Übung zur Vorlesung
Theoretische Physik IV

WiSe 2022/23 **Sheet 9** **06.01.2023**

Exercise 21 *Symmetric and antisymmetric states of quantum particles*

Let us consider a single particle with spin S confined in a one-dimensional box. The Hamiltonian operator $\hat{H}_1$ representing the particle then reads

$$\hat{H}_1 = \frac{\hat{p}^2}{2m} + V(\hat{x}), \tag{1}$$

where $V(x) = 0$ for $0 < x < L$ and $V(x) \to +\infty$ for any other case.

a) Determine the eigenenergies and eigenvectors of $\hat{H}_1$.

(1 point)

b) Considering now two uninteracting identical particles described by the joint Hamiltonian $\hat{H}_2 = \hat{H}_1 \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}_1$. Determine the form of the eigenenergy and eigenvectors when $S = 0$ (bosonic case). What is the ground state and its energy? Same question for the first excited state?

(1 point)

c) Considering now fermionic particles of spin $S = \frac{1}{2}$, determine the eigenvectors and corresponding eigenenergies. Which form take the ground state and first excited state?

(1 point)

Exercise 22 *Chemical potential of the ideal gas*

Let us consider a statistical ensemble with a chemical potential μ. The free energy changes by $\mu \mathrm{d}N$ when the number of particles changes by $\mathrm{d}N$, with temperature T and volume V remaining constant. From this follows the relation

$$\mathrm{d}A = -P\mathrm{d}V - S\mathrm{d}T + \mu \mathrm{d}N \tag{2}$$

and the chemical potential derives from the Maxwell relation

$$\mu = \left(\frac{\partial A}{\partial N}\right)_{V,T} \tag{3}$$

Determine from the partition function of the ideal gas

$$Q_N = \frac{1}{h^{3N} N!} \int \mathrm{d}q \mathrm{d}p \exp\left[-\beta \sum_{i=1}^{N} p_i^2/(2m)\right], \ \lambda = \sqrt{2\pi\hbar^2/(mk_BT)} \tag{4}$$

the free energy A and the chemical potential μ. The results are

$$A = -k_B T N \left[\ln \left(\frac{V}{N\lambda^3} \right) + 1 \right], \tag{5}$$

$$\mu = k_B T \ln(\lambda^3 n), \tag{6}$$

with the density n.

(1 point)