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Quantum Optics and Cold Atoms

SoSe 2014　　　　**Sheet 4 (Part 1)**　　　　**20 June 2014**

(Due on 1. July 2014)

Question 1　*Jaynes-Cummings Hamiltonian*

Consider the interaction between the two level atom and the cavity field described by the Jaynes-Cummings Hamiltonian:

$$H = \hbar\omega_0\sigma^\dagger\sigma + \hbar\omega_c a^\dagger a + \hbar g(a^\dagger\sigma + \sigma^\dagger a), \tag{1}$$

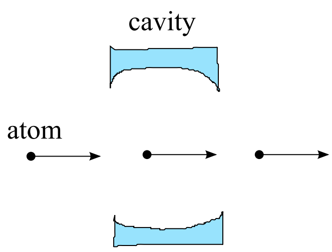
where the coupling constant g is real, ω_0 is the atomic transition frequency and ω_c is the cavity resonance frequency. The rising and lowering operators between the ground state $|g\rangle$ and the excited state $|e\rangle$ are defined as $\sigma^\dagger = |e\rangle\langle g|$ and $\sigma = |g\rangle\langle e|$ and $a^\dagger$, a are the photon creation and annihilation operators.

a) Solve the Schrödinger equation

$$i\hbar\partial_t|\psi\rangle_t = H|\psi\rangle_t \tag{2}$$

for the initial condition $|\psi\rangle_0 = |g\rangle\sum_{n=0}^{\infty} c_n|n\rangle$ with $\sum_{n=0}^{\infty}|c_n|^2 = 1$. Discuss the spectrum of the eigenvalues and the eigenvectors od the Hamiltonian H as the functions of the detuning $\delta = \omega_c - \omega_0$ and of the photon number n. *(2 Point)*

Question 2　*Master equations for a damped harmonic oscillator*



Consider an optical resonator crossed by a beam of atoms as shown on the picture. The light-atom interaction time τ is much smaller than the inverse injection rate $1/r$ and the inverse coupling strength $(g\sqrt{\langle n\rangle})^{-1}$, where $\langle n\rangle$ in a mean photon number, i.e. the following inequalities are fulfilled: $r\tau \ll 1$ and $g\sqrt{\langle n\rangle}\tau \ll 1$. In this case the field evolution can be described by the following master equation:

$$\partial_t\rho = \gamma_g(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a) + \gamma_e(a^\dagger\rho a - \frac{1}{2}aa^\dagger\rho - \frac{1}{2}\rho aa^\dagger), \tag{3}$$

where the oscillator damping and pumping rates $\gamma_g = r_g g^2\tau^2$ and $\gamma_e = r_e g^2\tau^2$ are proportional to the injection rate of the atoms in the ground state r_g or in the excited state r_e respectively.

a) For $\gamma_e > \gamma_g$ find a time scale t till which the dynamic evaluated from equation (3) is valid, given that the cavity is initially prepared in the vacuum state with density matrix $\rho(0) = |0\rangle\langle 0|$. *(1 Point)*

b) For $\gamma_g > \gamma_e$ show that in the steady state the density matrix for the cavity field is diagonal, i.e. $\rho_{nm} = 0$ for $n \neq m$. *(1 Point)*

c) Derive a master equation from the Jaynes-Cummings Hamiltonian (1) under the assumption that the atoms are injected with the rate r in the state $|\psi\rangle_{\mathrm{at}} = \alpha|g\rangle + \beta|e\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. *(3 Point)*