Quantum Optics

WS 2017/18 **Sheet 1** **03.11.2017**

Exercise 1 *An elastically bound electron*

a) Newton's equation of motion for a charged particle of mass m and charge e which is elastically bound to the origin is given by

$$m\ddot{\mathbf{r}} + f\mathbf{r} = 0 \tag{1}$$

where f us the spring constant, ω_0 is the oscillator eigenfrequency with $\omega_0^2 = f/m$ and $\mathbf{r} = \mathbf{r}(t)$ is the position of the particle. Write the energy W of the system and derive the general solution and the period τ of the motion. *(1 Point)*

b) The energy which is dissipated by the electron is given by

$$S = -\frac{dW}{dt} = \frac{2e^2}{3c^3}\ddot{\mathbf{r}}^2 \tag{2}$$

where c is the speed of light and $\ddot{\mathbf{r}}^2$ is the squared acceleration

$$\overline{\ddot{\mathbf{r}}^2} = \frac{1}{\tau}\int_0^\tau \ddot{\mathbf{r}}^2 dt.$$

Show that the average energy dissipated per period for quasiperiodic motion reads

$$\overline{\frac{dW}{dt}} = -\gamma\overline{W} \tag{3}$$

where we use the definition

$$\overline{A} = \frac{1}{\tau}\int_0^\tau dt A(t). \tag{4}$$

Determine the damping coefficient γ. *(1 Point)*

c) Assuming quasiperiodic motion, we now include the emitted radiation from the accelerating electron such that the new eqation of motion becomes

$$m\ddot{\mathbf{r}} + f\mathbf{r} = \boldsymbol{\mathcal{R}} \tag{5}$$

where $\boldsymbol{\mathcal{R}}$ is the force whicht gives rise to a change in the total energy.

Find $\boldsymbol{\mathcal{R}}$ using Eq. (2) in terms of $\dddot{\mathbf{r}}$ by multiplying Eq. (5) by $\dot{\mathbf{r}}$. Then using $\mathbf{r} = \mathbf{U}e^{i\omega t}$, where $\mathbf{U}$ is a complex vector, find a compact expression for the frequency of these oscillations. Where the time after which the energy of the emitting atom decreased to $1/e$ of its initial value is given by

$$T = \gamma^{-1} = 4 \times 10^{-7}\text{s} \tag{6}$$

and $\omega_0 = 2\pi \times 10^{14}\text{s}^{-1}$.

Exercise 2 *A sinusoidal perturbation*

Consider a physical system with Hamiltonian $\hat{H}_0$ such that

$$\hat{H}_0|\phi_n\rangle = E_n|\phi_n\rangle \tag{7}$$

with eigenvalues and eigenvectors E_n and $|\phi_n\rangle$, respectively and $\langle\phi_m|\phi_n\rangle = \delta_{mn}$. At $t = 0$ a perturbation is applied to the system. Its Hamiltonian now becomes

$$\hat{H}(t) = \hat{H}_0 + \lambda\hat{W}(t) \tag{8}$$

where λ is a real dimensionless parameter much smaller than 1 and $\hat{W}(t)$ is an observable of the same order of magnitude as $\hat{H}_0$ and which is zero for $t < 0$.
Now assume that $\hat{W}(t)$ has the form

$$\hat{W}(t) = \hat{W}\cos(\omega t) \tag{9}$$

where ω is a constant angular frequency and $\hat{W}$ is a time independent observable.

a) The system is assumed to be initially in a the stationary state $|\phi_i\rangle$ which is an eigenstate of $\hat{H}_0$ of eigenvalues E_i. Calculate the state vector $|\psi(t)\rangle$ to first order in λ and then calculate the probability $P_{if}(t) = |\langle\phi_f|\psi(t)\rangle|^2$ of finding the system in another eigenstate $|\phi_f\rangle$ of $\hat{H}_0$ at time t. *(1 Point)*

b) What is the transition probability induced by a constant perturbation? (i.e. $\omega = 0$) *(1 Point)*

c) Discuss the validity of the perturbative expansion. *(1 Point)*