Aufgabe A)

A.1)$ges:\ \beta\ |_{U_{CE}=0} = \frac{\underline{I}_C}{\underline{I}_B}$

$$\underline{I}_b = \frac{g_{be}}{g_{be}+j\omega C_{be}}\underline{I}_B$$
$$\underline{I}_C = \beta_0 \frac{g_{be}}{g_{be}+j\omega C_{be}}\underline{I}_B$$
$$\Rightarrow \frac{\underline{I}_C}{\underline{I}_B} = \frac{\beta_0 g_{be}}{g_{be}+j\omega C_{be}} = \frac{\beta_0}{1+j\omega\frac{C_{be}}{g_{be}}} = \underline{\beta} \qquad \frac{C_{be}}{g_{be}} = C_{be} r_e \beta_0 = \frac{1}{\omega_\beta}$$

$$|\underline{\beta}| = \frac{|\beta_0|}{\left|1+\frac{j\omega}{\omega_\beta}\right|}$$

A.2) ges: 3dB Grenzfrequenz
3dB entspricht Faktor $\frac{1}{\sqrt{2}}$

$$\left|1+\frac{j\omega}{\omega_\beta}\right| = \sqrt{2}$$
$$1+\frac{\omega^2}{\omega_\beta^2} = 2$$
$$\left|\frac{\omega}{\omega_\beta}\right| = 1 \ \Rightarrow\ \omega = \omega_\beta = \frac{g_{be}}{C_{be}}$$

A.3) Näherung für $\underline{\beta}$

$$\frac{1}{f} = T \ll \frac{C_{be}\beta_0}{g_m} = C_{be} r_e \beta_0 = \frac{1}{\omega_\beta}$$
$$\Rightarrow\ f \gg \omega_\beta \quad \text{Daraus folgt in noch besserer Näherung}$$
$$\omega = 2\pi f \gg \omega_\beta$$

$$\Rightarrow \underline{\beta} = \frac{\beta_0 \omega_\beta}{j\omega}$$

B.1) ges: komplexe Spannungsverstärkung $\underline{V}_U = \frac{\underline{U}_2}{\underline{U}_1}$
Ansatz: Zerlegung der Kollektor-Basis-Kapazität mittels Miller-Theorem in $C'_{cb} \| C_{be}$ und $C''_{cb} \| C_L$

$$\underline{V}_U = \frac{\underline{U}_2}{\underline{U}_1}$$
$$C'_{cb} = ?\ ;\ C''_{cb} = ?$$
$$\underline{U}_{be} = \underline{U}_2 - \frac{\underline{I}_{cb}}{j\omega C_{cb}}$$
$$\frac{1}{\underline{V}'_u} = \frac{\underline{U}_{be}}{\underline{U}_2} = 1 - \frac{\underline{I}_{cb}}{\underline{U}_2}\frac{1}{j\omega C_{cb}}$$
$$\left(1 - \frac{1}{\underline{V}'_u}\right) j\omega C_{cb} = \frac{\underline{I}_{cb}}{\underline{U}_2}$$

$$\left(1 - \frac{1}{\underline{V}'_u}\right) C_{cb} = C''_{cb}$$
$$\left(1 - \underline{V}'_u\right) C_{cb} = C'_{cb}$$

$$\frac{\underline{U}_{be}}{\underline{U}_1} = \frac{Z_b}{r_b + Z_b} \Rightarrow \underline{U}_{be} = \frac{Z_b}{r_b + Z_b}\underline{U}_1 = \frac{1}{r_b Y_b + 1}\underline{U}_1$$

mit $Y_b = g_{be} + j\omega(C_{be} + C'_{cb})$

$$\underline{U}_{be} = \frac{1}{1 + r_b g_{be} + j\omega r_b (C_{be} + C'_{cb})}\underline{U}_1 \quad \text{Näherung: } r_b g_{be} \ll 1$$
$$= \frac{1}{1 + j\omega r_b (C_{be} - \underline{V}'_u C_{cb})}\underline{U}_1$$

$$\underline{U}_2 = -g_m U_{be} \frac{1}{g_0 + G_L + j\omega C'}$$

$$\Rightarrow \frac{\underline{U}_2}{\underline{U}_1} = \frac{-g_m}{g_0 + G_L + j\omega C'}\ \frac{1}{1 + j\omega r_b (C_{be} - \underline{V}'_u C_{cb})} = \underline{V}_u$$

B.2) ges: $\underline{Z}_{ein}$ und $\underline{Z}_{aus}$

$$\begin{aligned}
\underline{Z}_{ein} &= \frac{\underline{U}_1}{\underline{I}_1}|_{I_2=0} \\
&= r_b + \left(\frac{1}{g_{be}} \| \frac{1}{j\omega(C_{be} + C'_{cb})} \right) \\
&= r_b + \frac{\frac{1}{g_{be}} \frac{1}{j\omega(C_{be} + C'_{cb}}}{\frac{1}{g_{be}} + \frac{1}{j\omega(C_{be} + C'_{cb}}} \\
&= r_b + \frac{1}{g_{be} + j\omega(C_{be} + C'_{cb})}
\end{aligned}$$

$$\begin{aligned}
\underline{Z}_{aus} &= \frac{\underline{U}_2}{\underline{I}_2}|_{I_1=0} \\
&= g_0 \| C_L \| R_L = \frac{1}{g_0} \| \frac{R_L \frac{1}{j\omega C_L}}{R_L + \frac{1}{j\omega C_L}} \\
&= \frac{\frac{1}{g_0} \frac{R_L}{R_L j\omega C_L + 1}}{\frac{1}{g_0} + R_L j\omega C_L + 1} \\
&= \frac{R_L}{(R_L j\omega C_L + 1)(1 + g_0 R_L j\omega C_L + g_0}
\end{aligned}$$