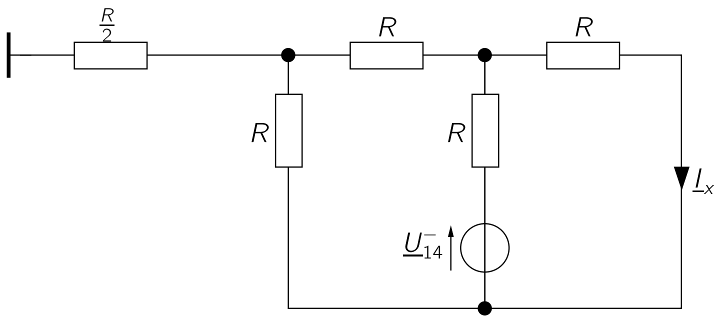
$$\underline{U}_a = \underline{U}_{14}^{+} \cdot \frac{\frac{2}{3}R}{\frac{2}{3}R + R} = \underline{U}_{14}^{+} \cdot \frac{2}{5}$$

$$\text{da } \frac{2R^2}{3R} = \frac{2}{3}R$$

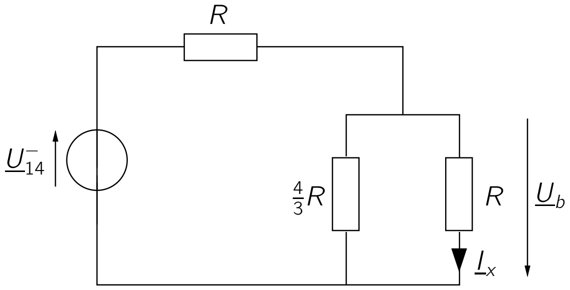
$$\Rightarrow \underline{I}_{x1}^{+} = \frac{2}{5}\frac{\underline{U}_{14}^{+}}{R}$$

$$\frac{\frac{R}{2} \cdot R}{R + \frac{R}{2}} = R \cdot \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}R$$

Gegentakt-ESB:



$$\Downarrow$$



$$\underline{U}_b = -\underline{U}_{14}^{-} \cdot \frac{\frac{4}{7}R}{(\frac{4}{7}+1)R} = -\underline{U}_{14}^{-} \cdot \frac{4}{11}$$

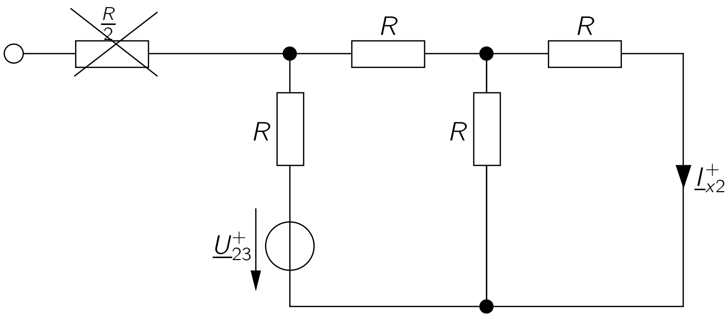
$$\frac{\frac{4}{3}R^2}{(\frac{4}{3}+1)R} = \frac{4}{7}R$$

$$\Rightarrow \underline{I}_{x1}^{-} = -\frac{4}{11}\frac{\underline{U}_{14}^{-}}{R}$$

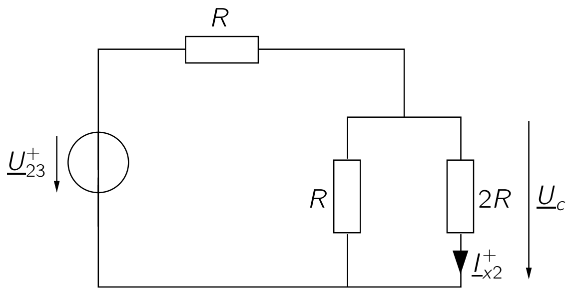
Fall 2:

$$\underline{U}_{23}^{+} = \frac{\underline{U}_2 + \underline{U}_3}{2} \qquad \underline{U}_{23}^{-} = \frac{\underline{U}_2 - \underline{U}_3}{2}$$

Gleichtakt-ESB:



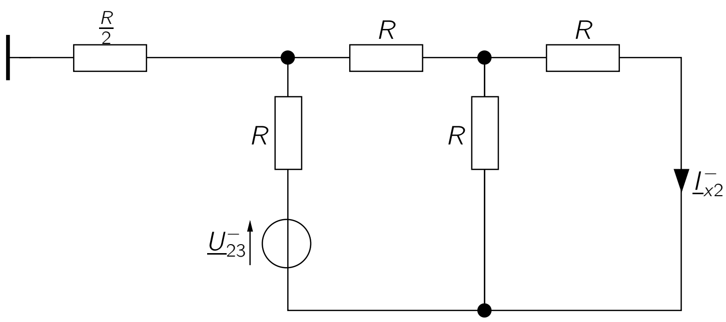
$$\Downarrow$$



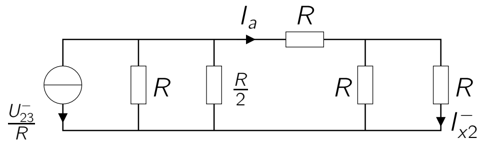
$$\underline{U}_c = \underline{U}_{23}^{+} \cdot \frac{\frac{R}{2}}{\frac{R}{2} + 2R} = \underline{U}_{23}^{+} \cdot \frac{1}{5}$$

$$\underline{I}_{x2}^{+} = \frac{1}{5R}\underline{U}_{23}^{+}$$

Gegentakt-ESB:



$$\Downarrow$$



$$I_a = -\frac{U_{23}^-}{R} \cdot \frac{\frac{1}{3}R}{\frac{1}{3}R + \frac{2}{3}R} = -\frac{U_{23}^-}{R} \cdot \frac{R}{R + \frac{9}{2}R} = -\frac{2}{11}\frac{U_{23}^-}{R}$$

$$I_{x2}^- = \frac{1}{2}I_a = -\frac{1}{11}\frac{U_{23}^-}{R}$$

Überlagerung:

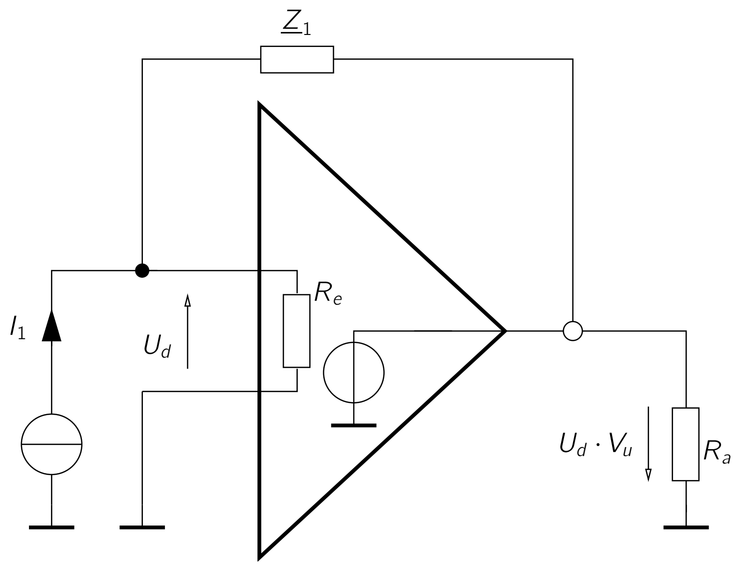
$$\begin{aligned} I_x &= I_{x1}^+ + I_{x1}^- + I_{x2}^+ + I_{x2}^- \\ &= \frac{1}{R}\left(\frac{2}{5}U_{14}^+ - \frac{4}{11}U_{14}^- + \frac{1}{5}U_{23}^+ - \frac{1}{11}U_{23}^-\right) \\ &= \frac{1}{R}\left(\frac{2}{5}\frac{U_1+U_4}{2} - \frac{4}{11}\frac{U_1-U_4}{2} + \frac{1}{5}\frac{U_2+U_3}{2} - \frac{1}{11}\frac{U_2-U_3}{2}\right) \\ &= \frac{1}{55R}(U_1 + 3U_2 + 8U_3 + 21U_4) \end{aligned}$$

b)

Berechnung über Gleichtakt-/Gegentaktanalyse bringt den Vorteil, dass durch Ausnutzen der Symmetrie nur die Hälfte des Netzwerks betrachtet werden muss. ⇒ deutlich kürzere Rechnung

Aufgabe 7

a)



$$1) U_d + (\frac{U_d}{R_e} + I_1) Z_1 = -U_a$$

$$2) U_d = \frac{U_a}{V_u}$$

$$(2) \text{ in } (1) \quad U_a(\frac{1}{V_u} + \frac{Z_1}{R_e V_u} + 1) = -I_1 Z_1$$

$$U_a(\frac{R_e + Z_1}{R_e V_u} + 1) = -I_1 Z_1$$

b)

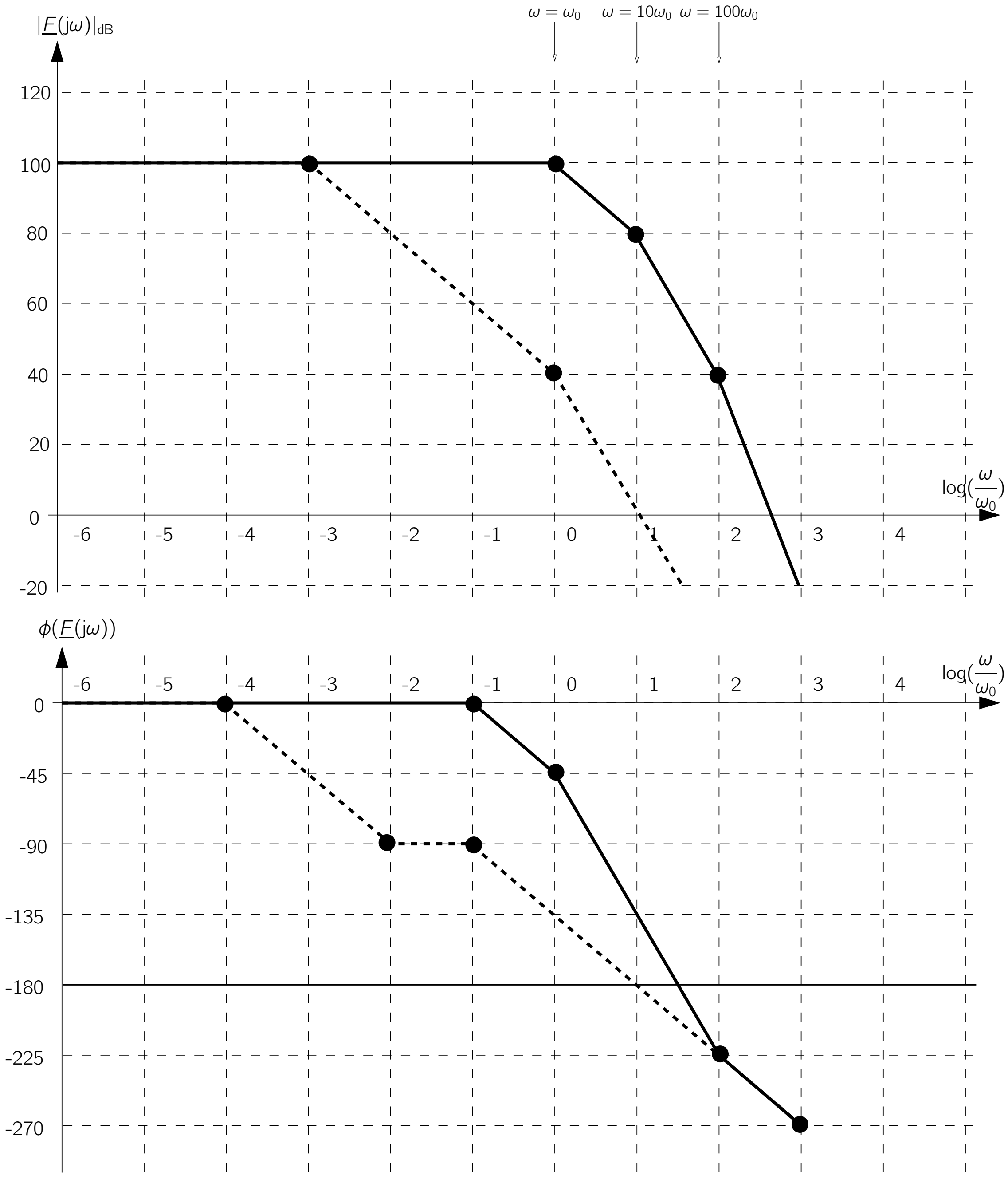
$$\frac{U_a}{I_1} = -\frac{Z_1}{1 + \frac{R_e + Z_1}{R_e V_u}} = \frac{-\frac{Z_1 R_e V_u}{R_e + Z_1}}{1 + \frac{R_e V_u}{R_e + Z_1}}$$

$$F_a = \frac{-Z_1 R_e V_u}{R_e + Z_1}$$

$$F_2 = \frac{R_e V_u}{R_e + Z_1} \cdot \frac{R_e + Z_1}{-Z_1 R_e V_u} = -\frac{1}{Z_1}$$

c)

$$F_aF_2 = \frac{R_e}{R_e + Z_1} \cdot \frac{v_0}{(1+\frac{j\omega}{\omega_0})(1+\frac{j\omega}{100\omega_0})}$$
$$= \frac{1}{1+\frac{j\omega}{10\omega_0}} \cdot \frac{v_0}{(1+\frac{j\omega}{\omega_0})(1+\frac{j\omega}{100\omega_0})}$$



d)

Phasenreserve: $180° - \phi(F(s))$ bei der Frequenz, für die gilt: $F(s) = 1$
$\Rightarrow -45° - \frac{2}{3} \cdot 45° \approx -75°$
$\Rightarrow$ INSTABIL

e)

$\omega_x \geq \omega_0$:
Für $\omega_x = \omega_0$ gilt $|F_a F_2| = 0$ dB bei $\log(\frac{\omega}{\omega_0}) \approx 2,3$. Aufgrund der doppelten Polstelle gilt für die Phase jedoch $\phi(F_a F_2) = -180°$ ab $\log(\frac{\omega}{\omega_0}) = 1$.
$\Rightarrow$ instabil (bzw. grenzstabil)

$\omega_x < \omega_0$:
Durch den ersten Pol bei ω_x und dem zweiten Pol bei ω_0 gilt $\phi(F_a F_2) = -180°$ ab $\log(\frac{\omega}{\omega_0}) = 1$
$\Rightarrow$ instabil, falls $|F_a F_2| = 0$ bei $\log(\frac{\omega}{\omega_0}) > 1$
$\Rightarrow$ $|F_a F_2| = 0$ muss demnach für Frequenzen gelten mit $\log(\frac{\omega}{\omega_0}) < 1$, die Steigung im Betragsgang beträgt in diesem Bereich $-40\,\frac{\text{dB}}{\text{dec}}$ (zwei Polstellen). Ab dem Erreichen der zweiten Polstelle bei $\log(\frac{\omega}{\omega_0}) = 0$ ändert sich die Steigung zu $-20\,\frac{\text{dB}}{\text{dec}}$
$\Rightarrow$ Schnitt mit dem stationären Wert von 100 dB ergibt für die Position der ersten Polstelle (vergleiche gestrichelter Graph im Bode-Diagramm):

$$\begin{aligned} \log(\frac{\omega_x}{\omega_0}) &= -3 \\ \Leftrightarrow \qquad \omega_x &= 0.001\,\omega_0\,. \end{aligned}$$