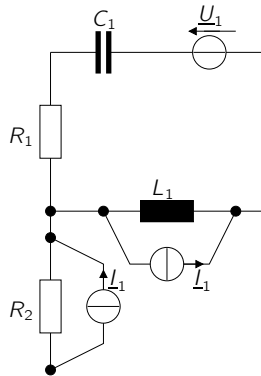
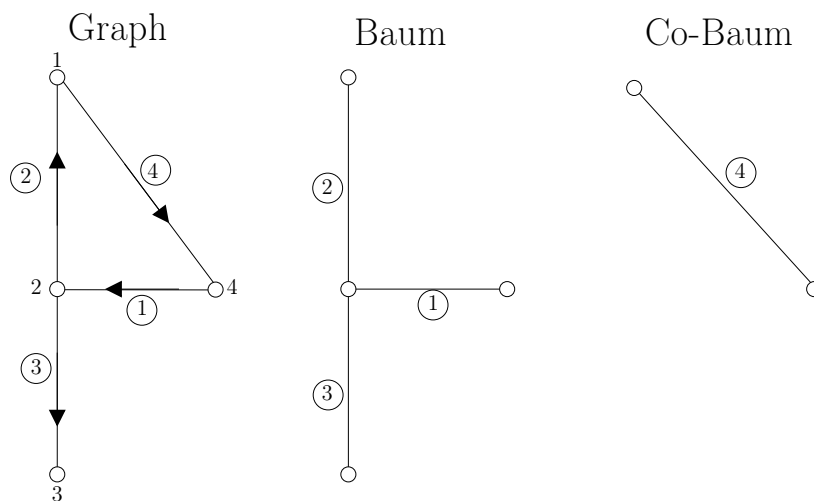


Aufgabe 1

a)



b)



c)

$$Y = \begin{pmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_1} & 0 & 0 \\ 0 & 0 & \frac{1}{R_2} & 0 \\ 0 & 0 & 0 & sC \end{pmatrix}$$

d)

$$A = K \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \end{matrix}$$

Knoten 2 als Bezugsknoten \rightarrow Streichen 2. Zeile:

$$\Rightarrow A = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

e)

$$\begin{aligned} Y_n = AYA^T &= \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_1} & 0 & 0 \\ 0 & 0 & \frac{1}{R_2} & 0 \\ 0 & 0 & 0 & sC \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & \frac{1}{sL_1} \\ -\frac{1}{R_1} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 \\ sC & 0 & -sC \end{pmatrix} = \begin{pmatrix} \frac{1}{R_1} + sC & 0 & -sC \\ 0 & \frac{1}{R_2} & 0 \\ -sC & 0 & \frac{1}{sL_1} + sC \end{pmatrix} \end{aligned}$$

$$\begin{aligned} I_{qn} = A(I_g - YU_g) &= \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \left[\begin{pmatrix} I_1 \\ 0 \\ I_1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_1} & 0 & 0 \\ 0 & 0 & \frac{1}{R_2} & 0 \\ 0 & 0 & 0 & sC \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ U_1 \end{pmatrix} \right] \\ &= \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ 0 \\ I_1 \\ -sC \cdot U_1 \end{pmatrix} = \begin{pmatrix} -sC \cdot U_1 \\ I_1 \\ I_1 + sC \cdot U_1 \end{pmatrix} \end{aligned}$$

$$\begin{matrix} Y_n & & U_n & = & I_{qn} \\ \begin{pmatrix} \frac{1}{R_1} + sC & 0 & -sC \\ 0 & \frac{1}{R_2} & 0 \\ -sC & 0 & \frac{1}{sL_1} + sC \end{pmatrix} & & \begin{pmatrix} U_{n1} \\ U_{n3} \\ U_{n4} \end{pmatrix} & = & \begin{pmatrix} -sC \cdot U_1 \\ I_1 \\ I_1 + sC \cdot U_1 \end{pmatrix} \end{matrix}$$

 U_{n3} ist entkoppelt \Rightarrow streichen 2. Zeile und 2. Spalte:

$$\begin{pmatrix} \frac{1}{R_1} + sC & -sC \\ -sC & \frac{1}{sL_1} + sC \end{pmatrix} \begin{pmatrix} U_{n1} \\ U_{n4} \end{pmatrix} = \begin{pmatrix} -sC \cdot U_1 \\ I_1 + sC \cdot U_1 \end{pmatrix}$$

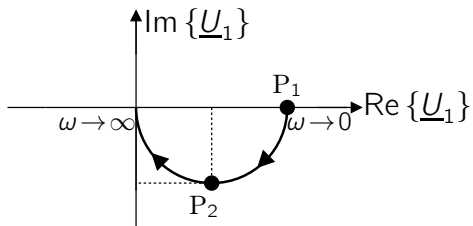
$$U_x = -U_{n1}$$

$$U_{n1} = \frac{\begin{vmatrix} -sC \cdot U & -sC \\ I_1 + sC \cdot U & \frac{1}{sL_1} + sC \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + sC & -sC \\ -sC & \frac{1}{sL_1} + sC \end{vmatrix}}$$

Aufgabe 2

a) + b)

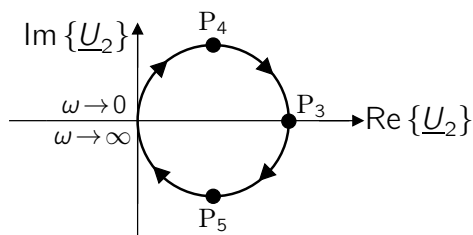
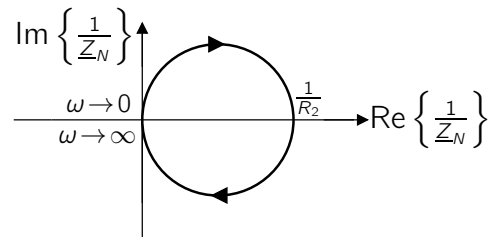
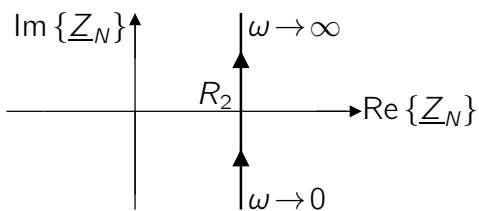
$$\underline{U}_1 = I_1 \cdot (R_1 \parallel C_1) = I_1 \cdot \frac{\frac{R_1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1} \cdot I_1$$



$$P_1 : \underline{U}_1 = R_1 I_1$$

$$P_2 : \underline{U}_1 = R_1 I_1 \left(\frac{1}{2} - j\frac{1}{2} \right)$$

$$\underline{U}_2 = \underline{U}_0 \cdot \frac{R_2}{\underbrace{R_2 + j\omega L_2 + \frac{1}{j\omega C_2}}_{Z_N}} = \frac{j\omega R_2 C_2}{j\omega R_2 C_2 - \omega^2 L_2 C_2 + 1} \cdot \underline{U}_0$$



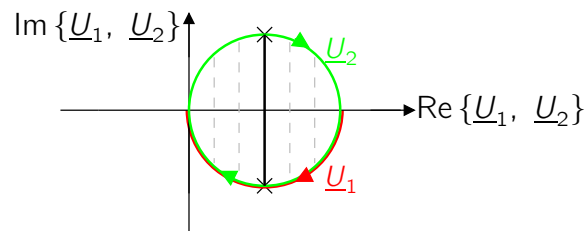
$$P_3 : \underline{U}_2 = \underline{U}_0$$

$$P_4, P_5 : \underline{U}_2 = \frac{\underline{U}_0}{2} (1 \pm j)$$

c)

$$\underline{U}_0 = R_1 I_1$$

d)

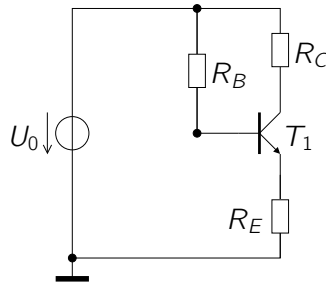


$$\begin{aligned} \underline{U}_1 &= \frac{R_1 I_1}{2} (1 - j) = \frac{R_1 I_1}{1 + j\omega R_1 C_1} \\ \Leftrightarrow \frac{1}{2} - \frac{j}{2} &= \frac{1}{1 + j\omega R_1 C_1} \\ \Leftrightarrow \frac{2}{1 - j} &= 1 + j\omega R_1 C_1 \\ \Leftrightarrow 2 \frac{1 + j}{2} &= 1 + j\omega R_1 C_1 \\ \Leftrightarrow \omega R_1 C_1 &= 1 \Leftrightarrow \omega = \frac{1}{R_1 C_1} \end{aligned}$$

$$\begin{aligned} \underline{U}_2 &= \frac{U_0}{2} (1 + j) = \frac{R_2}{R_2 + j\omega L_2 + \frac{1}{j\omega C_2}} U_0 \\ \Leftrightarrow 1 + j &= 2 \frac{R_2}{R_2 + j\omega L_2 + \frac{1}{j\omega C_2}} \\ \Leftrightarrow \frac{1}{1 + j} &= \frac{1}{2} \left(1 + j\omega \frac{L_2}{R_2} - j \frac{1}{\omega R_2 C_2} \right) \\ \Leftrightarrow \frac{1 - j}{2} &= \frac{1}{2} \left(1 + j \left(\frac{L_2}{R_2 R_1 C_1} - \frac{R_1 C_1}{R_2 C_2} \right) \right) \\ \Leftrightarrow \frac{L_2}{R_1 R_2 C_1} - \frac{R_1 C_1}{R_2 C_2} &= -1 \\ \Leftrightarrow L_2 &= -R_1 R_2 C_2 + \frac{R_1^2 C_1^2}{C_2} \end{aligned}$$

Aufgabe 3

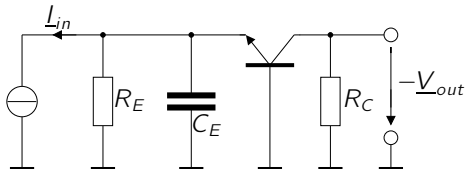
a)



b)

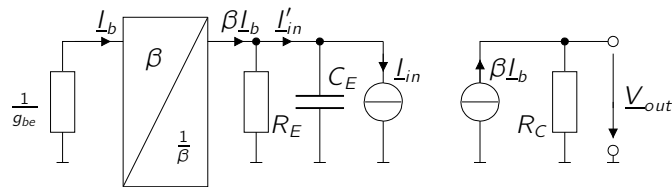
$$\begin{aligned}
 U_{CE} = U_{BE} &\Rightarrow U_{CB} = 0 = R_B \cdot I_B - R_C \cdot I_C \\
 &= R_B - R_C \cdot B \cdot I_B \\
 &\Leftrightarrow R_B = B \cdot R_C \\
 \frac{R_B}{R_C} &= B
 \end{aligned}$$

c)



Basisgrundsaltung, da Basis auf gemeinsamem Bezugspotential für Ein- und Ausgangssignal

d)



(2014-10-06: in Abb. vertauschtes β und $\frac{1}{\beta}$ im T-Operator korrigiert.)

$$V_{out} = \beta \cdot I_b \cdot R_C$$

$$\beta \cdot I_b = I'_{in} \frac{R_E}{R_E + \frac{1}{\beta \cdot g_{be}}} = I'_{in} \frac{R_E}{R_E + \frac{1}{g_m}}$$

$$I'_{in} = I_{in} \frac{\frac{1}{j\omega C_E}}{\frac{1}{j\omega C_E + R_E \parallel \frac{1}{\beta \cdot g_{be}}}} = I_{in} \frac{1}{1 + j\omega C_E \left(R_E \parallel \frac{1}{g_m} \right)}$$

$$\Rightarrow \underline{Z}_T = \frac{V_{out}}{I_{in}} = R_C \frac{R_E}{R_E + \frac{1}{g_m}} \cdot \frac{1}{1 + j\omega C_E \left(R_E \parallel \frac{1}{g_m} \right)}$$

e)

$$\underline{Z}_T = R_C \frac{R_E}{R_E + \frac{1}{g_m}} \cdot \frac{1}{1 + j\omega C_E \left(R_E \parallel \frac{1}{g_m} \right)}$$

$$1) |Z_T(\omega = 0)| = 0.99 \cdot R_C$$

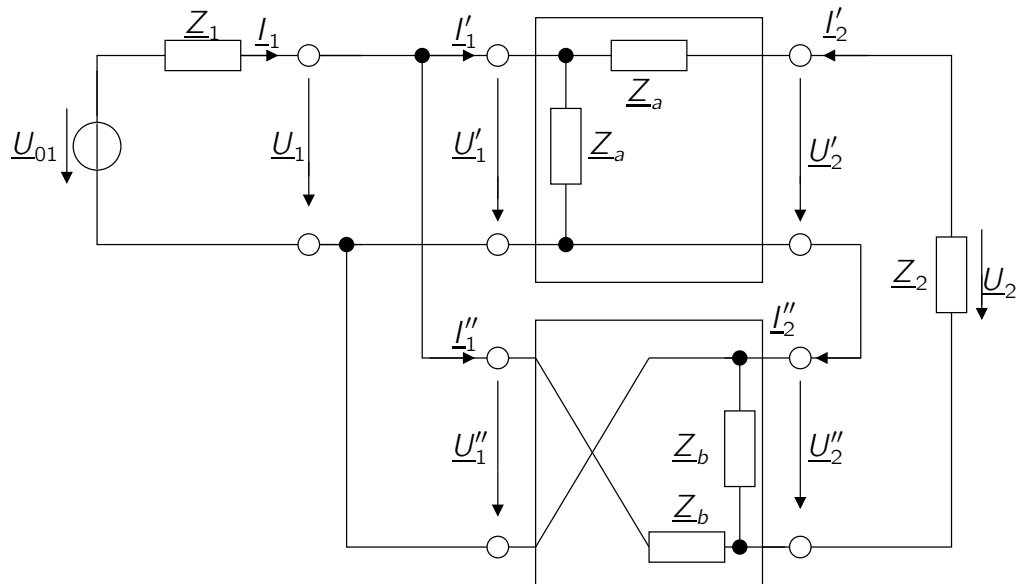
$$\frac{R_E}{R_E + \frac{1}{g_m}} = 0.99 \Leftrightarrow R_E \cdot g_m = \frac{0.99}{0.01} = 99$$

$$2) |Z_T(\omega = \omega_{3dB})| = \frac{1}{\sqrt{2}} |Z_T(\omega = 0)|$$

$$\omega_{3dB} = \frac{1}{C_E \left(R_E \parallel \frac{1}{g_m} \right)} = \frac{1}{C_E \frac{R_E}{1 + g_m \cdot R_E}} = \frac{1 + g_m \cdot R_E}{C_E \cdot R_E} = 10^8 \text{ Hz}$$

Aufgabe 4

a)



b)

i)

PSK

ii)

G-Parameter

$$\left. \begin{array}{l} U_1 = U'_1 = U''_1, \quad I_1 = I'_1 + I''_1 \\ I_2 = I'_2 = I''_2, \quad U_2 = U'_2 + U''_2 \end{array} \right\} \Rightarrow \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = [G] \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$$I_1 = G'_{11}U'_1 + G''_{11}U''_1 + G'_{12}I'_2 + G''_{12}I''_2 = (G'_{11} + G''_{11})U_1 + (G'_{12} + G''_{12})I_2$$

$$U_2 = G'_{21}U'_1 + G''_{21}U''_1 + G'_{22}I'_2 + G''_{22}I''_2 = (G'_{21} + G''_{21})U_1 + (G'_{22} + G''_{22})I_2$$

c)

$$\begin{aligned}
 \underline{G}'_{11} &= \left. \frac{i'_1}{u_1} \right|_{i'_2=0} = \frac{1}{\underline{Z}_a} & \underline{G}'_{12} &= \left. \frac{i'_1}{i'_2} \right|_{u_1=0} = -1 \\
 \underline{G}'_{21} &= \left. \frac{u_2}{u_1} \right|_{i'_2=0} = 1 & \underline{G}'_{22} &= \left. \frac{u_2}{i'_2} \right|_{u_1=0} = \underline{Z}_a \\
 \underline{G}''_{11} &= \left. \frac{i''_1}{u_1} \right|_{i''_2=0} = \frac{1}{2\underline{Z}_b} & \underline{G}''_{12} &= \left. \frac{i''_1}{i''_2} \right|_{u_1=0} = \frac{1}{2} \\
 \underline{G}''_{21} &= \left. \frac{u_2}{u_1} \right|_{i''_2=0} = -\frac{1}{2} & \underline{G}''_{22} &= \left. \frac{u_2}{i''_2} \right|_{u_1=0} = \frac{\underline{Z}_b}{2}
 \end{aligned}$$

$$[\underline{G}] = \begin{bmatrix} \frac{1}{\underline{Z}_a} + \frac{1}{2\underline{Z}_b} & -\frac{1}{2} \\ \frac{1}{2} & \underline{Z}_a + \frac{\underline{Z}_b}{2} \end{bmatrix}$$

d)

$$\begin{aligned}
 u_2 &= -\underline{Z}_2 i_2 \\
 u_2 &= \underline{G}_{21} u_1 + \underline{G}_{22} i_2 \Leftrightarrow -\underline{Z}_2 i_2 = \underline{G}_{21} u_1 + \underline{G}_{22} i_2 \\
 &\Leftrightarrow \underline{G}_{21} u_1 = -(\underline{G}_{22} + \underline{Z}_2) i_2 \\
 &\Leftrightarrow u_1 = -\frac{\underline{G}_{22} + \underline{Z}_2}{\underline{G}_{21}} i_2 \\
 \Rightarrow i_1 &= \underline{G}_{11} u_1 + \underline{G}_{12} i_2 = \left(\underline{G}_{12} - \underline{G}_{11} \frac{\underline{G}_{22} + \underline{Z}_2}{\underline{G}_{21}} \right) i_2 \\
 \Leftrightarrow i_1 &= \frac{\underline{G}_{12} \underline{G}_{21} - \underline{G}_{11} \underline{G}_{22} - \underline{G}_{11} \underline{Z}_2}{\underline{G}_{21}} i_2 \\
 \Leftrightarrow \underline{F}_I &= \frac{i_2}{i_1} = \frac{\underline{G}_{21}}{\underline{G}_{12} \underline{G}_{21} - \underline{G}_{11} (\underline{G}_{22} + \underline{Z}_2)} \\
 &= \frac{\frac{1}{2}}{-\frac{1}{2} \cdot \frac{1}{2} - \left(\frac{1}{\underline{Z}_a} + \frac{1}{2\underline{Z}_b} \right) \left(\underline{Z}_a + \frac{\underline{Z}_b}{2} + \underline{Z}_2 \right)} \\
 &= \frac{\frac{1}{2} \underline{Z}_a \underline{Z}_b}{-\frac{1}{4} \underline{Z}_a \underline{Z}_b - \left(\underline{Z}_b + \frac{\underline{Z}_a}{2} \right) \left(\underline{Z}_a + \frac{\underline{Z}_b}{2} + \underline{Z}_2 \right)}
 \end{aligned}$$

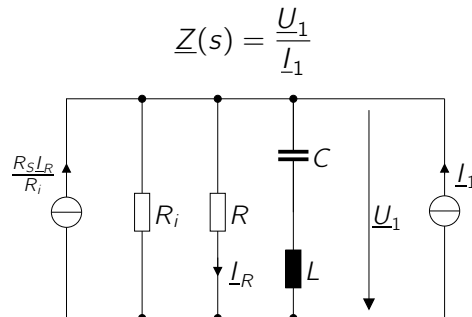
$$\begin{aligned}
 &= \frac{\frac{1}{2}Z_a Z_b}{-\frac{Z_a Z_b}{4} - Z_b Z_a - \frac{Z_b^2}{2} - \frac{Z_a^2}{2} - \frac{Z_a Z_b}{4} - Z_b Z_2 - \frac{Z_a Z_2}{2}} \\
 &= \frac{-Z_a Z_b}{Z_a^2 + 3Z_a Z_b + Z_b^2 + Z_2(2Z_b + Z_a)}
 \end{aligned}$$

$$\Rightarrow Z_a = 0 \vee Z_b = 0$$

$$|Z_a|, |Z_b|, |Z_2| \rightarrow \infty$$

Aufgabe 5

a)



$$U_1 = R I_R$$

$$I_R = \left(I_1 + \frac{R_s}{R_i} I_R \right) \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{R_i} + \frac{1}{sL + \frac{1}{sC}}}$$

$$= \left(I_1 + \frac{R_s}{R_i} I_R \right) \frac{R_i (s^2 LC + 1)}{(R_i + R)(s^2 LC + 1) + s R R_i C}$$

$$I_R = I_1 \frac{\alpha}{\beta} + \frac{R_s}{R_i} \frac{\alpha}{\beta} I_R$$

$$\Leftrightarrow I_R \left(1 - \frac{\alpha R_s}{\beta R_i} \right) = I_1 \frac{\alpha}{\beta}$$

$$\Leftrightarrow I_R = I_1 \frac{\frac{\alpha}{\beta}}{1 - \frac{R_s \alpha}{R_i \beta}} = \frac{\alpha}{\beta - \frac{R_s}{R_i} \alpha} I_1$$

$$= \frac{s^2 R_i LC + R_i}{(R_i + R)(s^2 LC + 1) + s R R_i C - R_s (s^2 LC + 1)} I_1$$

$$U_1 = R I_R = \frac{(s^2 LC + 1) R R_i}{(R_i + R)(s^2 LC + 1) + s R R_i C - R_s (s^2 LC + 1)} I_1$$

$$\Rightarrow \underline{Z} = \frac{(s^2 LC + 1) R R_i}{(s^2 LC + 1)(R + R_i - R_s) + s R R_i C}$$

b)

Oszillator soll Spannung \underline{U} erzeugen

→ hochohmige Last an Tor 1

→ Ansteuerung mit Stromquelle

$I \rightarrow 0 \Rightarrow$ hochohmige Last

c)

$$\frac{1}{LC} + \frac{RR_i}{L(R+R_i-R_s)}s + s^2 = 0$$

$$s_{1,2} = -\frac{RR_i}{2L(R+R_i-R_s)} \pm \sqrt{\frac{R^2R_i^2}{4L^2(R+R_i-R_s)^2} - \frac{1}{LC}}$$

entdämpft, falls $\text{Re}\{s_{1,2}\} \geq 0$

$$\Rightarrow R_s > R + R_i$$

d)

$$s_{1,2} = +\frac{R^2}{4LR} \pm \sqrt{\frac{R^2}{16L^2} - \frac{1}{LC}}$$

$$= \sigma \pm j\omega$$

$$\Rightarrow \sigma = \frac{R}{4L}$$

$$\omega = -\sqrt{\frac{1}{LC} - \frac{R^2}{16L^2}}$$

$$\text{, falls } \frac{1}{LC} > \frac{R^2}{16L^2}$$

$$16L > R^2C$$

e)

$$\underline{U}_1(s) = \underline{Z}(s) \cdot \underline{I}(s)$$

$$\underline{Z}(s) = -\frac{R^2(1+s^2LC)}{LC2R(s-s_1)(s-s_2)} = -\frac{R}{2LC} \frac{1+s^2LC}{(s-s_1)(s-s_2)}$$

$$\underline{I}(s) = 1$$

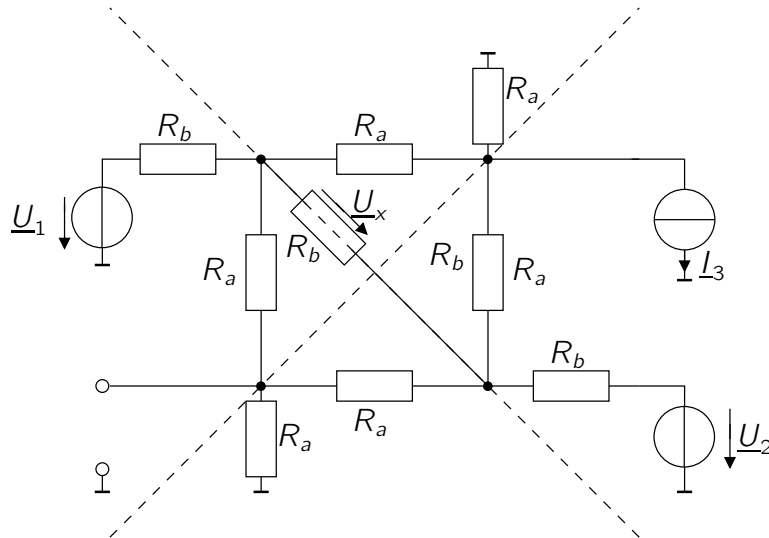
$$\underline{U}_1(s) = -\frac{R}{2LC} \frac{1+s^2LC}{(s-s_1)(s-s_2)}$$

$$u_1(t) = \sum_{i=1}^2 \frac{\underline{Z}(s)}{\underline{N}'(s)} e^{st} \Big|_{s=s_i}$$

$$= -\frac{R}{2LC} \left[\frac{1+s_1^2LC}{s_1-s_2} e^{s_1t} + \frac{1+s_2^2LC}{s_2-s_1} e^{s_2t} \right]$$

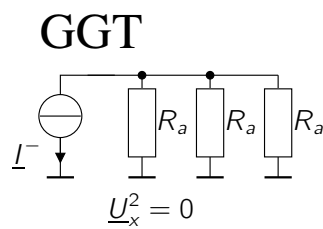
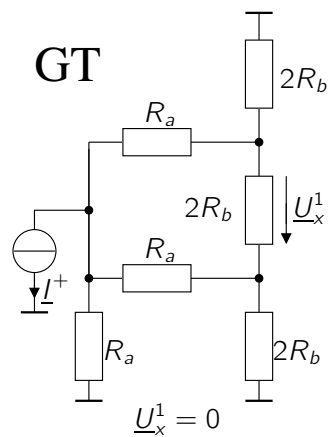
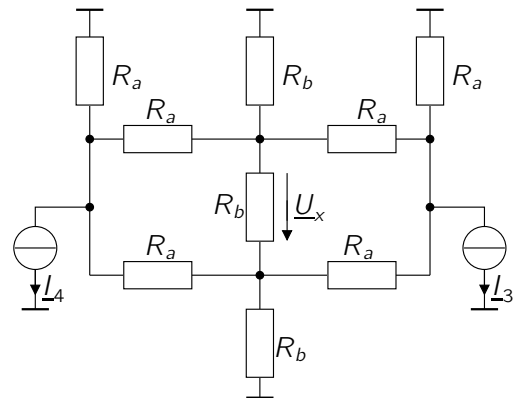
Aufgabe 6

a)



1)

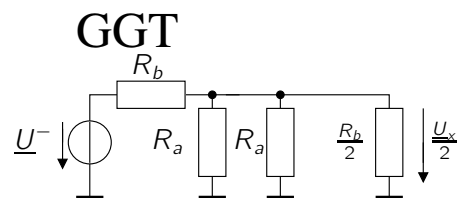
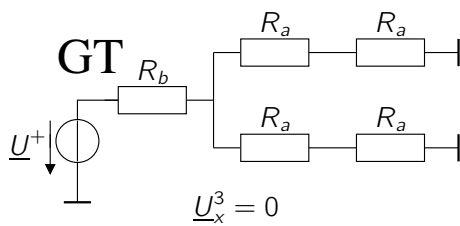
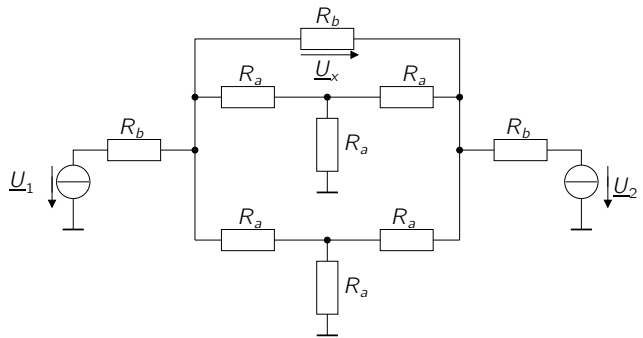
$$\begin{aligned} \underline{U}_1 = \underline{U}_2 = 0 \\ I_4 = I^+ + I^- = 0 \quad , \quad I_3 = I^+ - I^- \\ I^+ = \frac{I_3 + I_4}{2} = \frac{I_3}{2} \quad , \quad I^- = \frac{I_4 - I_3}{2} = -\frac{I_3}{2} \end{aligned}$$



2)

$$\begin{aligned} \underline{U}_1 &= \underline{U}^+ + \underline{U}^- & , & \quad \underline{U}_2 = \underline{U}^+ - \underline{U}^- \\ \underline{U}^+ &= \frac{\underline{U}_2 + \underline{U}_1}{2} & , & \quad \underline{U}^- = \frac{\underline{U}_1 - \underline{U}_2}{2} \end{aligned}$$

$$I_3 = 0$$



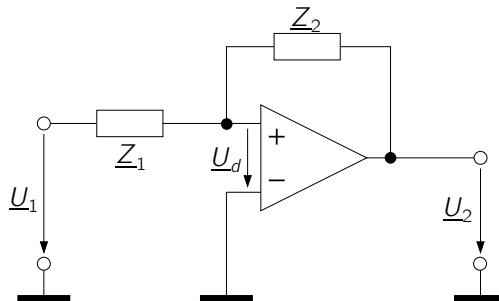
b)

$$\begin{aligned} \frac{\underline{U}_x}{2} &= \underline{U}^- \cdot \frac{R_a \parallel R_a \parallel \frac{R_b}{2}}{R_b + (R_a \parallel R_a \parallel \frac{R_b}{2})} \quad (*) = \frac{\frac{1}{2} R_a R_b}{R_b(R_a + R_b) + \frac{1}{2} R_a R_b} \underline{U}^- \\ \Rightarrow \underline{U}_x &= \frac{R_a}{3R_a + 2R_b} \cdot (\underline{U}_1 - \underline{U}_2) \end{aligned}$$

$$(*) \quad R_a \parallel R_a \parallel \frac{R_b}{2} = \frac{\frac{R_a}{2} \cdot \frac{R_a}{2}}{\frac{R_a}{2} + \frac{R_a}{2}} = \frac{1}{2} \frac{R_a R_b}{R_a + R_b}$$

Aufgabe 7

a)



$$\begin{aligned}
 U_2 &= v_u U_d \\
 U_d &= \frac{Z_2}{Z_1 + Z_2} U_1 + \frac{Z_1}{Z_1 + Z_2} U_2 \\
 \Leftrightarrow \frac{U_2}{v_u} &= \frac{Z_2}{Z_1 + Z_2} U_1 + \frac{Z_1}{Z_1 + Z_2} U_2 \\
 \Leftrightarrow U_2 \left(\frac{1}{v_u} - \frac{Z_1}{Z_1 + Z_2} \right) &= \frac{Z_2}{Z_1 + Z_2} U_1 \\
 \Leftrightarrow U_2 \frac{Z_1 + Z_2 - v_u Z_1}{v_u (Z_1 + Z_2)} &= \frac{Z_2}{Z_1 + Z_2} U_1 \\
 \Leftrightarrow F_u = \frac{U_2}{U_1} &= \frac{v_u Z_2}{Z_1 + Z_2 - v_u Z_1} = \frac{v_u \frac{Z_2}{Z_1 + Z_2}}{1 - v_u \frac{Z_1}{Z_1 + Z_2}}
 \end{aligned}$$

b)

$$Z'_1 = R_1; \quad Z'_2 = R_1 + \frac{1}{j\omega C_1}; \quad Z''_1 = \frac{1}{j\omega C_2}; \quad Z''_2 = R_2; \quad v'_u = v''_u = \alpha$$

$$\begin{aligned}
 \Rightarrow F_k &= \frac{v'_u Z'_2}{Z'_1 + Z'_2 - v'_u Z'_1} \cdot \frac{v''_u Z''_2}{Z''_1 + Z''_2 - v''_u Z''_1} \\
 &= \frac{\alpha \left(R_1 + \frac{1}{j\omega C_1} \right)}{R_1 + R_1 + \frac{1}{j\omega C_1} - \alpha R_1} \cdot \frac{\alpha R_2}{\frac{1}{j\omega C_2} + R_2 - \alpha \frac{1}{j\omega C_2}} \\
 &= \frac{1 + j\omega R_1 C_1}{\frac{1 + j\omega 2R_1 C_1}{\alpha} - j\omega R_1 C_1} \cdot \frac{j\omega R_2 C_2}{\frac{1 + j\omega R_2 C_2}{\alpha} - 1}
 \end{aligned}$$

$$\alpha \rightarrow \infty$$

$$\Rightarrow F_k = \frac{(1 + j\omega R_1 C_1) j\omega R_2 C_2}{j\omega R_1 C_1}$$

c)

 $0 \leq \omega \leq 10\omega_0 : 0\text{dB}, \quad \omega > 10\omega_0 : 20\text{dB pro Dekade}$

$$\Rightarrow R_1 C_1 = R_2 C_2 = \frac{1}{10\omega_0} \quad \Rightarrow \underline{E}_k = 1 + j \frac{\omega}{10\omega_0}$$

$$\left(\omega_\alpha = \omega_\mu = \omega_\gamma = \frac{1}{10\omega_0} \right)$$

d)

