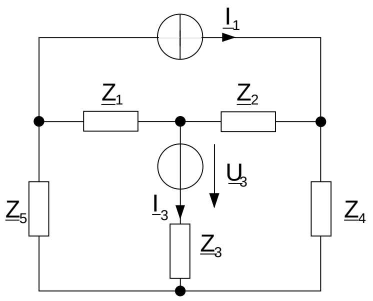
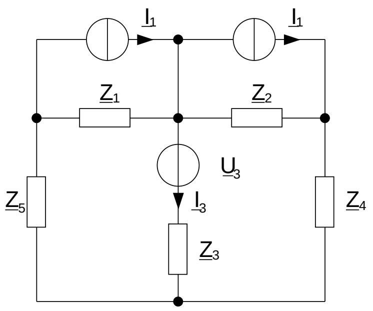
Klausur Elektronik II, WS 2006/2007

Lösungsvorschlag

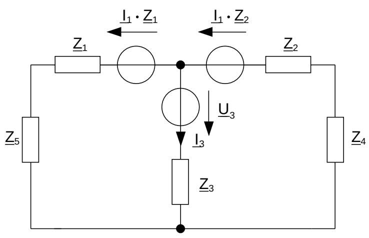
Aufgabe 1 (5 Punkte): Netzwerkberechnung



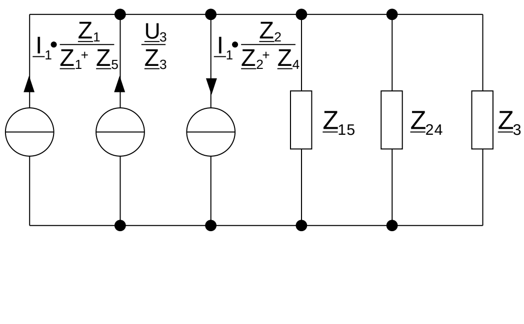
Stromquelle $\underline{I}_1$ verdoppeln



Umwandeln in Spannungsquellen



Umwandeln in Stromquellen



Zusammenfassen der Stromquellen und der Widerstände zu $\underline{I}_0$ und $\underline{Z}_0$

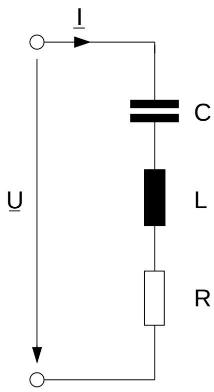
$$\underline{Z}_{15} = \underline{Z}_1 + \underline{Z}_5$$
$$\underline{Z}_{24} = \underline{Z}_2 + \underline{Z}_4$$

$$\underline{I}_0 = \underline{I}_1 \cdot \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_5} + \frac{\underline{U}_3}{\underline{Z}_3} - \underline{I}_1 \cdot \frac{\underline{Z}_2}{\underline{Z}_2 + \underline{Z}_4}$$

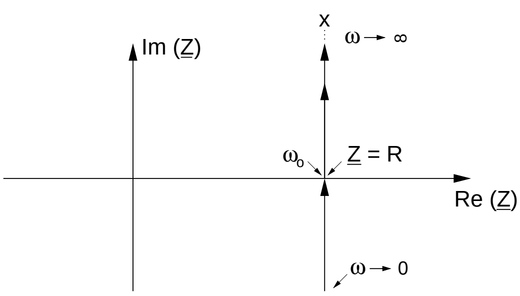
$$\underline{Z}_0 = \underline{Z}_{15} \| \underline{Z}_{24} = \frac{\underline{Z}_{15} \cdot \underline{Z}_{24}}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_4 + \underline{Z}_5}$$

Aufgabe 2 (5 Punkte): Ortskurve

$\underline{Z} = \frac{1}{\underline{Y}} = \frac{\underline{U}}{\underline{I}} \qquad \Rightarrow$



1) Eingangsimpedanz: $\underline{Z} = R + j(\omega L - \frac{1}{\omega C})$

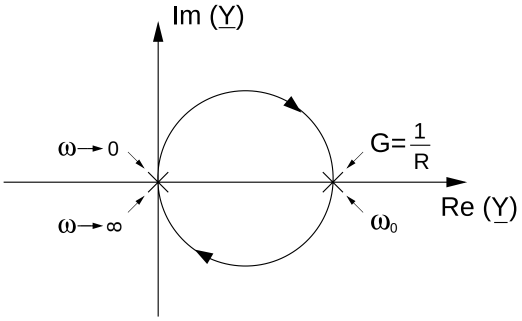


Schnittpunkt mit reeller Achse:

$\mathrm{Im}\ (\ \underline{Y}\ ) \stackrel{!}{=} 0$

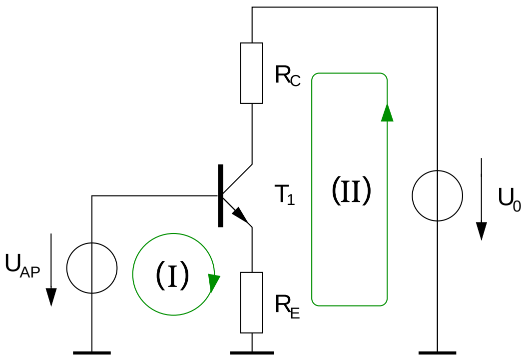
$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

2) Admittanz: $\underline{Y} = \frac{1}{\underline{Z}} = \frac{\underline{I}}{\underline{U}} = \frac{1}{R+j\cdot(\omega L-\frac{1}{\omega C})}$



Inversion der Ortskurve $\underline{Z}$

Aufgabe 3 (5 Punkte): Arbeitspunkt, Wärmewiderstand



$U_0 = 11\,\text{V}$

$U_{AP} = 2\,\text{V}$

$U_{BE} = 0,9\,\text{V}$

$\beta \rightarrow \infty$

1) ges: $\frac{R_C}{R_E}$ =?

$$P_V = \underbrace{I_B \cdot U_{BE}}_{\approx 0} + I_C \cdot U_{CE} \approx I_C \cdot U_{CE} := 0$$

$$\Rightarrow U_{CE} \overset{!}{=} 0$$

Maschenregel:

I: $-U_{AP} + U_{BE} + R_E \cdot I_E = 0$

$\Rightarrow R_E \cdot I_E = U_{AP} - U_{BE} = 2\ \text{V} - 0,9\ \text{V} = 1,1\,\text{V}$

II: $-U_0 + I_C \cdot R_C + U_{CE} + R_E \cdot I_E = 0$

$\Rightarrow I_C \cdot R_C = U_0 - U_{CE} - R_E \cdot I_E = 11\,\text{V} - 0\,\text{V} - 1,1\,\text{V} = 9,9\,\text{V}$

Knotenregel:

III: $I_B + I_C = I_E \Rightarrow I_E \approx I_C$

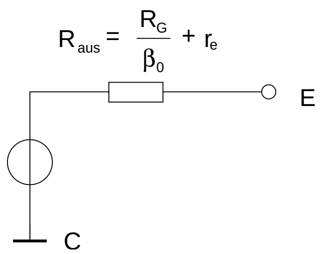
$$\Rightarrow \frac{I_C \cdot R_C}{I_E \cdot R_E} \approx \frac{I_C \cdot R_C}{I_C \cdot R_E} = \frac{R_C}{R_E} = \frac{9{,}9\,\mathrm{V}}{1{,}1\,\mathrm{V}} = 9$$

2) $\frac{R_C}{R_E} = 10$, $U_{AP} = 0\,\mathrm{V} \Rightarrow I_B = 0\,\mathrm{A}$

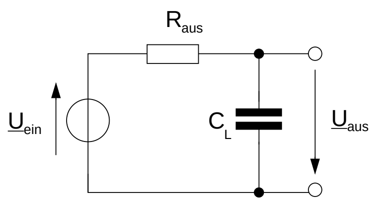
$\Rightarrow$ Transistor sperrt.
$\Rightarrow P_V = 0$ Watt

Aufgabe 4 (12 Punkte): Schaltungsberechnung, Dimensionierung, Arbeitspunkt

1) ges: Frequenzgang $\underline{F}(j\omega) = \frac{\underline{U}_{aus}(j\omega)}{\underline{U}_{ein}(j\omega)}$



WS - ESB:



$$\underline{U}_{ein} = \underline{I} \cdot \left(R_{aus} + \frac{1}{j\omega C_L} \right)$$

$$\underline{U}_{aus} = \underline{I} \cdot \frac{1}{j\omega C_L}$$

$$\underline{F}(j\omega) = \frac{\underline{U}_{aus}(j\omega)}{\underline{U}_{ein}(j\omega)} = \frac{\frac{1}{j\omega C_L}}{R_{aus} + \frac{1}{j\omega C_L}} = \frac{1}{1 + R_{aus} \cdot j\omega C_L}$$

$$= \frac{1}{1 + (\frac{R_G}{\beta} + r_e) \cdot j\omega C_L}$$

2) $\underline{F}(\omega = 0) = 1$

$|\underline{F}(\omega_0)| \stackrel{!}{=} \frac{1}{\sqrt{2}}, \qquad \omega_0 \cdot C_L \cdot R_{aus} = 1 \qquad \Leftrightarrow \qquad \omega_0 = \frac{1}{C_L \cdot R_{aus}}$

<u>Nun:</u>

$\frac{|\underline{F}(\omega_0)|}{|\underline{F}(\omega=0)|} = \frac{1}{\sqrt{2}} \cdot \frac{1}{1} = \frac{1}{\sqrt{2}} \quad \wedge \quad 20 \cdot \lg \frac{|\underline{F}(\omega_0)|}{|\underline{F}(\omega=0)|} = -3 \text{ dB}$

3) $\omega_0 = \frac{1}{C_L \cdot R_{aus}} = \frac{1}{C_L \cdot \left(\frac{R_G}{\beta_0} + r_e \right)}$

$$\begin{array}{llll} C_L & \downarrow & \Rightarrow \omega_0 & \uparrow \\ R_G & \downarrow & \Rightarrow \omega_0 & \uparrow \\ \beta_0 & \uparrow & \Rightarrow \omega_0 & \uparrow \\ I_0 & \uparrow & \Rightarrow \omega_0 & \uparrow \\ r_e & \downarrow & \Rightarrow \omega_0 & \uparrow \end{array}$$

$\big(\text{mit } g_m = \frac{I_C}{U_T} = \frac{I_0}{U_T} = \frac{1}{r_e}, \text{ da } I_0 = I_E \approx I_C\big).$

$\Rightarrow \quad \omega_0 = \frac{1}{C_L \cdot R_{aus}} = \frac{1}{C_L \cdot \left(\frac{R_G}{\beta_0} + \frac{U_T}{I_0} \right)}$

4) $\quad I_0 \gg \frac{U_T \cdot \beta_0}{R_G} \quad \vee \quad \frac{U_T}{I_0} \ll \frac{R_G}{\beta_0}$

5) <u>Amplitude der Wechselspannungsquelle</u>

$I_T = I_{CE} - I_{EC} = I_S \cdot \big(\underbrace{e^{\frac{U_{BE,0}}{U_T}}}_{\hookleftarrow} - \underbrace{e^{\frac{U_{BC}}{U_T}}}_{\hookrightarrow} \big)$ (siehe Skript: Elektronik I, WS 08/09)

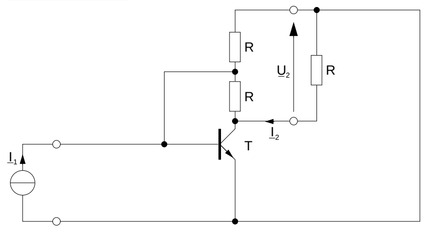
Bestimmt durch den Arbeitspunktstrom I_0.	Bestimmt durch die Wechselspannungsquelle $\lvert U_{ein} \rvert$.

Im normal-aktiven Bereich ist der Term $e^{\frac{U_{BC}}{U_T}}$ viel kleiner als der Term $e^{\frac{U_{BE,0}}{U_T}}$, d.h. der Transferstrom wird hauptsächlich durch den Arbeitspunktstrom I_0 bestimmt. Somit muss $|U_{ein}| = U_{BC} \ll U_{BE,0}$ sein.

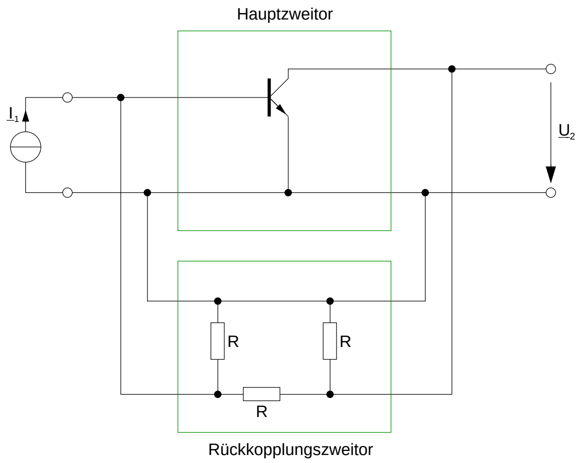
Aufgabe 5 (15 Punkte): Rückkopplung, Zweitor

$C_\infty \to \infty, \qquad \underline{Z}_C = 0 \qquad \Rightarrow \qquad$ Kurzschluss (KS)

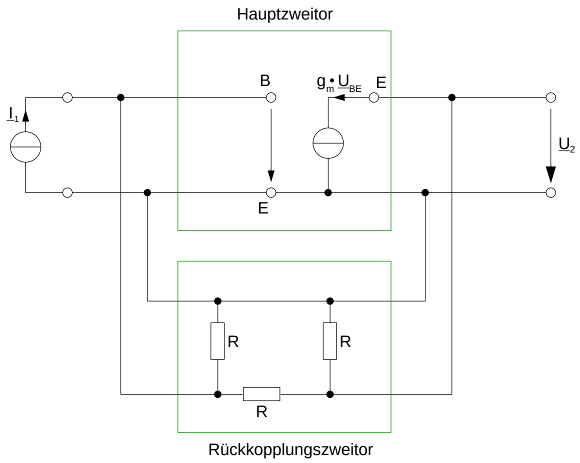
1) WS-ESB:



Emittergrundschaltung



Kleinsignal-ESB:



2) PPK-Rückkopplung: $[\underline{Y}] = [\underline{Y}^{(1)}] + [\underline{Y}^{(2)}]$

<u>Torbedingung erfüllt:</u>

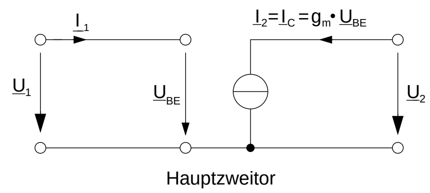
hineinfließende Ströme = herausfließende Ströme.

3) $\underline{I}_1 = \underline{Y}_{11} \cdot \underline{U}_1 + \underline{Y}_{12} \cdot \underline{U}_2$
$\underline{I}_2 = \underline{Y}_{21} \cdot \underline{U}_1 + \underline{Y}_{22} \cdot \underline{U}_2$

Matritzenschreibweise:

$$[\underline{Y}] = \underbrace{[\underline{Y}^{(1)}]}_{Hauptzweitor} + \underbrace{[\underline{Y}^{(2)}]}_{Rückkopplungszweitor} \quad \text{Admittanzmatrix}$$

4) <u>Hauptzweitor:</u>



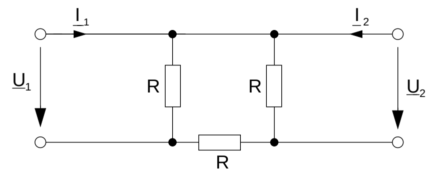
$$\underline{Y}_{11}^{(1)} = \left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{\underline{U}_2=0} = 0$$

$$\underline{Y}_{12}^{(1)} = \left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{\underline{U}_1=0} = 0$$

$$\underline{Y}_{21}^{(1)} = \left.\frac{\underline{I}_2}{\underline{U}_1}\right|_{\underline{U}_2=0} = g_m$$

$$\underline{Y}_{22}^{(1)} = \left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{U}_1=0} = 0 \qquad \Rightarrow \left[\underline{Y}^{(1)}\right] = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$$

Rückkopplungszweitor:



Rückkopplungszweitor

$$\underline{Y}_{11}^{(2)} = \left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{\underline{U}_2=0} = \frac{2}{R}$$

$$\underline{Y}_{12}^{(2)} = \left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{\underline{U}_1=0} = -\frac{1}{R}$$

$$\underline{Y}_{21}^{(2)} = \left.\frac{\underline{I}_2}{\underline{U}_1}\right|_{\underline{U}_2=0} = -\frac{1}{R}$$

$$\underline{Y}_{22}^{(2)} = \left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{U}_1=0} = \frac{2}{R} \qquad \Rightarrow \left[\underline{Y}^{(2)}\right] = \begin{bmatrix} \frac{2}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{2}{R} \end{bmatrix}$$

$$\Rightarrow \left[\underline{Y}\right] = \left[\underline{Y}^{(1)}\right] + \left[\underline{Y}^{(2)}\right] = \begin{bmatrix} \frac{2}{R} & -\frac{1}{R} \\ g_m - \frac{1}{R} & \frac{2}{R} \end{bmatrix}$$

5) $\underline{I}_1 = \underline{Y}_{11} \cdot \underline{U}_1 + \underline{Y}_{12} \cdot \underline{U}_2 = \frac{2}{R} \cdot \underline{U}_1 - \frac{1}{R} \cdot \underline{U}_2$

$\underline{I}_2 = \underline{Y}_{21} \cdot \underline{U}_1 + \underline{Y}_{22} \cdot \underline{U}_2 = (g_m - \frac{1}{R}) \cdot \underline{U}_1 + \frac{2}{R} \cdot \underline{U}_2$

$\underline{I}_2 \stackrel{!}{=} 0:$

$\Rightarrow$ Lösung LGS:

$$\underline{I}_1 = \left(\frac{\frac{4}{R^2}}{\frac{1}{R} - g_m} - \frac{1}{R} \right) \cdot \underline{U}_2$$

$$\Rightarrow \underline{Z}_T = \left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{\underline{I}_2=0} = \frac{1}{\left(\frac{\frac{4}{R^2}}{\frac{1}{R} - g_m} - \frac{1}{R} \right)} = \frac{\frac{1}{R} - g_m}{\frac{3}{R^2} + \frac{g_m}{R}} = \frac{R - g_m R^2}{3 + g_m R}$$

6) $\underline{Z}_T \stackrel{R, g_m \gg 1}{=} -R$

Aufgabe 6 (12 Punkte): Stabilität, Ortskurve

1) $H_0 = 1 \quad \wedge \quad H_R = 1$

$$\begin{aligned} \underline{H}_0(j \cdot 0) = 1 \underline{H}_0(j \cdot \omega_0) &= \frac{1}{1+j} = \frac{1}{2} - \frac{1}{2}j \\ \underline{H}_0(j \cdot 2\omega_0) = \frac{1}{1+2j} &= \frac{1}{5} - \frac{2}{5}j \\ \underline{H}_0(j \cdot \infty\omega_0) = \frac{1}{1+j\infty} &= 0 - j \cdot 0 \end{aligned}$$

2) Aus dem schwarzen Spiegelungsprinzip: $\underline{H}^\star(j\omega) = \underline{H}(-j\omega)$ folgt:

$$|\underline{H}(j\omega)| = |\underline{H}(-j\omega)| \quad \wedge \quad \arg(\underline{H}(j\omega)) = -\arg(\underline{H}(-j\omega))$$

3) $\underline{H}(s^*) = \frac{1}{1+\frac{-s}{\omega_0}} = \underline{H}(s)^* \quad$ mit $s = j\omega$

4) $\underline{H}(s) = \frac{H_0 \cdot (1+\frac{-s}{\omega_0}) + H_R}{(1+\frac{-s}{\omega_0})} = \frac{H_0 + H_R \frac{H_0 \cdot s}{\omega_0}}{1+\frac{s}{\omega_0}} \stackrel{!}{=} 0$

$\Rightarrow \quad s = -\frac{(H_0+H_R)\cdot\omega_0}{H_0}$

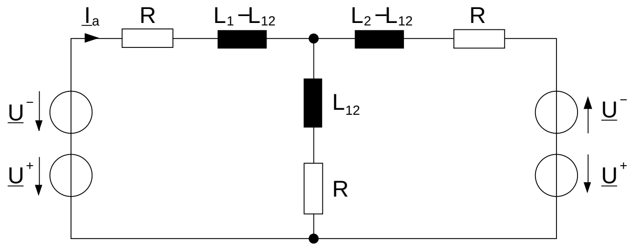
$$\begin{aligned} \underline{H}(0) &= H_0 + H_R \\ \underline{H}(j\omega_0) &= H_0 + \frac{H_R}{2} - j \cdot \frac{H_R}{2} \\ \underline{H}(j \cdot \infty) &\approx H_0 \end{aligned}$$

$$\Rightarrow \quad H_0 = H_0 + H_R$$

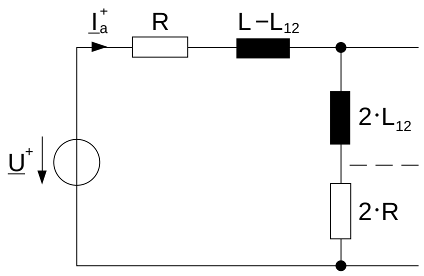
Pole von $\underline{H}(j\omega)$:
$s = -\omega_0$

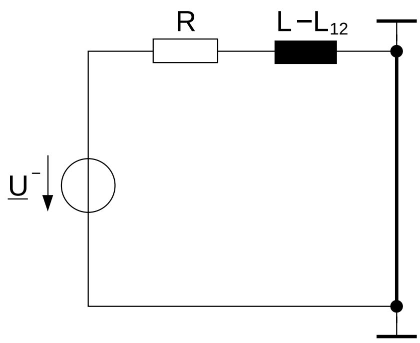
$\Rightarrow$ stabil, da Pol in LHE liegt (Nyquist-Kriterium).

Aufgabe 7 (8 Punkte): Gleichtakt-, Gegentaktzerlegung

1) 

$\underline{U}^+ = \frac{\underline{U}_a + \underline{U}_b}{2}$; $\quad \underline{U}^- = \frac{\underline{U}_a - \underline{U}_b}{2}$

2) 



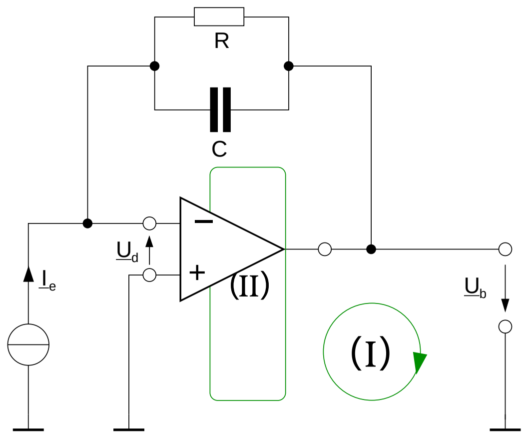
3) $\underline{U}_a = \underline{U}_b \quad \Rightarrow \quad \underline{U}^- = 0; \quad \underline{U}^+ = \underline{U}_a$

Gleichtakt: $\underline{I}^+ = \frac{\underline{U}_a}{R + 2R + j\omega L - j\omega L_{12} + 2j\omega L12} = \frac{\underline{U}_a}{3R + j\omega L + j\omega L_{12}}$
Gegentakt: $\underline{I}^- = 0$

$\underline{I} = \underline{I}^+ + \underline{I}^- = \frac{\underline{U}_a}{3R + j\omega L + j\omega L_{12}}$

Aufgabe 8 (12 Punkte): Operationsverstärker, Bode-Diagramm, Frequenzkompensation

1)



I: $\underline{U}_b(j\omega) = \underline{v}_u(j\omega) \cdot \underline{U}_d$

$\Rightarrow \quad \underline{U}_d = \frac{\underline{U}_b}{\underline{v}_u}$

II: $\underline{U}_d + \underbrace{\underline{I}_e \cdot \left(R \| \frac{1}{j\omega C}\right)}_{\underline{U}_{BC}} + \underline{U}_b = 0$

$\Rightarrow \quad \underline{U}_b \cdot (1 + \frac{1}{\underline{v}_u}) = \underline{I}_e \cdot \frac{\frac{-R}{j\omega C}}{R + \frac{1}{j\omega C}}$

$$\Leftrightarrow \quad \frac{\underline{U}_b}{\underline{I}_e} = -\frac{\frac{R}{j\omega C}}{\frac{1+j\omega RC}{j\omega C} \cdot \frac{1+\underline{v}_u}{\underline{v}_u}} = -\frac{\frac{R \cdot \underline{v}_u}{1+j\omega RC}}{1+\underline{v}_u}$$

2) $\boxed{\underline{F}_a(j\omega) = -\frac{R \cdot \underline{v}_u}{1+j\omega RC}}$

$\boxed{\underline{F}_2(j\omega) = -\frac{1+j\omega RC}{R}}$

Für die Schleifenverstärkung $\underline{F}_0$ gilt:

$$\underline{F}_0(j\omega) = \underline{F}_a(j\omega) \cdot \underline{F}_2(j\omega) = \underline{v}_u(j\omega)$$

3) $|\underline{v}_u| \rightarrow \infty$: $\quad \underline{v}_u \gg 1$

$$\Rightarrow \quad \underline{F}_(j\omega) = -\frac{R}{1+j\omega RC} = \frac{1}{\underline{F}_2(j\omega)}$$

4) Es gilt nun: $\frac{1}{RC} = \omega_{RC} = \frac{\omega_0}{10} \quad \wedge \quad v_0 = 10^4 \mathrel{\hat{=}} 80$ dB

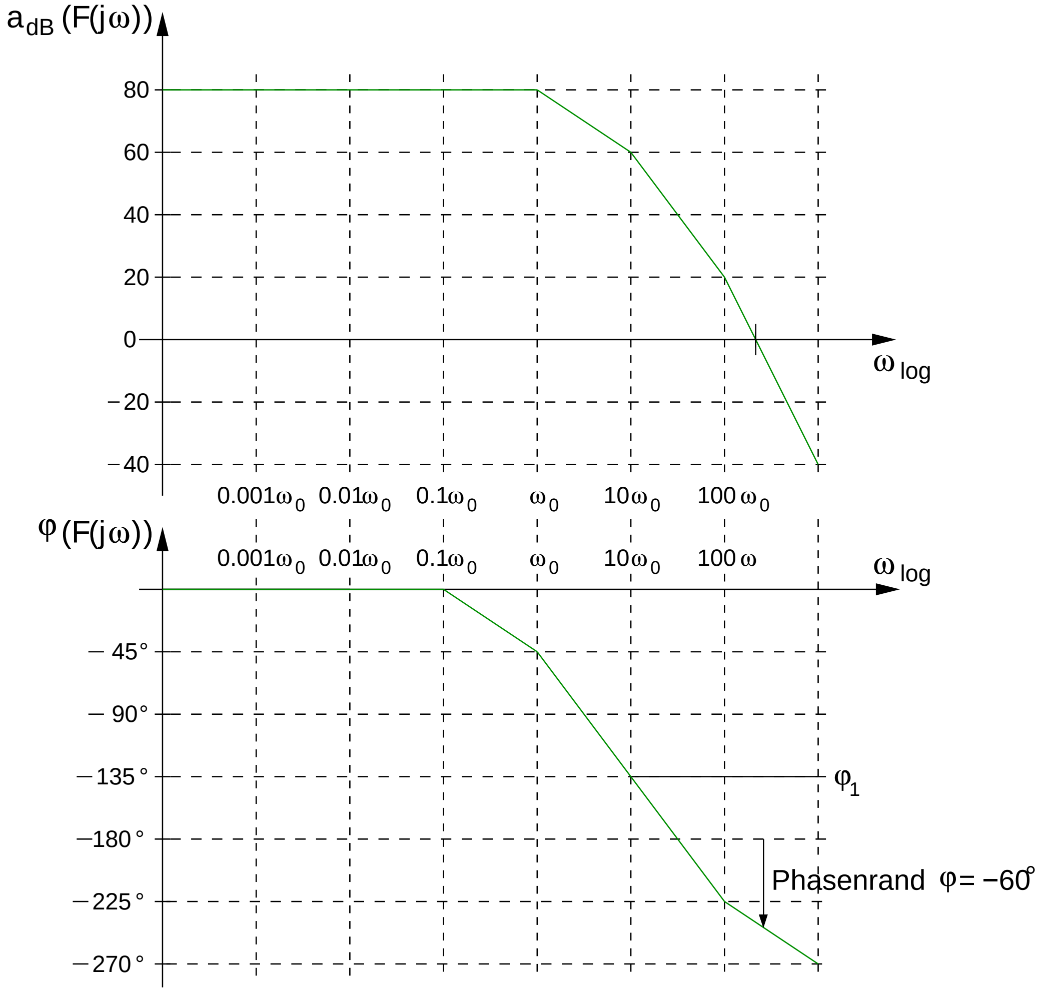
$$a_{dB}\left(\underline{F}_a(j\omega)\underline{F}_2(j\omega)\right) = 20 \cdot \log(v_0) - 20 \cdot \log\Big((1 + (\frac{\omega}{\omega_0})^2)^{\frac{1}{2}}\Big)$$
$$- 20 \cdot \log\Big((1 + (\frac{\omega}{10\omega_0})^2)^{\frac{1}{2}}\Big) - 20 \cdot \log\Big((1 + (\frac{\omega}{100\omega_0})^2)^{\frac{1}{2}}\Big)$$

$$\begin{aligned}
\omega = 0,01 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 0 \approx 80\text{dB} \\
\omega = 0,1 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 0,09\text{dB} \approx 80\text{dB} \\
\omega = \omega_0 : \quad a_{dB} &= 80\text{dB} - 3\text{dB} - 0,09\text{dB} \approx 80\text{dB} \\
\omega = 10 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 20\text{dB} \approx 60\text{dB} \\
\omega = 100 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 40\text{dB} - 20\text{dB} \approx 20\text{dB}
\end{aligned}$$

Für die Phasenreserve φ_r gilt: Abfall von 20 dB entsprechen einer Phasendrehung um $-45°$.

$$\begin{aligned}
\omega = 0,01 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 0 \approx 80\text{dB} \\
\omega = 0,1 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 0,09\text{dB} \approx 80\text{dB} \\
\omega = \omega_0 : \quad a_{dB} &= 80\text{dB} - 3\text{dB} - 0,09\text{dB} \approx 80\text{dB} \\
\omega = 10 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 20\text{dB} \approx 60\text{dB} \\
\omega = 100 \cdot \omega_0 : \quad a_{dB} &= 80\text{dB} - 40\text{dB} - 20\text{dB} \approx 20\text{dB}
\end{aligned}$$

Bode-Diagramm:



5) Phasenrand: $-45\ ^\circ$, d.h. Schaltung ist stabil.

6) $\varphi_1 = 45\ ^\circ$

$\Rightarrow \quad v_0 = 10^1 = 10$, da $a_{dB}(v_u(j \cdot 10\omega_0)) \overset{!}{=} 0$ (siehe Bode-Diagramm).