Quantum Optics

WS 2017/18 **Sheet 3** **08.12.2017**

Exercise 5

A Lindblad master equation is given by

$$\frac{\partial \hat{\rho}}{\partial t} = \mathcal{L}\hat{\rho} \tag{1}$$

with Lindbladian

$$\mathcal{L}\hat{\rho} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \sum_{j=1}^{N} \gamma_j([\hat{V}_j, \hat{\rho}V_j^\dagger] + [\hat{V}_j\hat{\rho}, \hat{V}_j^\dagger]), \tag{2}$$

where $\gamma_j \geq 0$ for all j, $\hat{H} = \hat{H}^\dagger$ is Hermitian and $\hat{V}_j$ is an operator.

a) Calculate the adjoint $\mathcal{L}^\ddagger$ of $\mathcal{L}$ by using

$$\mathrm{Tr}(\hat{A}(\mathcal{L}\hat{\rho})) = \mathrm{Tr}((\mathcal{L}^\ddagger\hat{A})\hat{\rho}). \tag{3}$$

(1 Point)

b) Assume $\gamma_j = 0$ for $j \geq 2$. When is $\mathcal{L}$ Hermitian? Give an example for an Hermitian and a non-Hermitian $\mathcal{L}$ you know from the lecture. *(1 Point)*

c) Assume now $\gamma_1 = -\gamma < 0$, $\hat{V}_1 = \hat{\sigma}_z$ and $\gamma_j = 0$ for $j \geq 2$. Why can $\mathcal{L}$ not describe the evolution of a density matrix? *(1 Point)*

Exercise 6

The optical Bloch equations for a two-level system driven by a laser are given by

$$\frac{d\mathbf{S}}{dt} = \begin{pmatrix} -\frac{\Gamma}{2} & -\Delta & 0 \\ \Delta & -\frac{\Gamma}{2} & -2\Omega \\ 0 & 2\Omega & -\Gamma \end{pmatrix} \mathbf{S} + \begin{pmatrix} 0 \\ 0 \\ -\Gamma \end{pmatrix},$$

with $\mathbf{S} = (S_x, S_y, S_z)^T$ and $S_x = \langle e|\hat{\rho}|g\rangle + \langle g|\hat{\rho}|e\rangle$, $S_y = i(\langle g|\hat{\rho}|e\rangle - \langle e|\hat{\rho}|g\rangle)$, $S_z = \langle e|\hat{\rho}|e\rangle - \langle g|\hat{\rho}|g\rangle$. Here Ω is the Rabi frequency, Γ is the decay rate of the excited state and $\Delta = \omega - \omega_0$ is the detuning between laser frequency and the atomic transition frequency.

a) Derive the steady state of the system. *(1 Point)*

b) Solve the system in the case of $\Delta = 0$. *(1 Point)*