Exercises for
Theoretical physics IV

WiSe 2022/23 **Sheet 3** **08.11.2022**

Exercise 4 *Volume dependence of the internal energy for various equations of state*

In this exercise it should be investigated how the internal energy for some equations of state $f(p, V, T) = 0$ with reversible changes of state depends on the volume V.

a) In the following we consider the internal energy U as a function of the temperature T and the volume V and denote with $dS = \delta Q/T = \frac{1}{T}\,dU + \frac{p}{T}\,dV$ the change in entropy S. Derive the relation from the integrability criterion for the complete differential dS

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p.$$

(2 points)

b) How does the internal energy $U(T, V)$ for the ideal gas $pV = NkT$ or for the van der Waals gas

$$\left(p + \frac{\alpha}{(V/N)^2}\right)\left(\frac{V}{N} - 4v_0\right) = kT$$

depend on the volume?

(2 points)

c) What is the relationship between pressure p and temperature T for the equation of state of a gas in which the internal energy U only depends on the temperature, but not on the volume?

(1 point)

Exercise 5 *Cooling with the help of the Carnot cycle*

The Carnot cycle discussed in the lecture can be used to cool the colder reservoir of temperature $T_1 < T_2$ by doing mechanical work $-L > 0$ on the system in a reversible manner (analogous to the lecture, L is the work done by the system). In contrast to the lecture, the isotherms and adiabats of the cycle are not clocked clockwise ($A \to B \to D \to C \to A$), but counterclockwise ($A \to C \to D \to B \to A$).
Repeat the reasoning from the lecture in the event that the Carnot cycle is run through counterclockwise. Then, based on the adiabatic expansion of the ideal gas, show that the amount of heat Q_1 that is withdrawn from the colder reservoir is given by the equation

$$Q_1 = \frac{(-L)}{\dfrac{T_2}{T_1} - 1}.$$

(3 points)