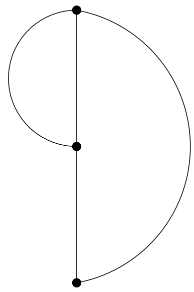
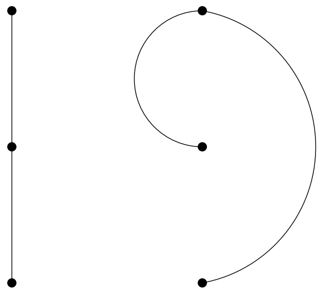
Aufgabe 1

a)

zu betrachtendes Netzwerk:

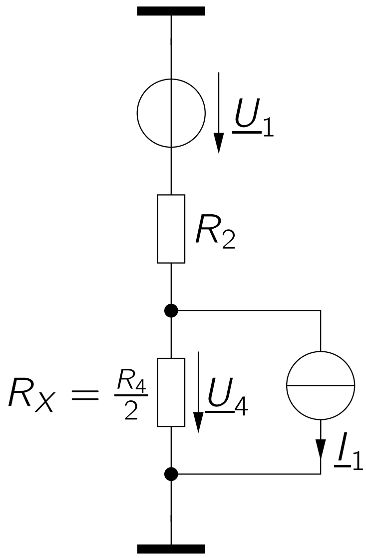


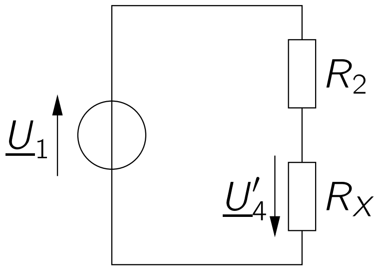
Baum und Co-Baum:



b)

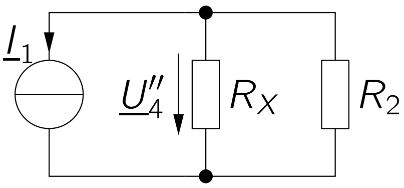
Vereinfachtes Netzwerk:





$I_1 = 0:$

$$U_4' = -\frac{R_X}{R_2 + R_X} \cdot U_1$$



$U_1 = 0:$

$$U_4'' = -\frac{R_X R_2}{R_2 + R_X} \cdot I_1$$

Berechnung der Spannung U_4 durch Superposition:

$$\Rightarrow U_4 = U_4' + U_4'' = -\frac{R_X}{R_2 + R_X}\left(U_1 + R_2 I_1\right)$$

c)

Berechung der Verlustleistung (über abgegebene Leistung der Quellen):

$$\begin{aligned}
P_\Sigma &= I_1 \cdot U_{V1} + I_{V1} \cdot U_1 = -I_1 \cdot U_4 - \left(\frac{U_4}{R_X} + I_1\right) \cdot U_1 \\
&= -U_4 \left(I_1 + \frac{U_1}{R_X}\right) - U_1 I_1 \\
&= \frac{R_X}{R_2 + R_X}(U_1 + R_2 I_1) I_1 + \frac{1}{R_2 + R_X}(U_1 + R_2 I_1) U_1 - U_1 I_1 \\
&= \left(\frac{R_X}{R_2 + R_X} + \frac{R_2}{R_2 + R_X} - 1\right) U_1 I_1 + \frac{R_2 R_X}{R_2 + R_X} I_1^2 + \frac{1}{R_2 + R_X} U_1^2 \\
&= \frac{1}{R_2 + \frac{R_4}{2}} \left(R_2 \frac{R_4}{2} I_1^2 + U_1^2\right)
\end{aligned}$$

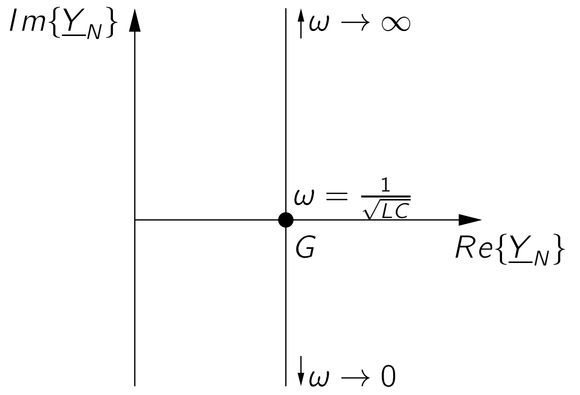
Aufgabe 2

a)

$$\frac{\underline{I}_1}{\underline{I}_0} = \frac{G}{G + j\omega C + \frac{1}{j\omega L_1}} = \frac{j\omega G L_1}{1 + j\omega G L_1 - \omega^2 L_1 C}$$

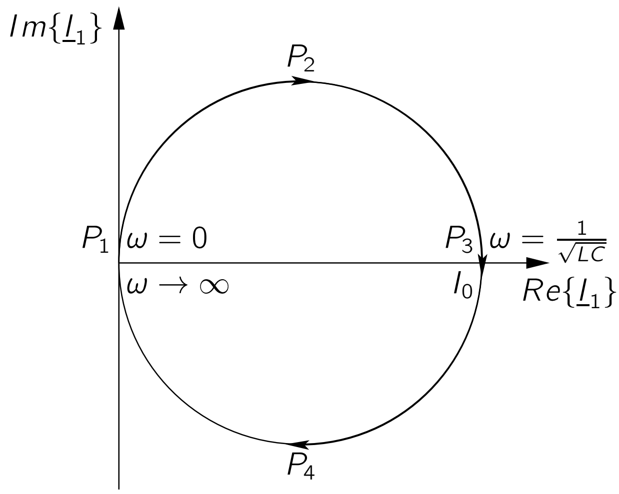
$$\frac{\underline{I}_2}{\underline{I}_0} = \frac{\frac{1}{jwL_2}}{\frac{1}{j\omega L_2} + \frac{1}{j\omega L_1}} = \frac{1}{1 + \frac{L_2}{L_1}} = \frac{L_1}{L_1 + L_2}$$

b)



$$Y_N = G + j\omega C + \frac{1}{j\omega L_1}$$

$$= G + j\omega C - j\frac{1}{\omega L_1}$$



$P_1 : I_1 = 0 \qquad \omega = 0, \omega = \infty$

$P_2 : I_1 = \frac{I_0}{2} + j\frac{I_0}{2}$

$P_3 : I_1 = I_0 \qquad \omega = \frac{1}{\sqrt{L_1 C}}$

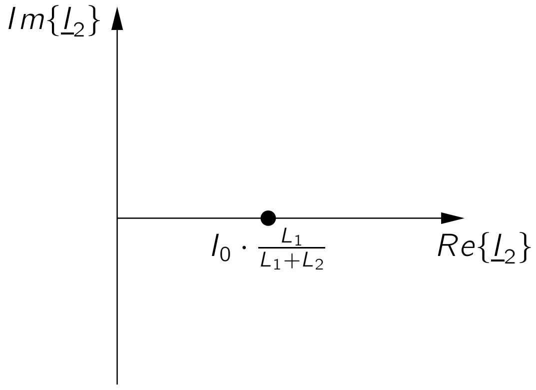
$P_4 : I_1 = \frac{I_0}{2} - j\frac{I_0}{2}$

Realteil bei P_3 maximal:

$$\omega C = \frac{1}{\omega L} \Leftrightarrow \omega^2 = \frac{1}{L_1 C}$$

$$\Leftrightarrow \omega = \sqrt{\frac{1}{L_1 C}}$$

c)

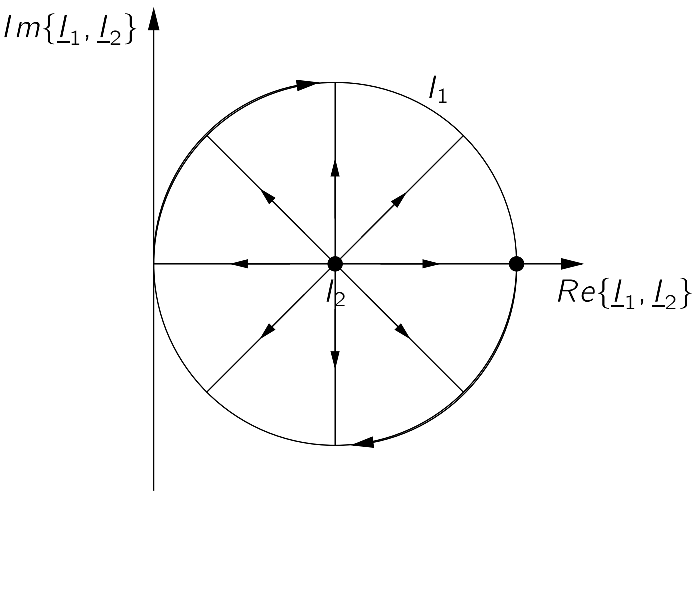


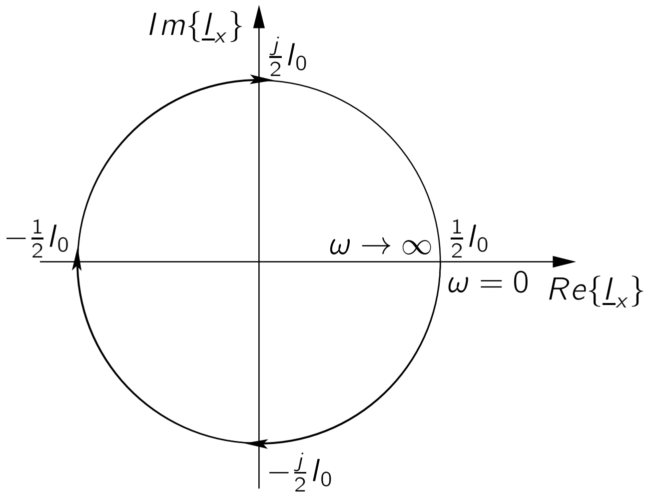
d)

$I_X = const.$ wenn Kreis aus b) den Mittelpunkt aus c) $\left(\frac{L_1}{L_1+L_2} \cdot I_0\right)$ umschliesst.

$$\Rightarrow I_0 \cdot \frac{L_1}{L_1 + L_2} = \frac{I_0}{2} \Rightarrow L_1 = L_2$$

e)



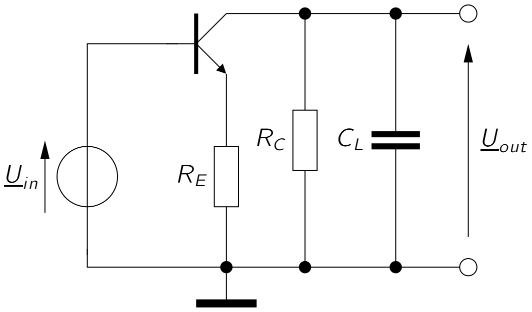

$Im\{\underline{I}_x\}$
$\frac{j}{2}I_0$
$-\frac{1}{2}I_0$
$\omega \to \infty$
$\frac{1}{2}I_0$
$\omega = 0$
$Re\{\underline{I}_x\}$
$-\frac{j}{2}I_0$

Aufgabe 3

a)

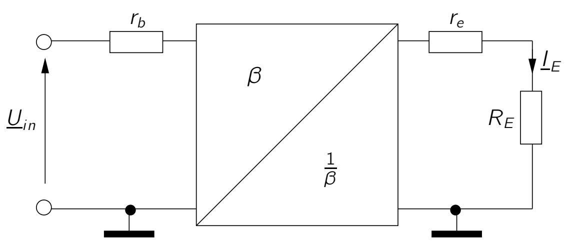
$$I_C \approx I_E = \frac{U_0 - U_{BE,0} - U_{in}}{R_E}$$
$$g_m = \frac{I_E}{U_T} = \frac{U_0 - U_{BE,0} - U_{in}}{U_T R_E}$$

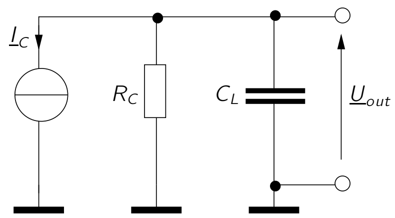
b)



$\Rightarrow$ Emittergrundschaltung

c)





$$\underline{U}_{out} = \beta \underline{I}_B \cdot \left(\frac{R_C}{1 + \frac{j\omega}{\omega_0}} \right) \qquad \text{mit } \omega_0 = \frac{1}{R_C C_L}$$

$$= -\beta \cdot \frac{\underline{U}_{in}}{\beta \left(r_e + R_E \right) + r_b} \cdot \frac{R_C}{1 + \frac{j\omega}{\omega_0}}$$
$$\frac{\underline{U}_{out}}{\underline{U}_{in}} = \underline{V}_U = -\frac{R_C}{r_e + R_E + \frac{r_b}{\beta}} \cdot \frac{1}{1 + \frac{j\omega}{\omega_0}}$$

d)

$$\left.\frac{\underline{U}_{out}}{\underline{U}_{in}}\right|_{\omega=0} = -\frac{R_C}{\underbrace{R_E + r_e + \frac{r_b}{\beta}}_{R_E \text{ lt. Text}}} \cdot \frac{1}{1+\frac{j\omega}{\omega_0}} = -\frac{R_E}{R_E + R_E} = -\frac{1}{2}$$

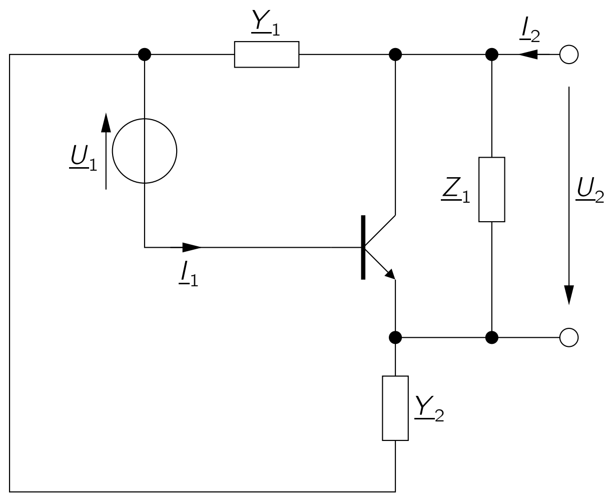
$$\left|\left.\frac{\underline{U}_{out}}{\underline{U}_{in}}\right|\right|_{\omega_{-3dB}} = \frac{1}{2} \cdot \frac{1}{1+\frac{j\omega_{-3dB}}{\omega_0}} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \left|1 + \frac{j\omega_{-3dB}}{\omega_0}\right| = \sqrt{2}$$

$$\Leftrightarrow \omega_{-3dB} = \omega_0$$

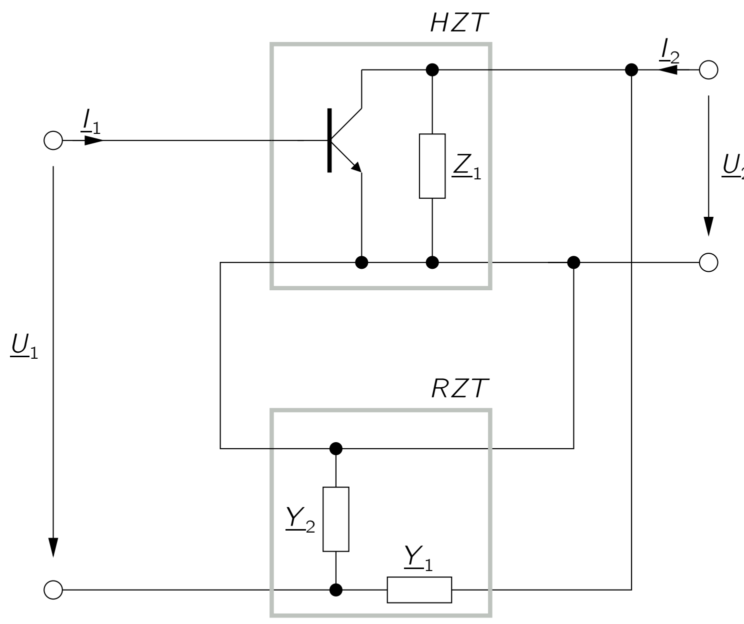
Aufgabe 4

a)



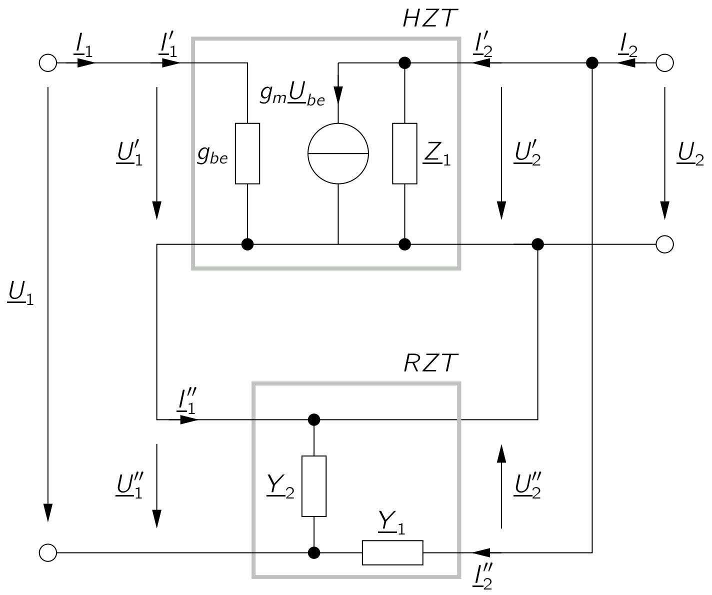
$\underline{Y}_1$
$\underline{I}_2$
$\underline{U}_1$
$\underline{Z}_1$
$\underline{U}_2$
$\underline{I}_1$
$\underline{Y}_2$

b)



HZT
$\underline{I}_2$
$\underline{I}_1$
$\underline{Z}_1$
$\underline{U}_2$
$\underline{U}_1$
RZT
$\underline{Y}_2$
$\underline{Y}_1$

c)



d)

i) Seriell-Parallel-Kopplung

ii) H-Hybrid-Matrix

$$\begin{pmatrix} \underline{U}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix}$$

$$= \begin{pmatrix} \underline{U}'_1 \\ \underline{I}'_2 \end{pmatrix} + \begin{pmatrix} \underline{U}''_1 \\ \underline{I}''_2 \end{pmatrix}$$

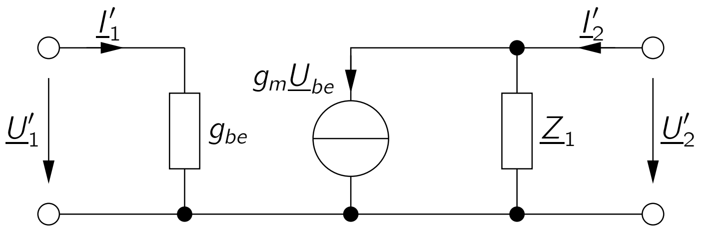
$$= \underline{H}' \begin{pmatrix} \underline{I}'_1 \\ \underline{U}'_2 \end{pmatrix} + \underline{H}'' \begin{pmatrix} \underline{I}''_1 \\ \underline{U}''_2 \end{pmatrix}$$

$$= \underline{H}' \begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix} + \underline{H}'' \begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix}$$

$$= \left[\underline{H}' + \underline{H}''\right] \begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix}$$

e)

HZT:

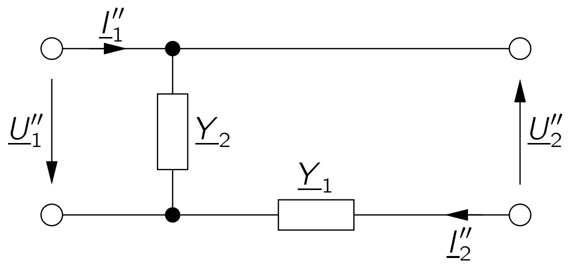


$$\underline{H}'_{11} = \left.\frac{\underline{U}'_1}{\underline{I}'_1}\right|_{\underline{U}'_2=0} = \frac{1}{g_{be}} \qquad \underline{H}'_{12} = \left.\frac{\underline{U}'_1}{\underline{U}'_2}\right|_{\underline{I}'_1=0} = 0$$

$$\underline{H}'_{21} = \left.\frac{\underline{I}'_2}{\underline{I}'_1}\right|_{\underline{U}'_2=0} = \frac{g_m}{g_{be}} \qquad \underline{H}'_{22} = \left.\frac{\underline{I}'_2}{\underline{U}'_2}\right|_{\underline{I}'_1=0} = \frac{1}{\underline{Z}_1}$$

$$\Rightarrow \underline{H}' = \begin{pmatrix} \frac{1}{g_{be}} & 0 \\ \frac{g_m}{g_{be}} & \frac{1}{\underline{Z}_1} \end{pmatrix}$$

RZT:



$$\underline{H}''_{11} = \frac{1}{\underline{Y}_1 + \underline{Y}_2} \qquad \underline{H}''_{12} = -\frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2}$$

$$\underline{H}''_{21} = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2} \qquad \underline{H}''_{22} = \frac{1}{\frac{1}{\underline{Y}_1} + \frac{1}{\underline{Y}_2}}$$

$$\Rightarrow \underline{H}'' = \begin{pmatrix} \frac{1}{\underline{Y}_1+\underline{Y}_2} & -\frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2} \\ \frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2} & \frac{\underline{Y}_1\underline{Y}_2}{\underline{Y}_1+\underline{Y}_2} \end{pmatrix}$$

Gesamtmatrix:

$$\Rightarrow \underline{H} = \underline{H}' + \underline{H}'' = \begin{pmatrix} \frac{1}{g_{be}} + \frac{1}{\underline{Y}_1+\underline{Y}_2} & -\frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2} \\ \frac{g_m}{g_{be}} + \frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2} & \frac{1}{\underline{Z}_1} + \frac{\underline{Y}_1\underline{Y}_2}{\underline{Y}_1+\underline{Y}_2} \end{pmatrix}$$

f)

$$\underline{Z}_1 = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{I}_2=0}$$

$$\overset{\underline{I}_2=0}{\Rightarrow} \quad \underline{H}_{21}\underline{I}_1 + \underline{H}_{22}\underline{U}_2 = 0$$

$$\Leftrightarrow \quad \underline{U}_2 = -\frac{\underline{H}_{21}}{\underline{H}_{22}}\underline{I}_1$$

liefert eingesetzt:

$$\Rightarrow \underline{U}_1 = \underline{H}_{11}\underline{I}_1 - \frac{\underline{H}_{21}\underline{H}_{12}}{\underline{H}_{22}}\underline{I}_1$$

$$\Leftrightarrow \frac{\underline{U}_1}{\underline{I}_1} = \underline{H}_{11} - \frac{\underline{H}_{21}\underline{H}_{12}}{\underline{H}_{22}}$$

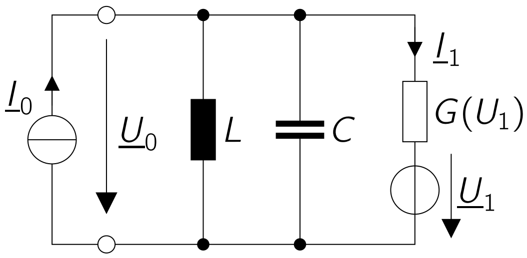
$$= \frac{1}{g_{be}} + \frac{1}{\underline{Y}_1 + \underline{Y}_2} + \frac{\left(\frac{g_m}{g_{be}} + \frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2}\right)\left(\frac{\underline{Y}_1}{\underline{Y}_1+\underline{Y}_2}\right)}{\frac{1}{\underline{Z}_1} + \frac{\underline{Y}_1\underline{Y}_2}{\underline{Y}_1+\underline{Y}_2}}$$

Aufgabe 5

a)

$$\underline{Z}_{in} = \frac{\underline{U}_0}{\underline{I}_0} = \frac{1}{\frac{1}{j\omega L} + j\omega C + G(U_1)}$$

b)



Jede WF kann herangezogen werden. $V_I = \frac{\underline{I}_1}{\underline{I}_0}$, $\underline{I}_0$ ist grösste Ursache, da NW-Topologie unverändert für $\underline{I}_0 \rightarrow 0$

c)

Polstellen von $\underline{Z}_{in}$:

$$s^2 + s\frac{G}{C} + \frac{1}{LC} = 0$$

$$\Leftrightarrow \qquad s = \pm\sqrt{\frac{1}{4}\left(\frac{G}{C}\right)^2 - \frac{1}{LC}} - \frac{G}{2C}$$

aufklingend: $Re\{s\} > 0$

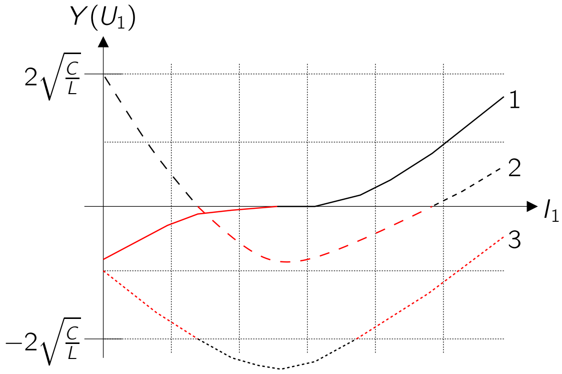
sinusförmig: $Im\{s\} \neq 0$

$$\frac{1}{4}\left(\frac{G}{C}\right)^2 - \frac{1}{LC} < 0$$

$$\Rightarrow G(U_1) < 0$$

$$\Rightarrow |G(U_1)| < 2\sqrt{\frac{C}{L}}$$

d)



e)

$$\underline{U}_0(t) = \sum_k \frac{Z(s_k)}{N'(s_k)} e^{s_k t} \qquad , s_k \text{ Polstellen}$$

$$Z(s_k) = sL \qquad N'(s_k) = 2sLC + LG$$

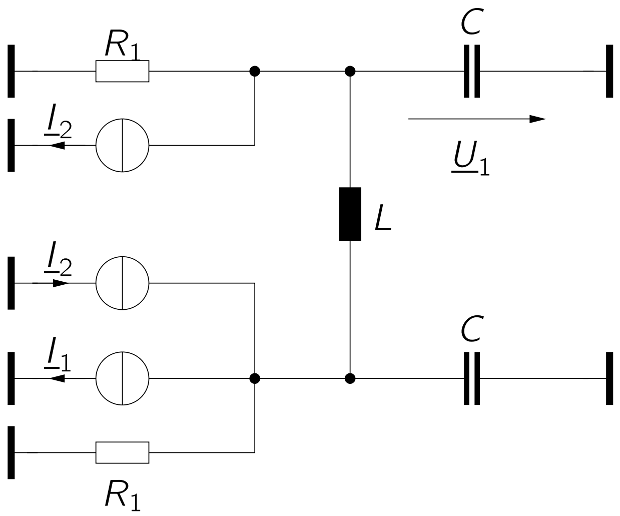
$$\Rightarrow \underline{U}_0(t) = c_1 e^{st} + c_2 e^{s^* t}$$

$$= \underbrace{e^{\sigma t}}_{\text{aufklingend}} \left(c_1 \underbrace{e^{j\omega t} + c_2 e^{-j\omega t}}_{\text{sinusförmig}}\right)$$

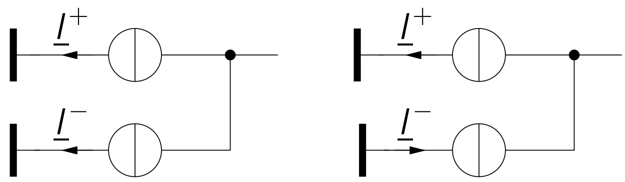
Aufgabe 6

a)

Quellen anders anordnen (äquivalente Umformung):



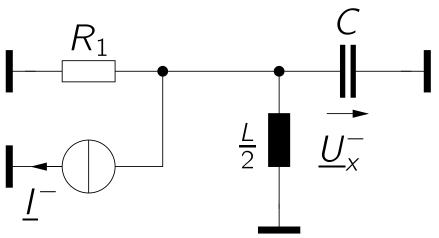
Darstellen durch Gleich- und Gegentaktquellen:



$$\underline{I}_1 - \underline{I}_2 = \underline{I}^+ - \underline{I}^-$$
$$\underline{I}_2 = \underline{I}^+ + \underline{I}^-$$
$$\Leftrightarrow \quad \underline{I}^+ = \frac{\underline{I}_1}{2}$$
$$\underline{I}^- = \underline{I}_2 - \frac{\underline{I}_1}{2}$$

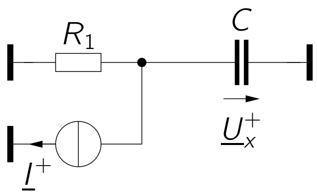
b)

Einphasiges Gegentakt-Ersatzschaltbild:



$$\underline{U}_x^- = -\underline{I}^- \left(R_1 \left|\left| \frac{L}{2} \right|\right| C \right) = -\frac{\underline{I}^-}{\frac{1}{R_1} + \frac{1}{j\omega\frac{L}{2}} + j\omega C}$$

Einphasiges Gleichtakt-Ersatzschaltbild:



$$\underline{U}_x^+ = -\frac{\underline{I}^+}{\frac{1}{R_1} + j\omega C}$$

c)

$$\underline{U}_x = \underline{U}_x^- + \underline{U}_x^+ = -\frac{\underline{I}^-}{\frac{1}{R_1} + \frac{1}{j\omega\frac{L}{2}} + j\omega C} - \frac{\underline{I}^+}{\frac{1}{R_1} + j\omega C}$$

$$= -\frac{\underline{I}_2 - \frac{\underline{I}_1}{2}}{\frac{1}{R_1} + \frac{1}{j\omega\frac{L}{2}} + j\omega C} - \frac{\underline{I}_1}{2\left(\frac{1}{R_1} + j\omega C\right)}$$

Aufgabe 7

a)

$$\underline{U}^+ = \frac{j\omega L}{R_1 + j\omega L} \cdot \underline{U}_1 = \frac{j\omega \frac{L}{R_1}}{1 + j\omega \frac{L}{R_1}} \cdot \underline{U}_1$$

$$\underline{U}^- = \frac{R_1}{R_1 + R_2} \cdot \underline{U}_2$$

$$\Rightarrow \underline{U}_2 = \underline{v}_u(j\omega)(\underline{U}^+ - \underline{U}^-) = \underline{v}_u(j\omega) \left(\frac{j\omega \frac{L}{R_1}}{1 + j\omega \frac{L}{R_1}} \cdot \underline{U}_1 - \frac{R_1}{R_1 + R_2} \cdot \underline{U}_2 \right)$$

$$\Rightarrow \underline{F}(j\omega) = \frac{\underline{U}_2}{\underline{U}_1} = \frac{\underline{v}_u(j\omega) \frac{j\omega \frac{L}{R_1}}{1+j\omega \frac{L}{R_1}}}{1 + \underline{v}_u(j\omega) \frac{R_1}{R_1+R_2}} = \frac{j\omega \frac{L}{R_1}}{1 + j\omega \frac{L}{R_1}} \cdot \frac{1}{\frac{1}{\underline{v}_u(j\omega)} + \frac{R_1}{R_1+R_2}}$$

b)

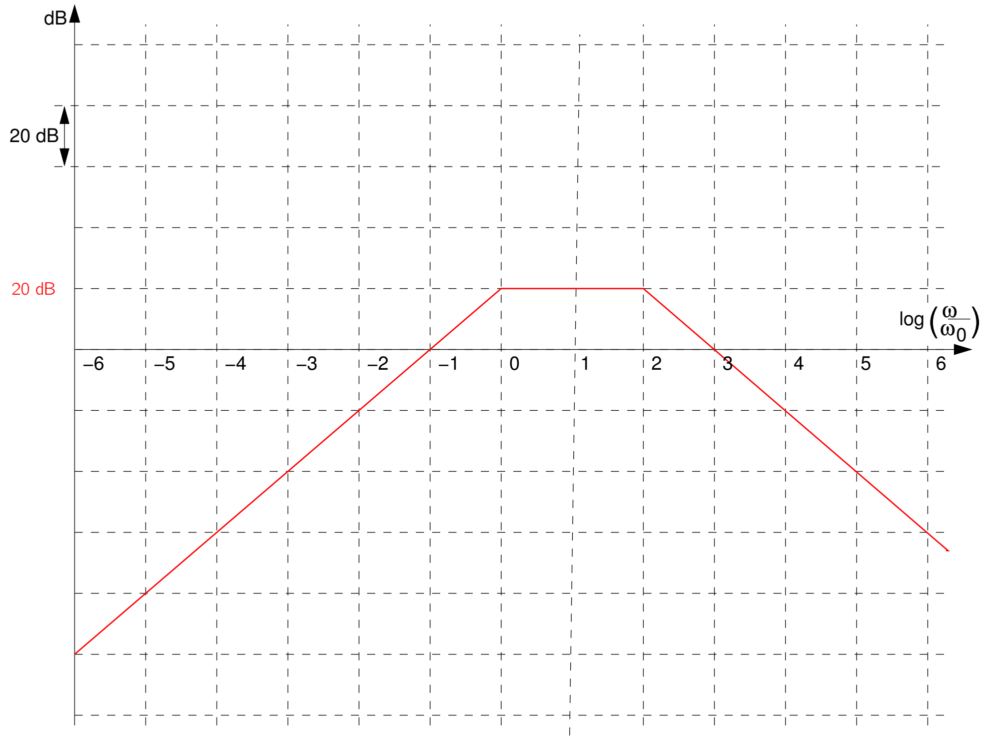
$$\lim_{\underline{v}_u \to \infty} \underline{F}(j\omega) = \frac{j\omega \frac{L}{R_1}}{1 + j\omega \frac{L}{R_1}} \cdot \frac{R_1 + R_2}{R_1}$$

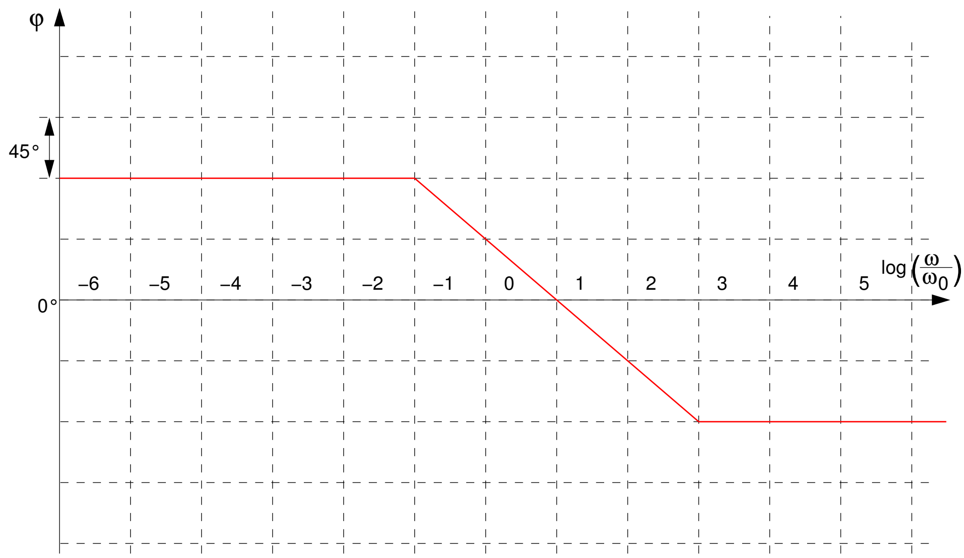
c)

$$\underline{F}(j\omega) = \frac{j\omega \frac{L}{R_1}}{1 + j\omega \frac{L}{R_1}} \cdot \frac{1}{\frac{1+\frac{j\omega}{\omega_0}}{v_0} + \frac{R_1}{R_1+R_2}} = \frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}} \cdot \frac{1000}{1 + \frac{j\omega}{\omega_0} + 99}$$

$$= \frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}} \cdot \frac{1000}{100 + \frac{j\omega}{\omega_0}} = 10 \frac{\frac{j\omega}{\omega_0}}{1 + \frac{j\omega}{\omega_0}} \frac{1}{1 + \frac{j\omega}{100\omega_0}}$$

d)





e)

Der Betragsgang schneidet die 0-dB-Achse bei $\frac{1}{10}\omega_0$ und bei $1000\omega_0$.