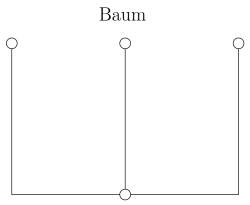
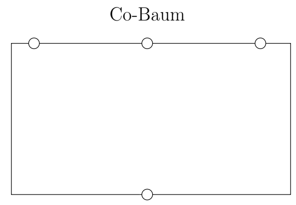
Aufgabe 1

a)



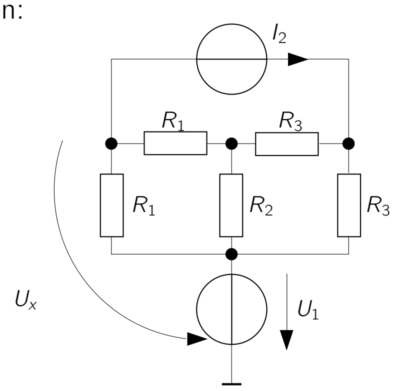


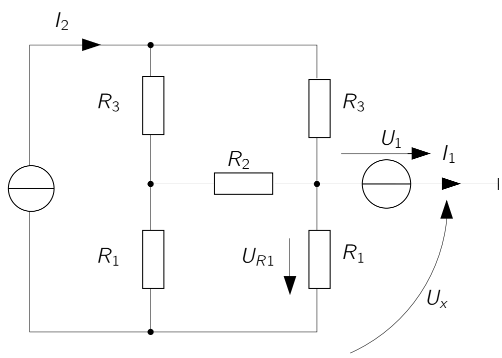
b)

4 Co-Baum-Ströme und unabhängig davon 2 bekannte Ströme (Stromquellen)
$\Rightarrow$ 2 unbekannte Ströme

c)

umzeichnen:





$$U_{R1} = \frac{R_1 I_2}{2} \text{ (symmetrie)}$$

$$U_x = U_1 - U_{R1} = U_1 - \frac{R_1 I_2}{2} = 2R_0 I_2 - \frac{R_1 I_2}{2} = \frac{3}{2} R_0 I_2 = \frac{3}{4} U_1$$

d)

$$I_1 = 0 \Rightarrow U_1 I_1 = 0$$

$$U_2 = \frac{I_2}{2}(R_1 + R_2) = \frac{U_1}{4R_0} = U_1$$

$$\Rightarrow P = 0 + I_2 U_1 = \frac{U_1}{2R_0} U_1 = \frac{1}{2} \frac{U_1^2}{R_0}$$

Aufgabe 2

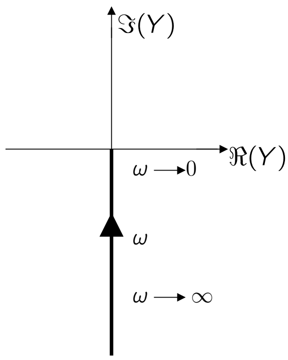
a)

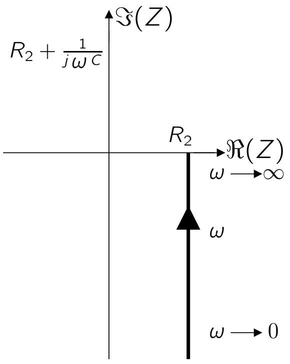
$$Y_x = \frac{1}{j\omega L_k} = -j\frac{1}{\omega L_k}$$

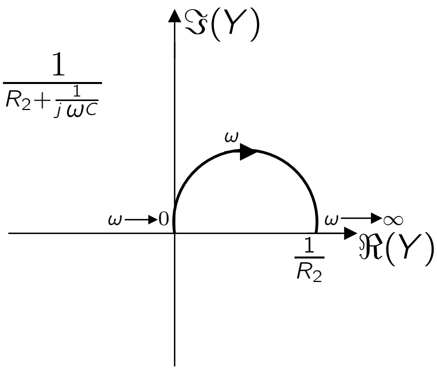
$$Y_y = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{j\omega C}}}\right)^{-1} = \left(\frac{R_1 + (R_2 + \frac{1}{j\omega C})}{R_1 + R_2 + \frac{1}{j\omega C}}\right)^{-1}$$

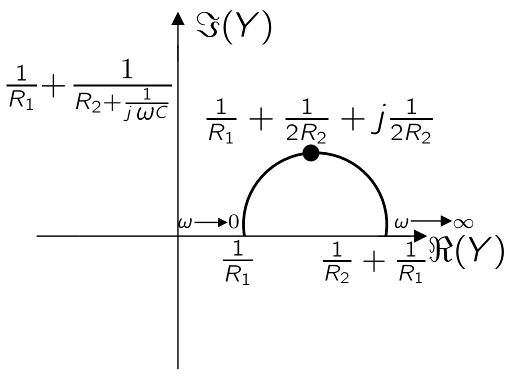
$$= \frac{R_1 + R_2 + \frac{1}{j\omega C}}{R_1 R_2 + \frac{R_1}{j\omega C}} = \frac{(R_1 + R_2) j\omega C + 1}{R_1 R_2 j\omega C + R_1}$$

b)









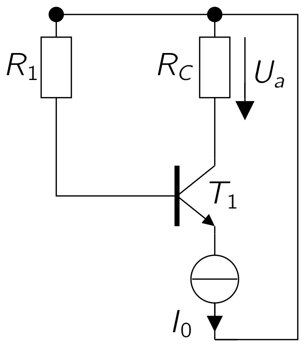
c)

$$Im(Y_x) = -Im(Y_y)$$

$$\frac{1}{\omega L_R} = \frac{1}{2R_2} \quad \Leftrightarrow \quad L_R = \frac{2R_2}{\omega} \Rightarrow L_k = \frac{2R_2}{\omega_x}$$

Aufgabe 3

a)



b)

Dimensioniere

$$U_{CE} = 10U_{BE}\ ,\ U_a = 5U_{BE}$$

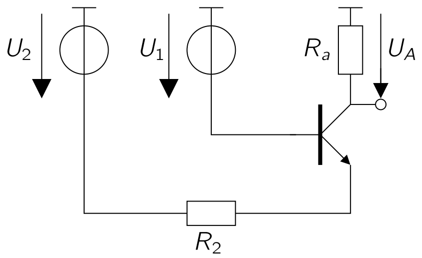
1.) $I_B R_1 = U_a + U_{CB} =$
$U_a + 9U_{BE} = 5U_{BE} + 10U_{BE} - U_{BE} = 14U_{BE}$

2.) $I_C = BI_B = \dfrac{B}{B+1} I_E = \dfrac{B}{B+1} I_0$

$\Rightarrow R_1 = 14U_{BE}\dfrac{B+1}{I_0}$

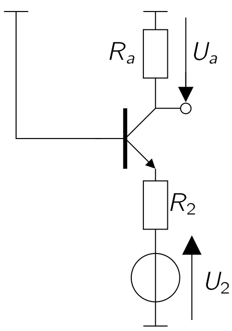
3.) $R_a = \dfrac{U_a}{I_C} = 5U_{BE}\dfrac{B+1}{BI_0}$

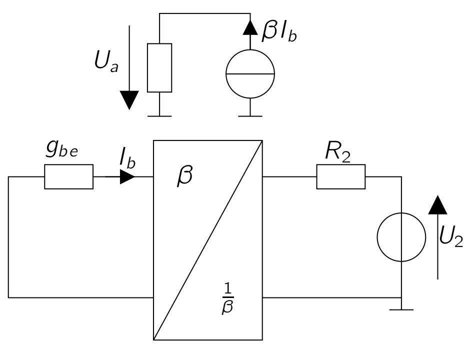
c)



d)

$U_1 = 0;$

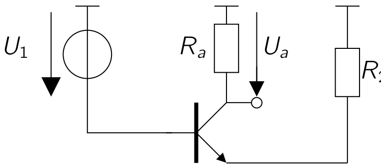


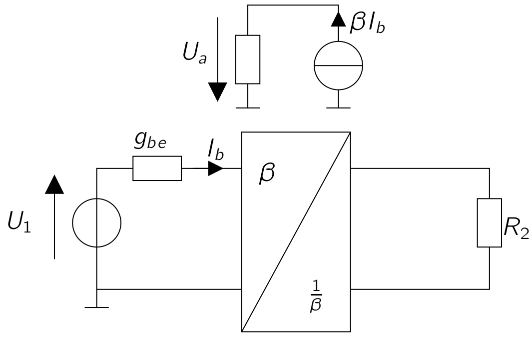


$$U_a = \beta I_b R_a = \beta \frac{U_2}{\frac{1}{g_{be}} + \beta R_2} R_a$$

$$\left.\frac{U_a}{U_2}\right|_{U_1=0} = \frac{\beta R_a}{\frac{1}{g_{be}} + \beta R_2} = \frac{R_a}{\frac{1}{\beta g_{be}} + R_2} = \frac{R_a}{\frac{1}{g_m} + R_2} = \frac{R_a}{r_e + R_2}$$

$U_2 = 0$





$$U_a = \beta I_b R_a = \beta \cdot \left(-\frac{U_1}{\frac{1}{g_{be}} + \beta R_2}\right) R_a$$

$$\left.\frac{U_a}{U_1}\right|_{U_2=0} = \frac{\beta R_a}{\frac{1}{g_{be}} + \beta R_2} = \frac{R_a}{r_e + R_2} \quad \text{(s.o)}$$

$$\Rightarrow U_a = \frac{\beta R_a}{\frac{1}{g_{be}} + \beta R_2}(U_2 - U_1)$$

e)

$$\frac{\beta R_a}{\frac{\beta}{g_{me}} + \beta R_2} = \frac{R_a}{\frac{1}{g_m} + R_2} \stackrel{!}{=} 1$$

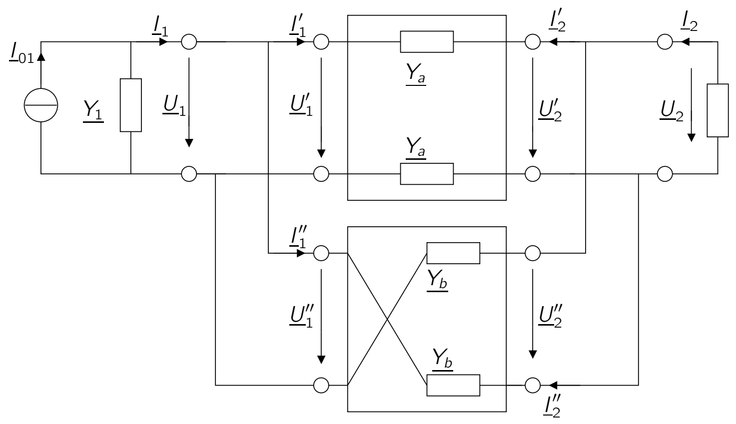
$$\Rightarrow R_a = R_2 + \frac{1}{g_m} = R_2 + \frac{U_T}{I_c} \;\Rightarrow \frac{U_a}{I_c} = R_2 + \frac{U_T}{T_c}$$

$$\Rightarrow U_a = I_c R_2 + U_T \;\Rightarrow 5U_{BE} = I_0 \frac{B}{B+1} R_2 + U_T$$

$$\Rightarrow I_0 = \frac{5U_{BE} - U_T}{\frac{B}{B+1} R_2}$$

Aufgabe 4

a)



b)

i)PPK
ii)$\underline{Y}$-Parameter , weil

$$\underline{U}'_1 = \underline{U}''_1 = \underline{U}_1; \quad \underline{U}'_2 = \underline{U}''_2 = \underline{U}_2; \quad \underline{I}_1 = \underline{I}'_1 + \underline{I}''_1; \quad \underline{I}_2 = \underline{I}'_2 + \underline{I}''_2;$$

$$\begin{aligned}\underline{I}_1 = \underline{Y}'_{11}\underline{U}'_1 + \underline{Y}''_{11}\underline{U}''_1 + \underline{Y}'_{12}\underline{U}'_2 + \underline{Y}''_{12}\underline{U}''_2 &= \underline{Y}'_{11}\underline{U}_1 + \underline{Y}''_{11}\underline{U}_1 + \underline{Y}'_{12}\underline{U}_2 + \underline{Y}''_{12}\underline{U}_2 \\ &= (\underline{Y}'_{11} + \underline{Y}''_{12})\underline{U}_1 + (\underline{Y}'_{12} + \underline{Y}''_{12})\underline{U}_2\end{aligned}$$

$$\begin{aligned}\underline{I}_2 = \underline{Y}'_{21}\underline{U}'_1 + \underline{Y}''_{21}\underline{U}''_1 + \underline{Y}'_{22}\underline{U}'_2 + \underline{Y}''_{22}\underline{U}''_2 &= \underline{Y}'_{21}\underline{U}_1 + \underline{Y}''_{21}\underline{U}_1 + \underline{Y}'_{22}\underline{U}_2 + \underline{Y}''_{22}\underline{U}_2 \\ &= (\underline{Y}'_{21} + \underline{Y}''_{22})\underline{U}_1 + (\underline{Y}'_{22} + \underline{Y}''_{22})\underline{U}_2\end{aligned}$$

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}''$$

c)

$$\underline{Y}'_{11} = \left.\frac{\underline{I}'_1}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = \frac{\underline{Y}_a}{2} \qquad \underline{Y}'_{12} = \left.\frac{\underline{I}'_1}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = -\frac{\underline{Y}_a}{2}$$

$$\underline{Y}'_{21} = \left.\frac{\underline{I}'_2}{\underline{U}'_1}\right|_{\underline{U}'_2=0} = -\frac{\underline{Y}_a}{2} \qquad \underline{Y}'_{22} = \left.\frac{\underline{I}'_2}{\underline{U}'_2}\right|_{\underline{U}'_1=0} = \frac{\underline{Y}_a}{2}$$

$$\Rightarrow \underline{Y}' = \begin{pmatrix} \frac{\underline{Y}_a}{2} & -\frac{\underline{Y}_a}{2} \\ -\frac{\underline{Y}_a}{2} & \frac{\underline{Y}_a}{2} \end{pmatrix}$$

$$\underline{Y}''_{11} = \left.\frac{\underline{I}''_1}{\underline{U}''_1}\right|_{\underline{U}''_2=0} = \frac{\underline{Y}_b}{2} \qquad \underline{Y}''_{12} = \left.\frac{\underline{I}''_1}{\underline{U}''_2}\right|_{\underline{U}''_1=0} = \frac{\underline{Y}_b}{2}$$

$$\underline{Y}''_{21} = \left.\frac{\underline{I}''_2}{\underline{U}''_1}\right|_{\underline{U}''_2=0} = \frac{\underline{Y}_b}{2} \qquad \underline{Y}''_{22} = \left.\frac{\underline{I}''_2}{\underline{U}''_2}\right|_{\underline{U}''_1=0} = \frac{\underline{Y}_b}{2}$$

$$\Rightarrow \underline{Y}' = \begin{pmatrix} \frac{\underline{Y}_b}{2} & \frac{\underline{Y}_b}{2} \\ \frac{\underline{Y}_b}{2} & \frac{\underline{Y}_b}{2} \end{pmatrix}$$

$$\Rightarrow \underline{Y} = \underline{Y}' + \underline{Y}'' = \begin{pmatrix} \frac{\underline{Y}_b+\underline{Y}_a}{2} & \frac{\underline{Y}_b-\underline{Y}_a}{2} \\ \frac{\underline{Y}_b-\underline{Y}_a}{2} & \frac{\underline{Y}_b+\underline{Y}_a}{2} \end{pmatrix}$$

d)

$$\underline{U}_2 = -\frac{\underline{I}_2}{\underline{Y}_2}$$

$$\underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 = \underline{Y}_{11}\underline{U}_1 - \frac{\underline{Y}_{12}}{\underline{Y}_2}\underline{I}_2 \quad \Longleftrightarrow \quad \underline{U}_1 = \frac{1}{\underline{Y}_{11}}\underline{I}_1 + \frac{\underline{Y}_{12}}{\underline{Y}_2\underline{Y}_{11}}\underline{I}_2$$

$$\Rightarrow \underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 = \frac{\underline{Y}_{21}}{\underline{Y}_{11}}\underline{I}_1 + \frac{\underline{Y}_{12}\underline{Y}_{21}}{\underline{Y}_{11}\underline{Y}_2}\underline{I}_2 - \frac{\underline{Y}_{22}}{\underline{Y}_2}\underline{I}_2$$

$$\Longleftrightarrow I_2\left(1 - \frac{\underline{Y}_{12}\underline{Y}_{21}}{\underline{Y}_{11}\underline{Y}_2} + \frac{\underline{Y}_{22}}{\underline{Y}_2}\right) = \frac{\underline{Y}_{21}}{\underline{Y}_{11}}$$

$$\Longleftrightarrow I_2\frac{\underline{Y}_{11}\underline{Y}_2 - \underline{Y}_{12}\underline{Y}_{21} + \underline{Y}_{22}\underline{Y}_{11}}{\underline{Y}_{11}\underline{Y}_2} = \frac{\underline{Y}_{21}}{\underline{Y}_{11}}$$

$$\Longleftrightarrow \underline{F}_I = \frac{\underline{I}_2}{\underline{I}_1} = \frac{\underline{Y}_{21}\underline{Y}_2}{\underline{Y}_{11}\underline{Y}_2 - \underline{Y}_{12}\underline{Y}_{21} + \underline{Y}_{22}\underline{Y}_{11}}$$

$$\underline{F}_I = 0 \text{ für } \underline{Y}_{21} = 0 \Longleftrightarrow \frac{\underline{Y}_b-\underline{Y}_a}{2} = 0 \Longleftrightarrow \underline{Y}_a = \underline{Y}_b$$

Aufgabe 5

a)

$$Y_{out} = \frac{1}{(\frac{1}{sC} + \frac{RsL}{R+sL} + r_x)}$$

b)

Oszillation:

Stromfluss muss möglich sein,
Anregung mit Spannungsquelle die mit t $\mapsto \infty$ verschwindet.

$$Y = \frac{I}{U}\Big|_{u \mapsto \infty}$$

c)

$$Y_{out} = \frac{1}{\frac{1}{sC} + \frac{RsL}{R+sL} + r_x} = \frac{sC(R+sL)}{R + sL + RsLsC + r_x sC(R+sL)}$$

$$\Rightarrow \; s^2(LCr_x + RCL) + s(L + RCr_x) + R \stackrel{!}{=} 0$$

$$\Rightarrow \; s^2 + s \cdot \frac{L + RCr_x}{LC(R + r_x)} + \frac{R}{LC(R + r_x)} = 0$$

$$s_{1\;2} = -\frac{1}{2} \cdot \frac{L + RCr_x}{LC(R + r_x)} \; \frac{+}{-} \; \sqrt{\left(\frac{L + RCr_x}{2LC(R + r_x)}\right)^2 - \frac{R}{LC(R + r_x)}}$$

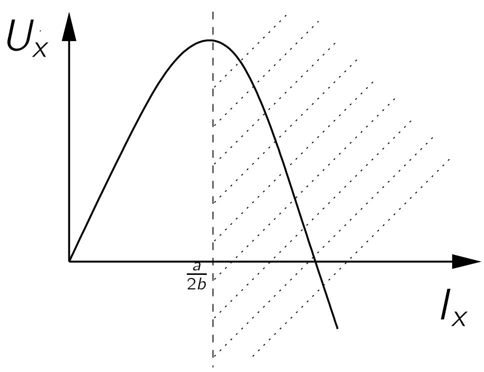
d)

konstante Amplitude wenn $\Re(s) = 0 = \sigma \Rightarrow \; r_x = -\frac{L}{RC}$

e)

Schwingfrequenz: $s = \sigma + j\omega = 0 + j\sqrt{\frac{R}{LC(R+(-\frac{L}{RC}))}}$ $\; mit \;\; R - \frac{L}{RC} > 0 \rightarrow \;\; R^2 > \frac{L}{C}$

f)



g)

$$U = aI - bI^2$$

$$\frac{dU}{dI}\Big|_{AP} = rx = a - 2bI$$

$$\Rightarrow \quad \cdot)\; r_x < 0 \;\; \leftrightarrow \;\; a - 2bI < 0 \rightarrow \;\; I > \frac{a}{2b}$$

$$\cdot)\; rx \stackrel{!}{=} -\frac{L}{RC}$$

$$\leftrightarrow \;\; a - 2bI = -\frac{L}{RC} \;\; \rightarrow \;\; I = +\frac{1}{2b}(\frac{L}{RC} + a)$$

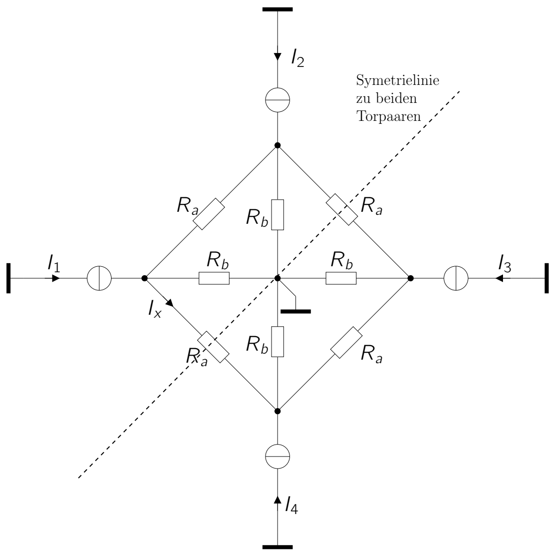
Aufgabe 6

a)

Torpaar 1 : I_1, I_4

Torpaar 2 : I_2, I_3

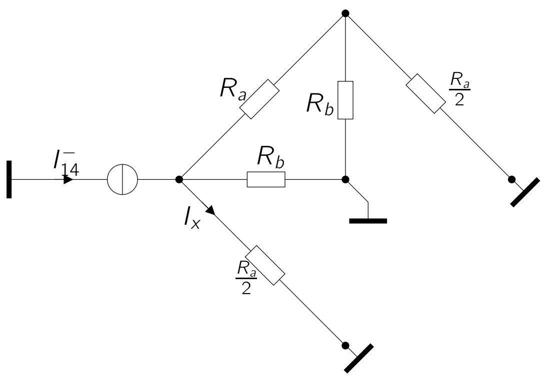
$$I_{14}^{+} = \frac{I_1+I_4}{2} \qquad I_{14}^{-} \frac{I_1-I_4}{2}$$
$$I_{23}^{+} = \frac{I_2+I_3}{2} \qquad I_{23}^{-} \frac{I_2-I_3}{2}$$

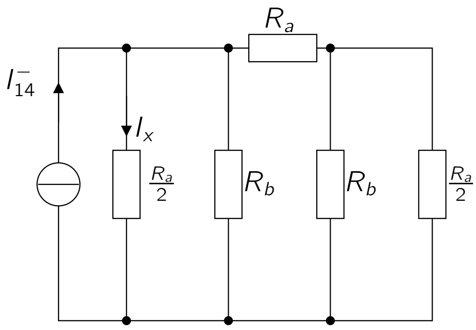


b)

betrachte Torpaar 1 , $I_2 = I_3 = 0$
Gleichtakt: $I_x = 0$ (“*freies Ende*“ *bei* $\frac{R_a}{2}$)

Gegentakt:

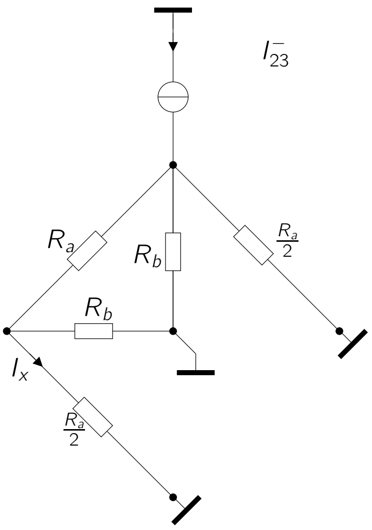


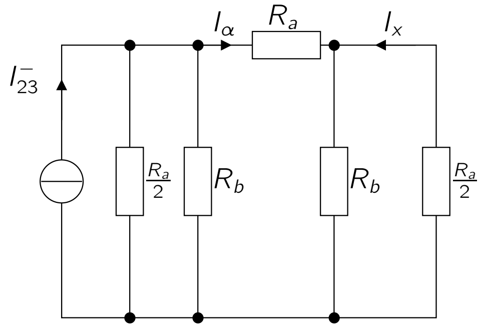


$$\frac{I_x}{I_{14}^{-}} = \frac{\frac{2}{R_a}}{\frac{2}{R_a} + \frac{1}{R_b} + \frac{1}{R_a + \frac{\frac{R_a}{2} R_b}{\frac{R_a}{2} + R_b}}} = \frac{\frac{2}{R_a}}{\frac{2}{R_a} + \frac{1}{R_b} + \frac{1}{R_a + \frac{R_a R_b}{R_a + 2R_b}}} = H_{14}$$

betrachte Torpaar 2 , $I_1 = I_4 = 0$
Gleichtakt: $I_x = 0$ $(s.o.)$

Gegentakt:





$$\frac{I_\alpha}{I_{23}^{-}} = -\frac{\frac{R_a}{2}//R_b}{R_a + 2(R_a//R_b)} \qquad \frac{I_x}{I_\alpha} = \frac{R_b}{\frac{R_a}{2} + R_b} = \frac{2R_b}{2R_b + R_a}$$

$$\Rightarrow \quad \frac{I_x}{I_{23}^{-}} = \frac{\frac{R_a}{2}//R_b}{R_a + 2(\frac{R_a}{2}//R_b)} \cdot \frac{2R_b}{2R_b + R_a} = H_{23}$$

$$I_x = H_{14} I_{14}^{-} + H_{23} I_{23}^{-} = \frac{H_{14}}{2}(I_1 - I_4) + \frac{H_{23}}{2}(I_2 - I_3)$$

Aufgabe 7

a)

$$\frac{\underline{U}_p}{\underline{U}_1} = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}$$

$$\underline{U}_p = \underline{U}_d + \underline{U}_e \Leftrightarrow \underline{U}_d = \underline{U}_p - \underline{U}_e$$

$$\underline{U}_2 = v_u \underline{U}_d \frac{\underline{Z}_e}{R_a+\underline{Z}_e} \Leftrightarrow \underline{U}_2 = v_u(\underline{U}_p - \underline{U}_2)\frac{\underline{Z}_e}{R_a+\underline{Z}_e}$$

$$\Rightarrow \underline{U}_2(1 + v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}) = \underline{U}_p v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}$$

$$\frac{\underline{U}_2}{\underline{U}_p} = \frac{v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}{1+v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}$$

$$\Rightarrow \underline{F} = \frac{\underline{U}_p}{\underline{U}_1}\frac{\underline{U}_2}{\underline{U}_p} = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \cdot \frac{v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}{1+v_u \frac{\underline{Z}_e}{R_a+\underline{Z}_e}}$$

b)

$$\underline{F}_k = \frac{\underline{Z}_2'}{\underline{Z}_1'+\underline{Z}_2'} \frac{\underline{v}_u' \frac{\underline{Z}_e'}{R_a+\underline{Z}_e'}}{1+\underline{v}_u' \frac{\underline{Z}_e'}{R_a+\underline{Z}_e'}} \cdot \frac{\underline{Z}_2''}{\underline{Z}_1''+\underline{Z}_2''} \frac{\underline{v}_u'' \frac{\underline{Z}_e''}{R_a+\underline{Z}_e''}}{1+\frac{\underline{Z}_e''}{R_a+\underline{Z}_e''}}$$

$$\underline{Z}_1' = R_1; \underline{Z}_2' = j\omega L_2;\ \underline{Z}_1'' = j\omega L_1;\ \underline{Z}_2'' = R_2;\ |\underline{Z}_e| \mapsto \infty;$$
$$\underline{Z}_e' = \underline{Z}_1'' + \underline{Z}_2'' = R_2 + j\omega L_1;\ \underline{v}_u' = \underline{v}_u'' = \alpha \in \mathbb{R},\ \alpha >> 1$$

$$\underline{F}_k = \frac{j\omega L_2}{R_1+j\omega L_2} \frac{\alpha \frac{R_2+j\omega L_1}{R_a+R_2+j\omega L_1}}{1+\alpha \frac{R_2+j\omega L_1}{R_a+R_2+j\omega L_1}} \cdot \frac{R_2}{R_2+j\omega L_1} \frac{\alpha}{1+\alpha}$$

$$\overset{|\underline{Z}_e''|\mapsto\infty}{\approx} \frac{1}{1+j\omega \frac{L_2}{R_1}} \cdot \frac{\alpha(1+j\omega \frac{L_1}{R_2})}{(1+\alpha)(1+j\omega \frac{L_1}{R_2} + \frac{R_a}{(1+\alpha)R_2})} \cdot \frac{j\omega \frac{L_1}{R_2}}{1+j\omega \frac{L_1}{R_2}} \cdot \underbrace{\frac{\alpha}{1+\alpha}}_{\to 1}$$

$$= \frac{1}{1+j\frac{\omega}{\omega_1}} \cdot \frac{\alpha(1+j\frac{\omega}{\omega_2})}{(1+\alpha)(1+j\frac{\omega}{\tilde{\omega}})} \cdot \frac{j\frac{\omega}{\omega_2}}{1+j\frac{\omega}{\omega_2}}$$

$$\omega_1 = \frac{R_1}{L_2}; \quad \omega_2 = \frac{R_2}{L_1};$$
$$\tilde{\omega} = \frac{1}{\frac{L_1}{R_2} + \frac{R_a}{\omega(1+\alpha)R_2}} = \frac{1}{\frac{1}{\omega_2} + \frac{R_a}{\omega(1+\alpha)R_2}}$$

c)

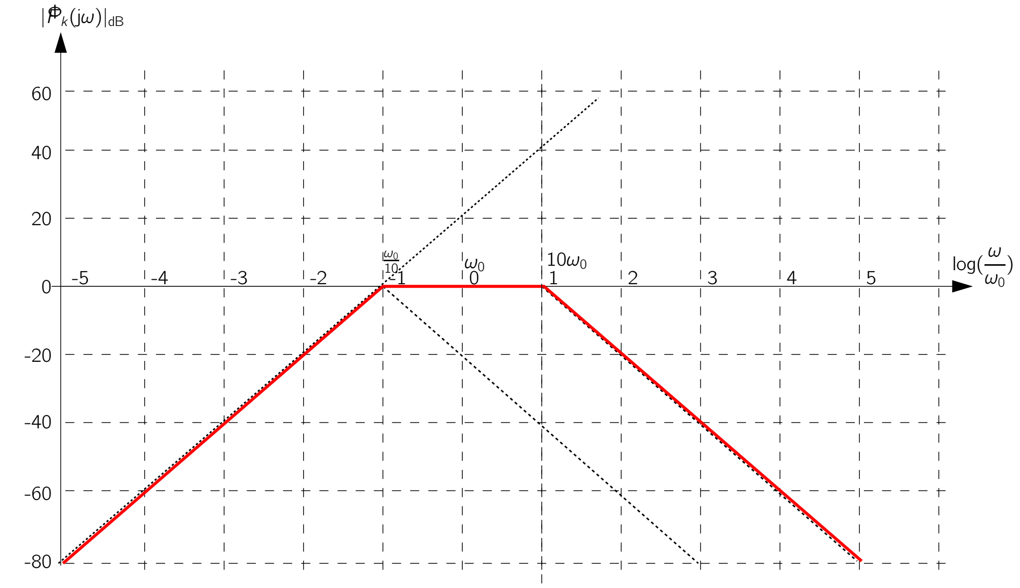
$$\alpha R_1 >> R_a \Rightarrow \tilde{\omega} = \frac{1}{\frac{1}{\omega_2}+0} = \omega_2$$

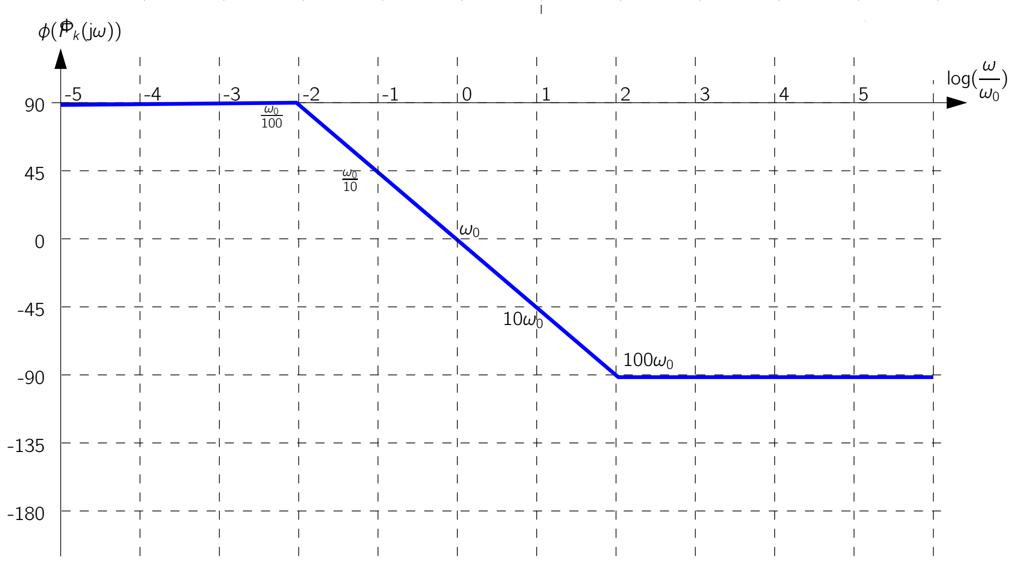
$$\underline{F}_k = \frac{1}{1+j\frac{\omega}{\omega_1}} \cdot \underbrace{\frac{\alpha\cancel{(1+j\frac{\omega}{\omega_2})}}{(1+\alpha)\cancel{(1+j\frac{\omega}{\omega_2})}}}_{\approx\frac{\alpha}{\alpha}\approx 0} \cdot \frac{j\frac{\omega}{\omega_2}}{1+j\frac{\omega}{\omega_2}}$$

$$= \frac{j\frac{\omega}{\omega_2}}{(1+j\frac{\omega}{\omega_1})(1+j\frac{\omega}{\omega_2})}$$

$$\omega_1 = 10\omega_0 = \frac{R_1}{L_1}; \quad \omega_2 = \frac{\omega_0}{10} = \frac{R_2}{L_1}$$

d)


$|\underline{F}_k(j\omega)|_{dB}$
60
40
20
0
-20
-40
-60
-80
-5
-4
-3
-2
-1
0
1
2
3
4
5
$\frac{\omega_0}{10}$
ω_0
$10\omega_0$
$\log(\frac{\omega}{\omega_0})$


$\phi(\underline{F}_k(j\omega))$
90
45
0
-45
-90
-135
-180
-5
-4
-3
-2
-1
0
1
2
3
4
5
$\frac{\omega_0}{100}$
$\frac{\omega_0}{10}$
ω_0
$10\omega_0$
$100\omega_0$
$\log(\frac{\omega}{\omega_0})$