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Quantum Optics and Cold Atoms

SoSe 2014 **Sheet 3** **29 May 2014**

(Due on 3. June 2014)

Question 1 *Squeezed states in number state representation*

Given a harmonic oscillator with annihilation and creation operators a and $a^\dagger$ such that $[a, a^\dagger] = 1$, let

$$|\xi\rangle = S(\xi)\,|0\rangle = \sum_n c_n\,|n\rangle \tag{1}$$

with $|0\rangle$ the vacuum state of a single mode and

$$S(\xi) = exp(\frac{1}{2}(\xi^* a^2 - \xi a^{\dagger 2})) \tag{2}$$

and choose $\xi = r \in \mathbb{R}$ where r is the squeezing parameter.

a) Use the relation

$$SaS^\dagger = a\cosh(r) + a^\dagger \sinh(r) \tag{3}$$

$$Sa^\dagger S^\dagger = a^\dagger \cosh(r) + a\sinh(r) \tag{4}$$

and demonstrate that

$$(a\cosh(r) + a^\dagger \sinh(r))\,|\xi\rangle = 0 \tag{5}$$

(1 Point)

b) Use this relation to show that

$$c_n = \langle n|\,\xi\rangle = 0\;, \quad \text{if } n \text{ odd} \tag{6}$$

$$c_{2n} = \langle 2n|\,\xi\rangle = \frac{(2n-1)!!}{(2n)!!}(-1)^n(\tanh(r))^n c_0 \tag{7}$$

with $c_0 \neq 0$ *(1 Point)*

c) Determine c_0 remembering that $\sum_n |c_n|^2 = 1$. *(0.5 Point)*

Question 2

The first-order correlation function for an electric field of a well defined potential is given by

$$g^{(1)}(r_1, t_1, r_2, t_2) = \frac{\langle \hat{E}^-(r_1, t_1)\hat{E}^+(r_2, t_2)\rangle}{[\langle \hat{E}^-(r_1, t_1)\hat{E}^+(r_1, t_1)\rangle\langle \hat{E}^-(r_2, t_2)\hat{E}^+(r_2, t_2)\rangle]^{1/2}} \tag{8}$$

where $\langle \ldots \rangle$ is evaluated over a given initial state. Here

$$\hat{E}^+ = \sum_\lambda E_\lambda^+ a_\lambda e^{i\mathbf{k}_\lambda \cdot \mathbf{r}} e^{-i\omega_\lambda t} \tag{9}$$

with $\omega_\lambda = c|\mathbf{k}_\lambda|$ and the sum runs over all modes with the same potential and $\hat{E}^- = (\hat{E}^+)^\dagger$. Note that

$$|g^{(1)}(r_1, t_1, r_2, t_2)| = \begin{cases} = 1 & \text{first-order coherent light} \\ = 0 & \text{incoherent light} \\ \neq 0 \text{ or } 1 & \text{partially coherent light} \end{cases} \tag{10}$$

a) Calculate $g^{(1)}(\tau)$ at $r_1 = r_2$ for a:

- single photon $|1_\lambda\rangle = a_\lambda^\dagger |vac\rangle$
- single photon $|\phi_f\rangle = \sum_k f_k a_k^\dagger |vac\rangle$
- single mode coherent state $|\alpha_k\rangle$
- coherent states $|\alpha_{k_1}, \alpha_{k_2}\rangle$

(2 Point)

b) Calculate now $g^{(1)}(\tau)$ for a single photon wavepacket and evaluate in the continuum limit the $g^{(1)}(\tau)$ for a gaussian wavepacket. (*Hint* use $\sum_k \to \frac{L}{2\pi}\int dk$ where L is the length of the quantisation volume and take $f_k \to f(k)$ with $\sqrt{\frac{L}{2\pi}} f(k) = \frac{e^{-\frac{(k-k_0)^2}{2\delta k^2}} e^{-ikx_0}}{(\pi \delta k^2)^{\frac{1}{4}}}$.)

(1 Point)

Question 3

The second order correlation function for an electric field of a well defined potential is given by

$$g^{(2)}(r_1, t_1, r_2, t_2; r_2, t_2, r_1, t_1) = \frac{\langle \hat{E}^-(r_1, t_1)\hat{E}^-(r_2, t_2)\hat{E}^+(r_2, t_2)\hat{E}^+(r_1, t_1)\rangle}{\langle \hat{E}^-(r_1, t_1)\hat{E}^+(r_1, t_1)\rangle\langle \hat{E}^-(r_2, t_2)\hat{E}^+(r_2, t_2)\rangle} \tag{11}$$

a) show that if the field is classical $g^{(2)}(0) \geq g^{(2)}(\tau)$, where $\tau = t_2 - t_1$ is a fixed time delay, and show that $\infty > g^{(2)}(0) \geq 1$.

(*Hint*: use Cauchy's inequality $2\bar{I}(t_1)\bar{I}(t_2) \leq \bar{I}(t_1)^2 + \bar{I}(t_2)^2$ and that

$$[\bar{I}(t_1)\bar{I}(t_1+\tau) + \ldots + \bar{I}(t_N)\bar{I}(t_N+\tau)]^2 \leq [\bar{I}(t_1)^2 + \ldots + \bar{I}(t_N)^2][\bar{I}(t_1+\tau)^2 + \ldots + \bar{I}(t_N+\tau)^2]$$

where $\bar{I}(t) = \frac{1}{2}\epsilon_0 c|E(t)|^2$ is the intensity)

(1 Point)

b) Calculate $g^{(2)}(\tau)$ for a:

- single photon $|1_\lambda\rangle = a_\lambda^\dagger |vac\rangle$
- single photon (gaussian wavepacket) $|\phi_f\rangle = \sum_k f_k a_k^\dagger |vac\rangle$
- single mode coherent state $|\alpha_k\rangle$
- coherent states $|\alpha_{k_1}, \alpha_{k_2}\rangle$

(2 Point)