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Quantum Optics and Cold Atoms

SoSe 2014 **Sheet 1** **16 April**

(Due on 28. April)

Question 1 *An elastically bound electron*

a) Newton's equation of motion for a charged particle of mass m and charge e which is elastically bound to the origin is given by

$$m\ddot{r} + fr = 0 \tag{1}$$

where f is the spring constant and the oscillator eigenfrequency is $\omega_0^2 = \frac{f}{m}$.

Write the energy W of the system and the general solution.

(1 Point)

b) The energy which is dissipated by the electron is given by

$$S = -\frac{dW}{dt} = \frac{2e^2}{3c^3}\overline{\ddot{r}^2} , \tag{2}$$

where c is the speed of light, $\ddot{r}$ is the acceleration and $\overline{\ddot{r}^2}$ is its time average $\overline{\ddot{r}^2} = \frac{1}{\tau}\int_0^\tau \ddot{r}^2(t)dt$, where τ is the period. Show that for quasiperiodic motion

$$\frac{dW}{dt} = -\gamma W , \tag{3}$$

and determine the damping coefficient γ. *(1 Point)*

c) Assuming quasi-periodic motion, we now include the emitted radiation from the accelerating electron such that the new equation of motion becomes

$$m\ddot{r} + fr = \mathcal{R} \tag{4}$$

where $\mathcal{R}$ is the force which gives rise to a change in the total energy.

Using Eq.(2) find $\mathcal{R}$ in terms of $\dddot{r}$ by first multiplying Eq.(4) by $\dot{r}$. Then using $r = Ue^{i\omega t}$, where U is a complex vector, find a compact expression for the frequency of these oscillations. Where the time after which the energy of the emitting atom is $1/e$ its initial value is given by

$$T = \frac{1}{\gamma} = 4 \times 10^{-7}\text{s} \tag{5}$$

and $\omega_0 = 2\pi \times 10^{14}\text{s}^{-1}$.

(1 Point)

Question 2 *A sinusoidal perturbation*

Consider a physical system with Hamiltonian H_0 such that

$$H_0|\varphi_n\rangle = E_n|\varphi_n\rangle \tag{6}$$

with eigenvalues and eigenvectors E_n and φ_n respectively and $\langle\varphi_m|\varphi_n\rangle = \delta_{mn}$. At $t = 0$ a perturbation is applied to the system. Its Hamiltonian now becomes

$$H(t) = H_0 + \lambda\hat{W}(t) \tag{7}$$

where λ is a real dimensionless parameter much smaller than 1 and $\hat{W}(t)$ is an observable of the same order of magnitude as H_0 and which is zero for $t < 0$.
Now assume that $\hat{W}(t)$ has the form

$$\hat{W}(t) = \hat{W}\cos(\omega t) \tag{8}$$

where ω is a constant angular frequency and $\hat{W}$ is a time independent observable.

a) The system is assumed to be initially in the stationary state $|\psi_i\rangle$ which is an eigenstate of H_0 of eigenvalue E_i. Calculate the state vector $|\psi(t)\rangle$ to first order in λ and then calculate the probability $P_{if} = |\langle\varphi_f|\psi(t)\rangle|^2$ of finding the system in another eigenstate $|\varphi_f\rangle$ of H_0 at time t. *(1 Point)*

b) What is the transition probability induced by a constant perturbation? (i.e. $\omega = 0$) *(1 Point)*

c) Discuss the validity of the perturbative expansion. *(1 Point)*