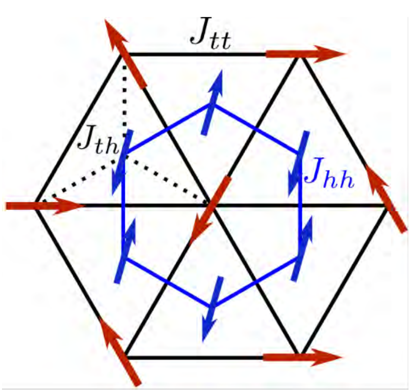
IOWA STATE UNIVERSITY

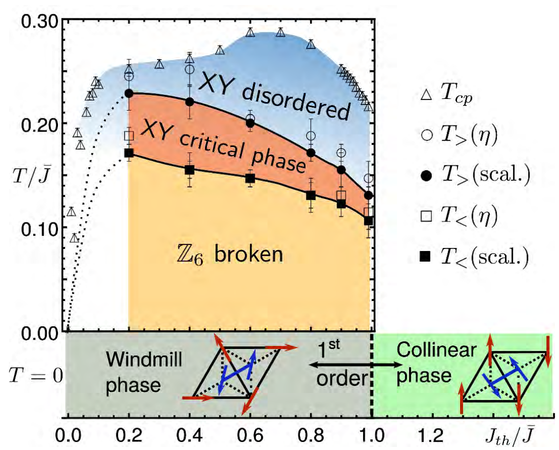
Department of Physics and Astronomy

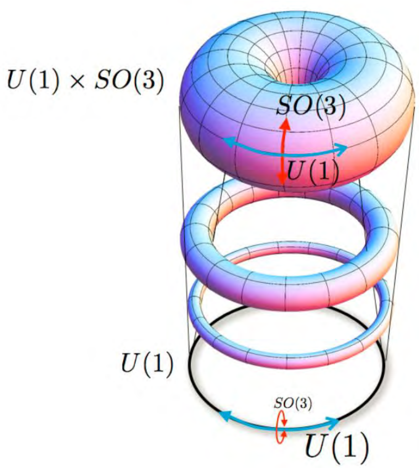
Intertwined electronic states of matter: emergent order in frustrated antiferromagnets

Peter P. Orth (Iowa State University)

Physics Colloquium, Grinnell College, Grinnell, IA, 24 October 2017







Collaborators

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(Rutgers)

P. Coleman
(Rutgers)

R. M. Fernandes
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(Minnesota)

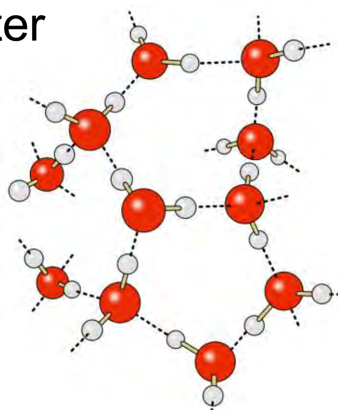
Experimental collaborators:

P. C. Canfield
(Iowa)
& his group

States of matter: symmetry, order and topology

- Condensed matter physics investigates states of matter = phases
  - Liquids, Solids: different translational symmetry

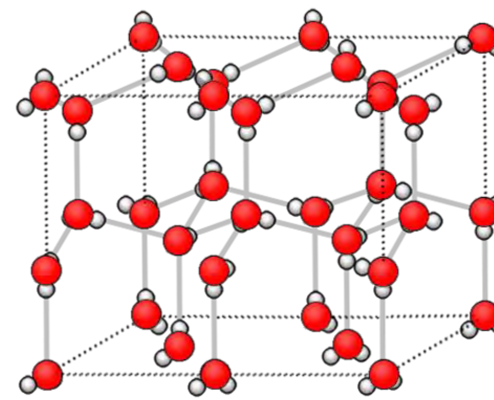
Water



Phase transition

Cooling/Heating
Driving West/East
(“Go West, young man”)

Ice



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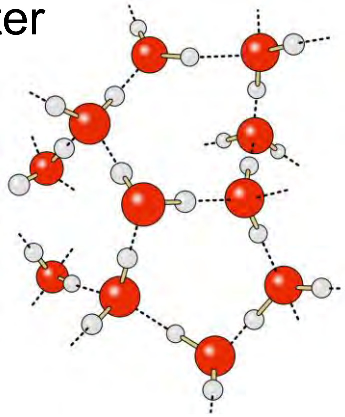

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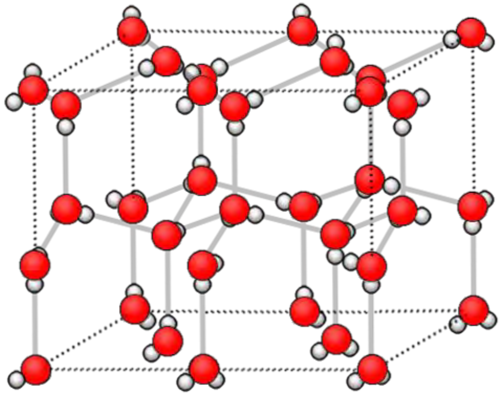

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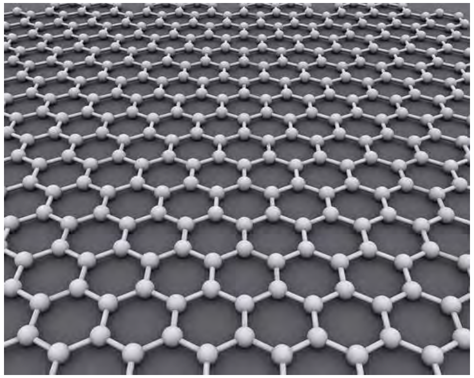
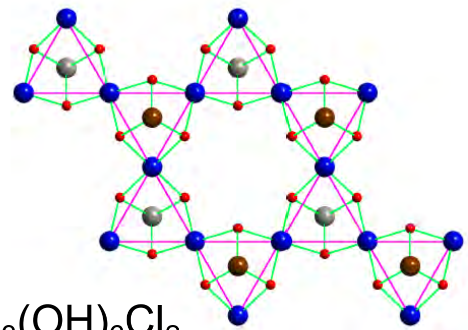
Graphene: 2D crystal

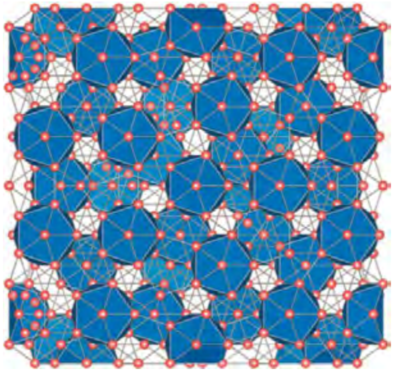


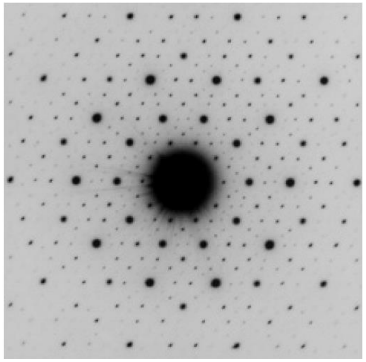
Herbertsmithite: Kagome lattice



$ZnCu_3(OH)_6Cl_2$

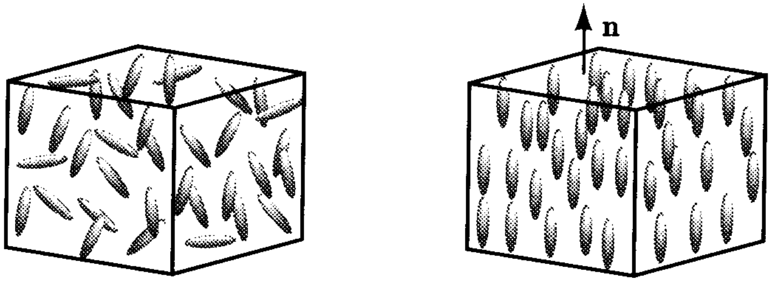
Quasicrystal: Au-Al-Yb





5-fold symmetry

Liquid crystals: nematic orientational order

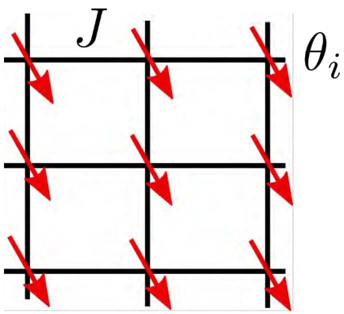


[1] Wikipedia; [2] M. R. Norman, RMP **88**, 041002 (2016); [3] P. M. Chaikin, T. C. Lubensky "Principles of Condensed Matter Physics"; [4] K. Deguchi *et al.,* Nat Mat. **11**, 1013 (2012).

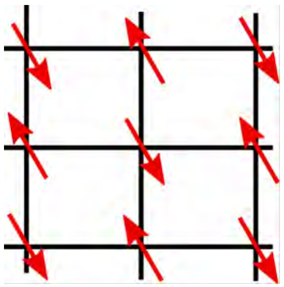
States of matter: symmetry, order and topology

- Condensed matter physics investigates states of matter = phases
  - Liquids, solids: crystals, quasi-crystals, amorphous matter, liquid crystals
  - Magnetic order: ferromagnetic, antiferro, spin-density waves, skyrmions
  - Paramagnets: low dimensional magnets, (quantum) spin liquids

Ferromagnets

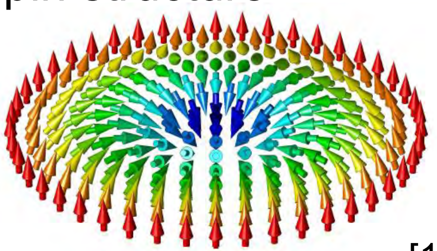


Antiferromagnets

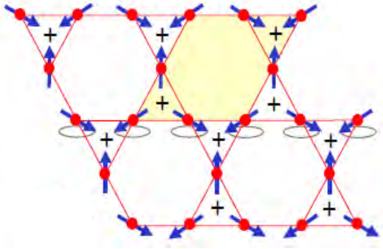


Fe_2O_3

Skyrmions: topological spin structure



Quantum spin liquids



Herbertsmithite

$ZnCu_3(OH)_6Cl_2$

[1] Wikipedia; [2] ScienceDaily.com; [3] ETH Zuerich Website

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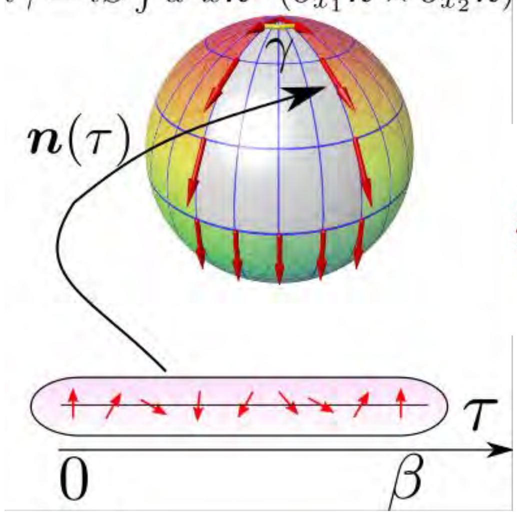
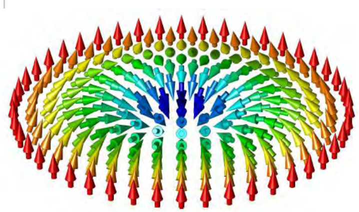
  - Electronic order: superconductivity, charge density waves
  - Topological order: topo. insulators, Weyl semimetals, skyrmions, toric code

Geometric Berry phase of electrons

$$i\gamma = iS \int d^2x \boldsymbol{n} \cdot (\partial_{x_1} \boldsymbol{n} \times \partial_{x_2} \boldsymbol{n})$$

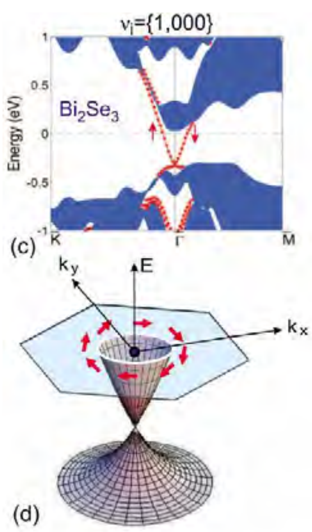




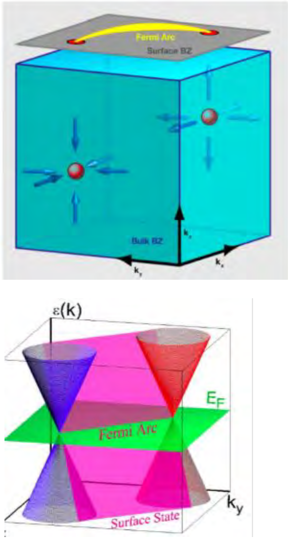
Skyrmions

Topological insulators and metals

TI
Bi_2Se_3



Weyl SM



[1] M. Z. Hasan, C. L. Kane, RMP **82** 3045 (2010); N. P. Armitage *et al.*, arXiv:1705:01111 (2017).

States of matter: emergent order

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  - Electronic order: superconductivity, charge density waves
  - Topological order: quantum (spin) Hall, Weyl semimetals, toric code

In all these examples:
Order occurs in elementary degrees of freedom: charge, spin.

Topic of this talk:
Order can also occur in composite objects such as higher order correlation functions (of charge and spin).

Emergent order

How to describe matter: Theory of Everything

- In contrast to particle physicist, we have a "Theory of Everything":

$$H = T_e + V_{ee} + T_i + V_{ii} + V_{ei} + H_{SO} + H_{\text{hyper}} + H_{\text{rel}} + H_{\text{ext}}$$

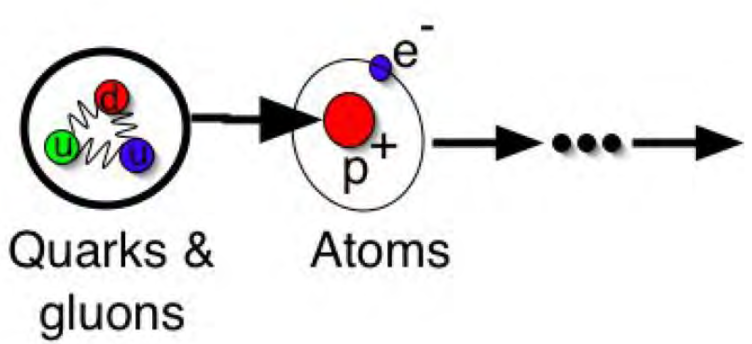
Electrons: $$T_e + V_{ee} = -\sum_j \frac{\hbar^2}{2m_e}\nabla_j^2 + \frac{1}{2}\sum_{j\neq k}\frac{e^2}{|r_j - r_k|}$$

Ions: $$T_i + V_{ii} = -\sum_J \frac{\hbar^2}{2m_J}\nabla_J^2 + \frac{1}{2}\sum_{J\neq K}\frac{Z_J Z_K e^2}{|R_J - R_K|}$$

Electron-ion interaction:

$$V_{ei} = -\sum_{j,J}\frac{Z_J e^2}{|r_j - R_J|}$$

Coarse graining



Fauna studies, botany, ecology

Fox squirrel

Many-body systems: Exponential wall

- Quantum many-body wavefunction $\Psi(r_1, r_2, \ldots, r_N)$
- Number of states grows exponentially:

$$B = q^{3N}, q \geq 3$$

Example: $\{|1,1\rangle, |1,2\rangle, |2,1\rangle, |2,2\rangle\}$

Bits needed to even record it.

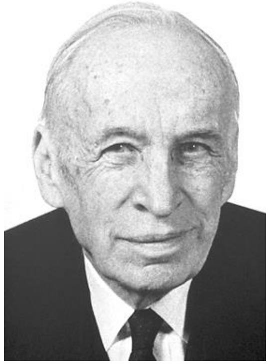
→ $q = 3, N = 1000, B = 10^{1500} \gg 10^{80}$

Largely exceeds number of baryons in the universe!

Van Vleck catastrophy: the many-electron wavefunction may not even be a legitimate scientific concept when $N \geq 1000$.

→
- Numerical techniques based on density (DFT)
- Effective models (at different scales)
- **Emergence** of new phenomena at larger scales, renormalization group

[1] W. Kohn (Nobel lecture).



J. H. Van Vleck

W. Kohn

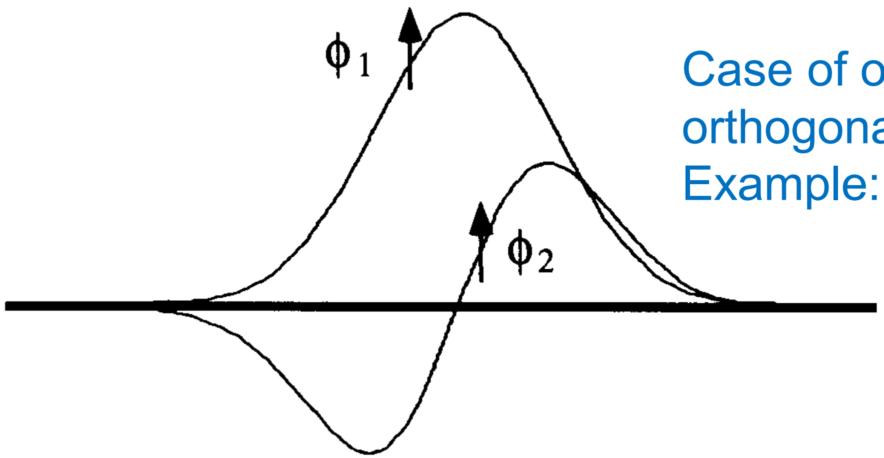
P. W. Anderson

How do we build effective models: magnetism

- Two electrons in different orbitals of same ion
  - Electronic spin: orbital and spin wavefunction

$$\Psi(r_1, r_2, s_1, s_2) = \Phi(r_1, r_2)\chi(s_1, s_2)$$

  - Coulomb repulsion
  - Pauli exclusion principle (Fermi statistics)



Case of overlapping and (almost) orthogonal orbitals
Example: $2p_x, 2p_y$

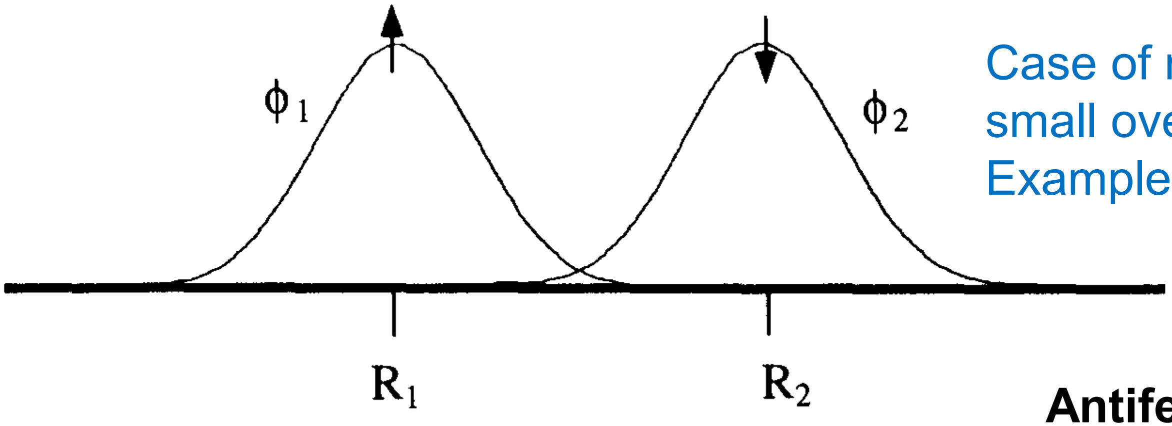
Ferromagnetic spin model

$$H = \sum_{i,j} (-J_{ij}) \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Ferromagnetic exchange interaction (Hund's rule)

How do we build effective models: magnetism

- Two electrons localized around nearby ions
  - Coulomb repulsion small
  - Maximizing kinetic energy via delocalization (Heisenberg uncertainty principle)
  - Pauli exclusion principle



Case of non-orthogonal orbitals with small overlap
Example: $2s, 2s$ on nearest-neighbors

Antiferromagnetic spin model

$$H = \sum_{i,j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

➡ **Antiferromagnetic exchange interaction**

Magnetism and phase transitions

- Ising spin model (discrete spins) $J > 0, S_i^z = \pm 1$

$$H = -J \sum_{\langle i,j \rangle} S_i^z S_j^z$$

Mean-field solution: $\sum_{\langle i,j \rangle} S_i^z S_j^z \to Nzm^2 + zm \sum_i S_i^z + \dots$

Self-consistency equation for the magnetization:

$$m = \frac{1}{Z} \sum_{S^z = \pm 1} S^z e^{-\beta H[S^z]} = \frac{e^{\beta Jzm} - e^{-\beta Jzm}}{e^{\beta Jzm} + e^{-\beta Jzm}} = \tanh \beta Jzm$$

⇨ Non-zero solution below $T_c = zJ$

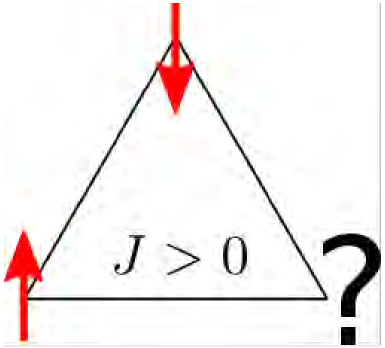
Phase transition from paramagnetic to magnetic state at T_c.

- Exact solution available in 1D (Ising) and 2D (Onsager):
$T_c(1D) = 0, T_c(2D) = 2J/(\ln(1+\sqrt{2}) = 2.27\ J$

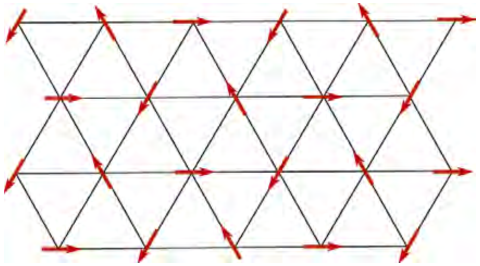
Magnetic frustration

- But, things can become much more interesting

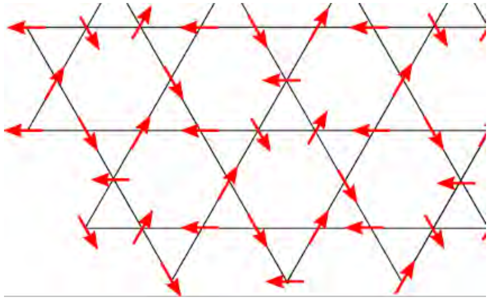
Frustration



Triangular lattice



Kagome lattice

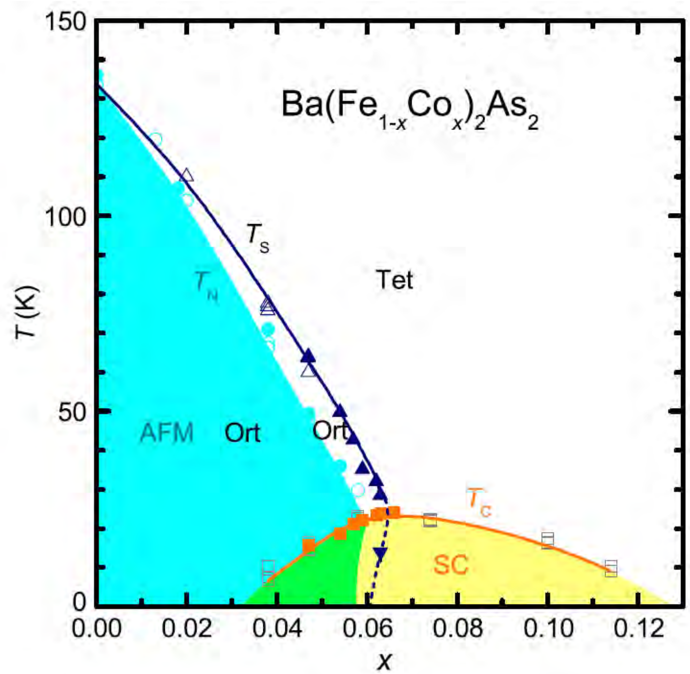


Herbertsmithite

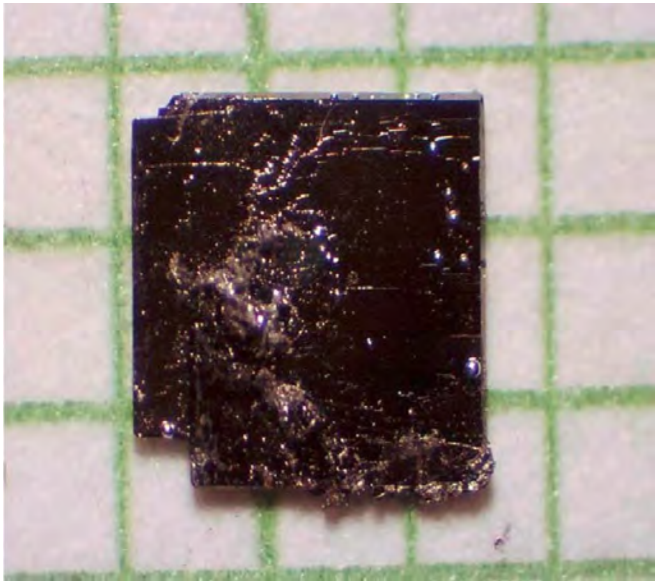
Rebecca Flint (ISU)

- Ground state degeneracy
- Quantum disordered states: spin liquids (talk in a few weeks by my colleague Rebecca Flint)

Order and phases of iron based superconductors (SCs)



$BaFe_2As_2$ crystal



Phase diagram of **iron based superconductor** $Ba(Fe_{1-x}Co_x)_2As_2$
(Canfield lab at Iowa State University)

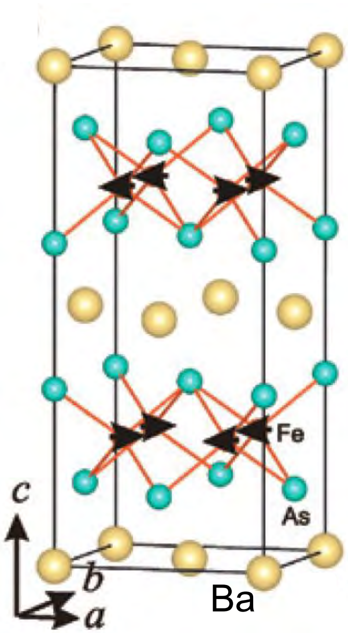
[1] S. Nandi *et al.*, PRL **104**, 057006 (2010); [2] P. C. Canfield, S. L. Budko, Annu. Rev. Cond. Mat. **1**, 27 (2010).

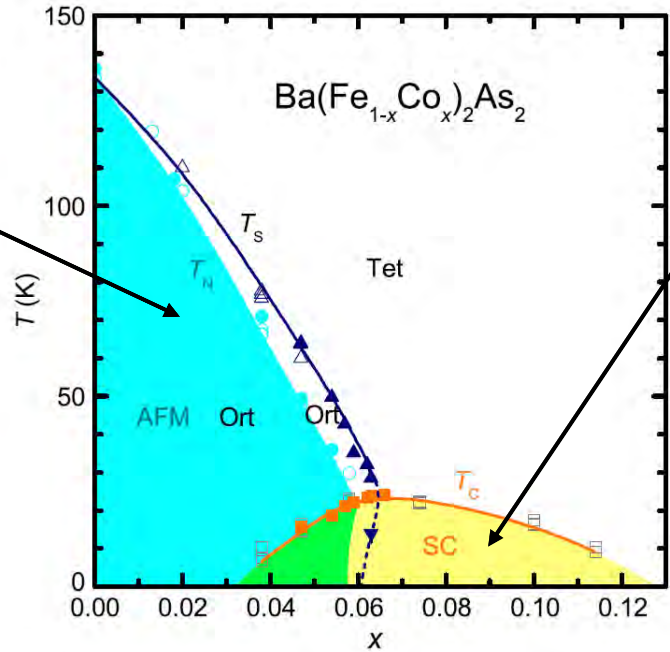
EMERGENT ORDER IN IRON BASED SUPERCONDUCTORS

Elementary order in iron based SC

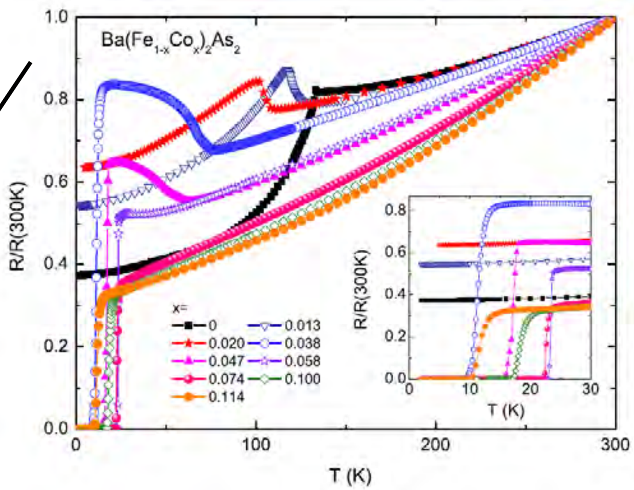
Phase diagram of iron based superconductor

Magnetic order (AFM collinear)

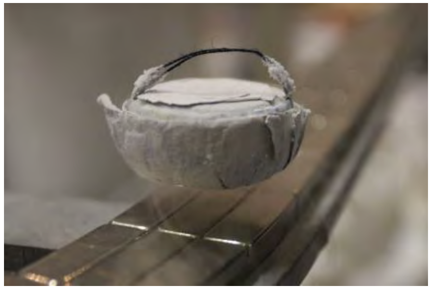




Superconductivity (SC)



Meissner effect

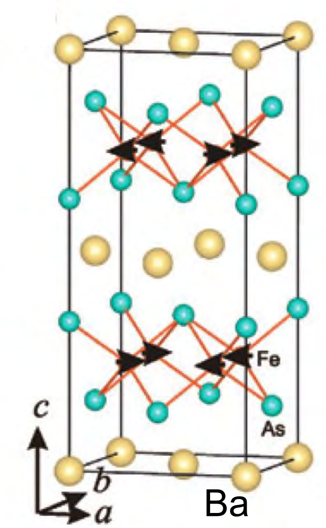


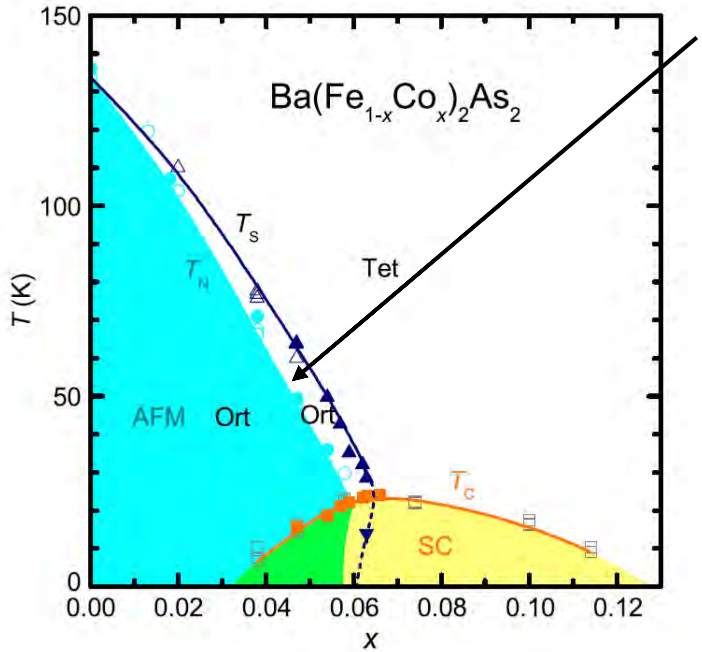
[1] S. Nandi *et al.*, PRL **104**, 057006 (2010); [2] D. Johnston, Adv. Phys. **59**, 803 (2010); [3] N. Ni *et al.*, PRB **78**, 214515 (2008).

Emergent order in iron based SC

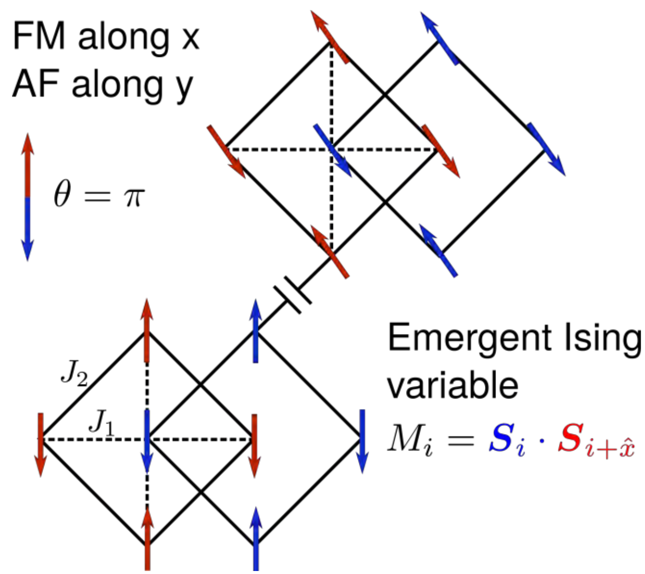
Phase diagram of iron based superconductor

Magnetic order (AFM)





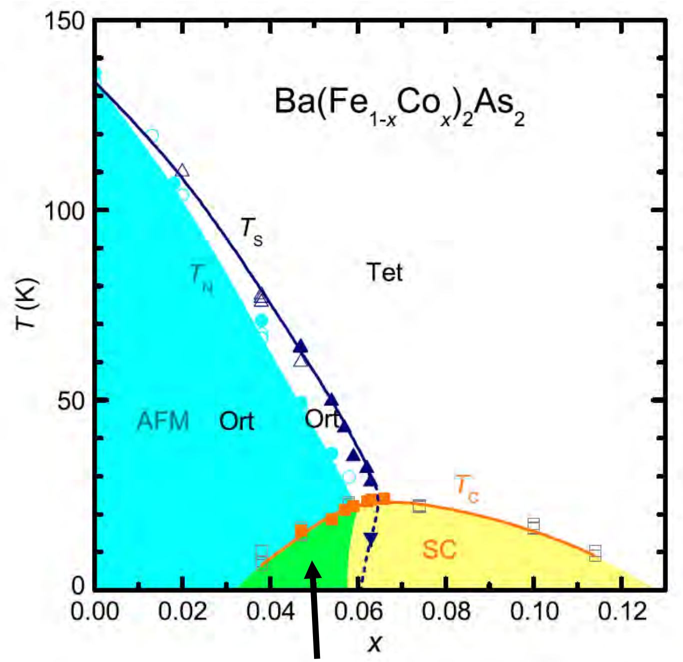
Emergent nematic order: tetragonal to orthorhombic crystal structure deformation



Emergent order in relative orientation of spins

Elementary and emergent order in iron based SC

Phase diagram of iron based superconductor

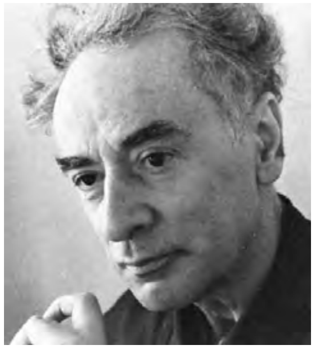


Coexistence of **intertwined** AFM,
SC & nematic **order**

Mutual impact of intertwined phases

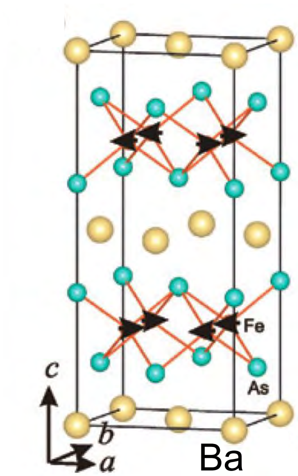
Broken symmetry classification

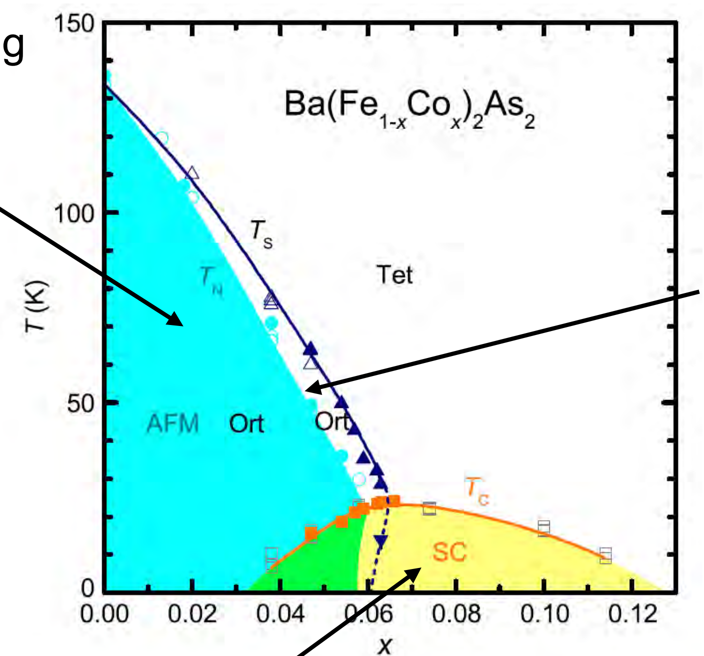
- Classify phases by broken symmetries (Landau paradigm)
- Order parameter $\phi \neq 0$ non-zero in symmetry broken phase
- Phase transitions = spontaneous change of symmetry



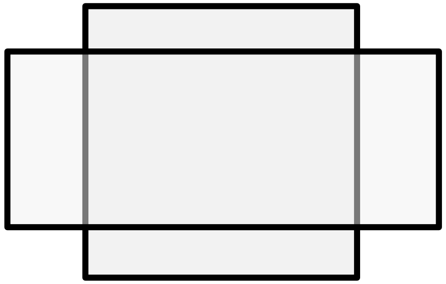
L. D. Landau (1908 – 1968)

Magnetic order (breaking of **spin rotational symmetry**)





Nematic order (**rotational symmetry** breaking from tetragonal C_4 to orthorhombic C_2)

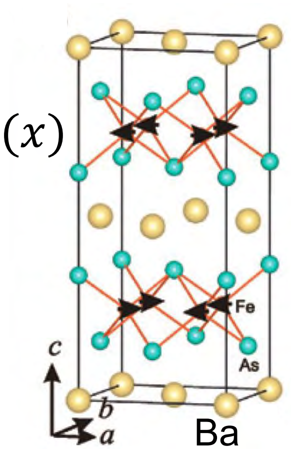


Superconductivity (breaking of **global U(1) symmetry**)

Spontaneous symmetry breaking at phase transition

- **Order parameter ϕ = magnetization/orthorhombicity of Fe sites/plaquettes**
- Expand free energy F close to phase transition in small $\phi(x) \propto M(x)$.
- Symmetry dictates form of expansion: $w = 0$ due to time-reversal (TR)

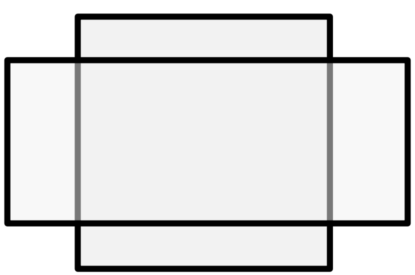
$\phi = M(x)$



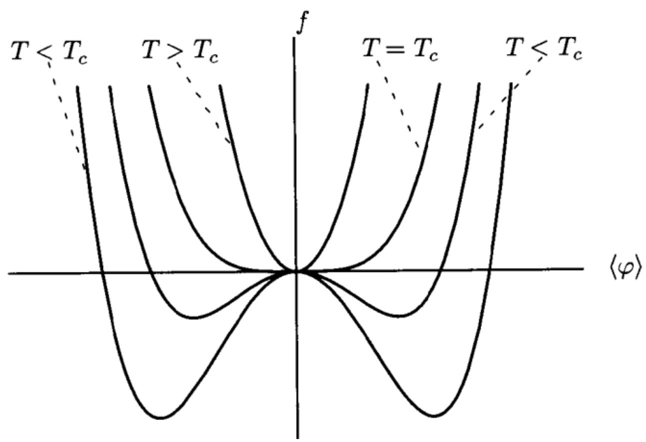
$$F = \frac{1}{2}\int d^d x \{c[\nabla\phi]^2 + r_0\phi^2 - w\phi^3 + u\phi^4\}$$

r_0 changes sign at $T = T_c$: $r_0 = a(T - T_c)$

$\phi = (a - b)/(a + b)$



Spontaneous symmetry breaking



$T > T_c : \langle\phi\rangle = 0$

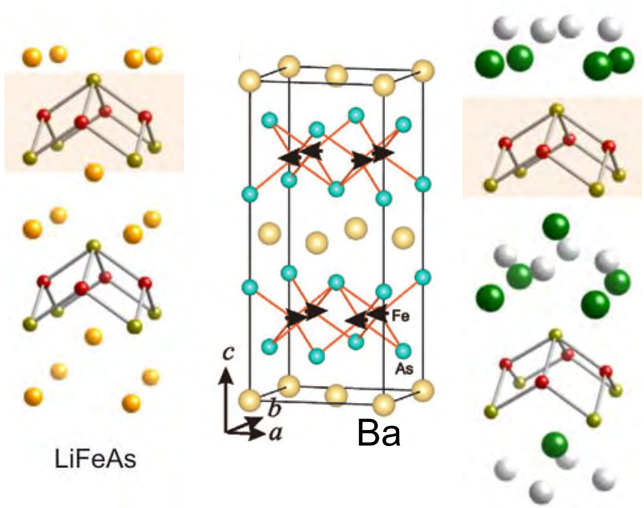
$T < T_c : \langle\phi\rangle \neq 0$

$\phi \to -\phi$

TR symmetry broken

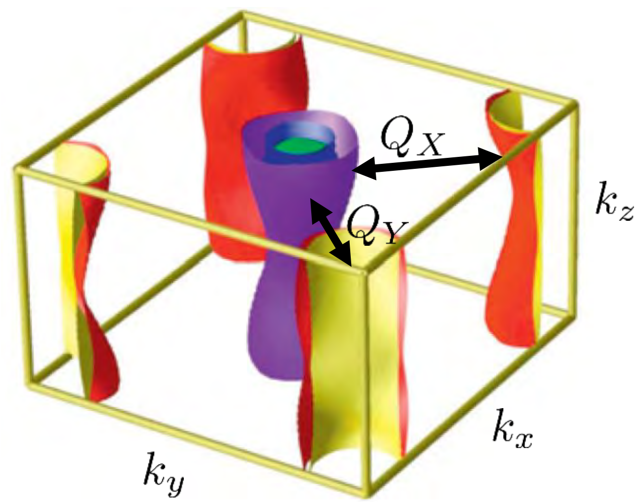
Free energy for iron based superconductors

- Tetragonal crystal lattice, FeAs-planes with Fe square lattices
- Magnetic fluctuations occur at two wavevectors: $Q_X = (\pi, 0), Q_Y = (0, \pi)$

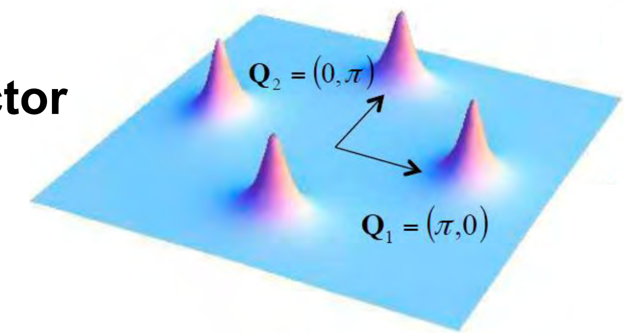


Crystal structures of FeSCs

Bandstructure of FeSCs



Magnetic structure factor (fluctuations)



[1] J. Paglione and R. Greene Nature Physics **6**, 645 (2010);
[2] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012).

Free energy for iron based superconductors

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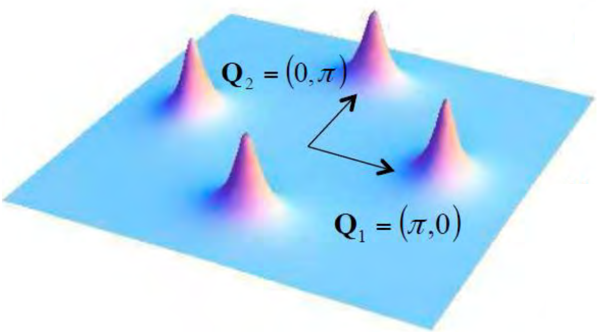
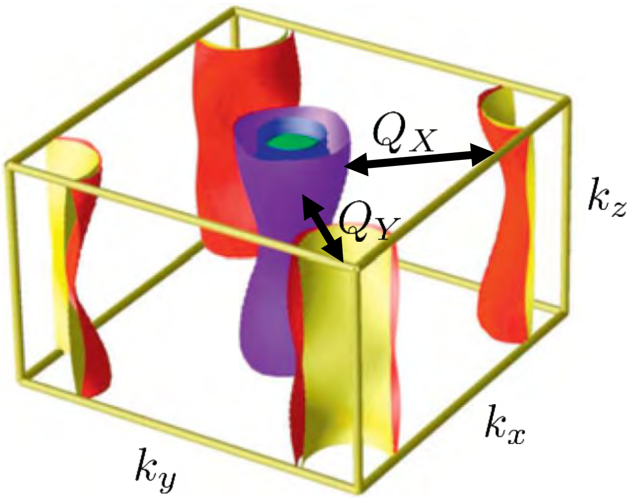
Magnetic structure factor (fluctuations):



Bandstructure of FeSCs



Magnetic order parameter (OP)

$$\boldsymbol{M}(\boldsymbol{x}) = \boldsymbol{M}_X \cos(\boldsymbol{Q}_X \cdot \boldsymbol{x}) + = \boldsymbol{M}_Y \cos(\boldsymbol{Q}_Y \cdot \boldsymbol{x})$$

[1] J. Paglione and R. Greene Nature Physics **6**, 645 (2010);
[2] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012).

Free energy for iron based superconductors

- Tetragonal crystal lattice, FeAs-planes with Fe square lattices
- Magnetic fluctuations occur at two wavevectors:

Free energy of iron based superconductors consistent with tetragonal and spin rotational symmetry

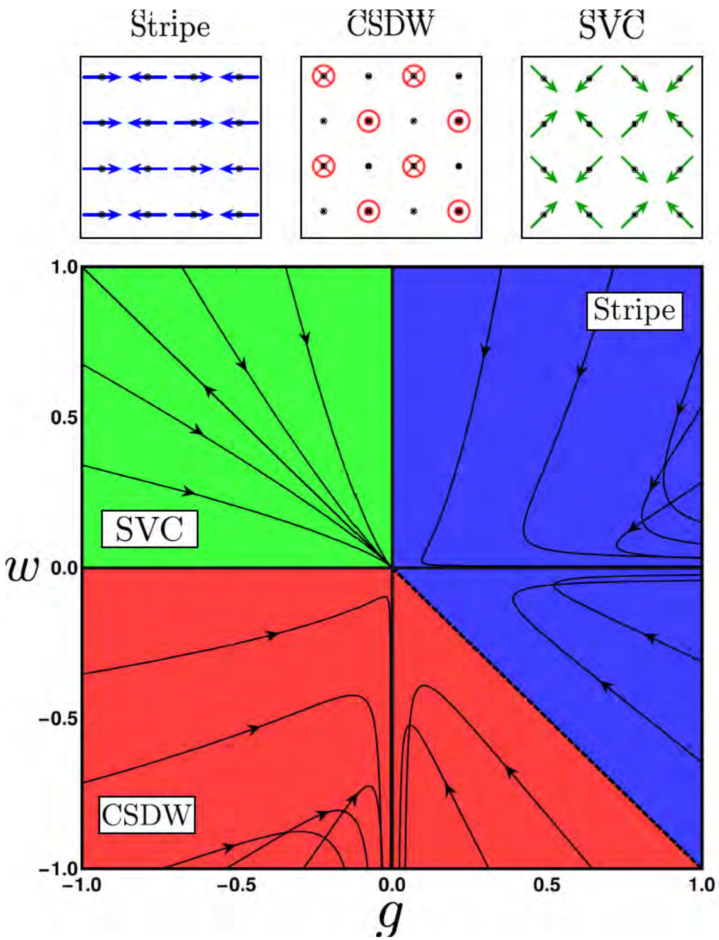
$$F = \int_q \chi_q^{-1}(\boldsymbol{M}_X^2 + \boldsymbol{M}_Y^2) + \frac{u}{2}\int_x (\boldsymbol{M}_X^2 + \boldsymbol{M}_Y^2)^2 - \frac{g}{2}\int_x (\boldsymbol{M}_X^2 - \boldsymbol{M}_Y^2)^2 + 2w\int_x (\boldsymbol{M}_X \cdot \boldsymbol{M}_Y)^2$$

Magnetic ground state minimizes free energy: depends on parameters u, g, w.

[1] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012).

Magnetic phases of iron based superconductors

- Three different types of magnetic order

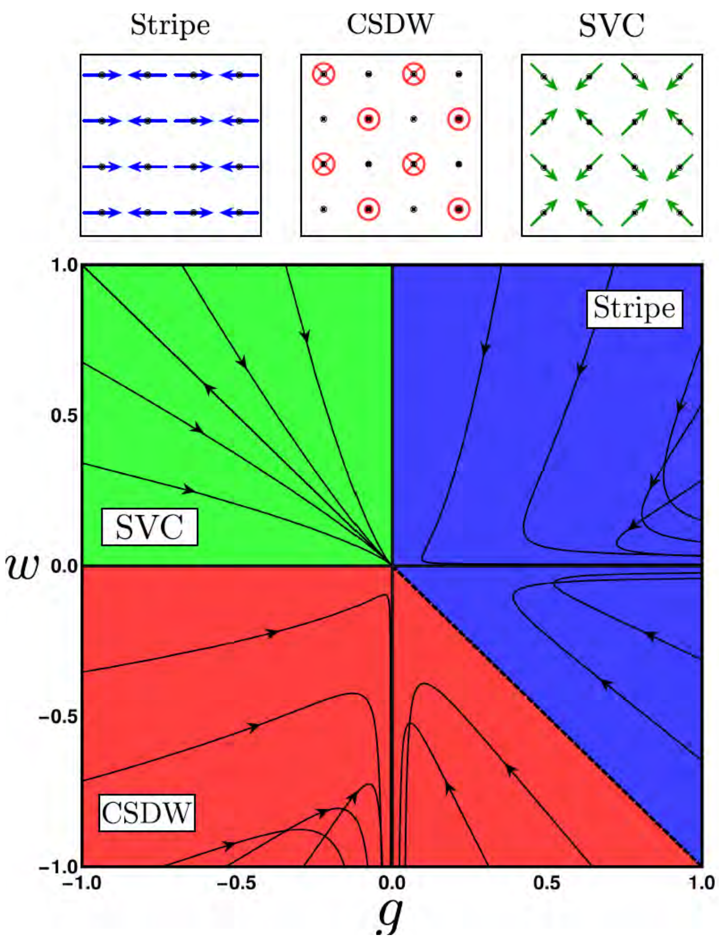


- **Stripe spin-density wave (SSDW)**
  - M_1 or $M_2 \neq 0$. Breaks
    - $O(3)$ spin rotation symmetry
    - $C_4 \rightarrow C_2$ crystal symmetry

[1] R. M. Fernandes *et* al., PRB **93**, 014511 (2016). [2] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Magnetic phases of iron based superconductors

- Three different types of magnetic order

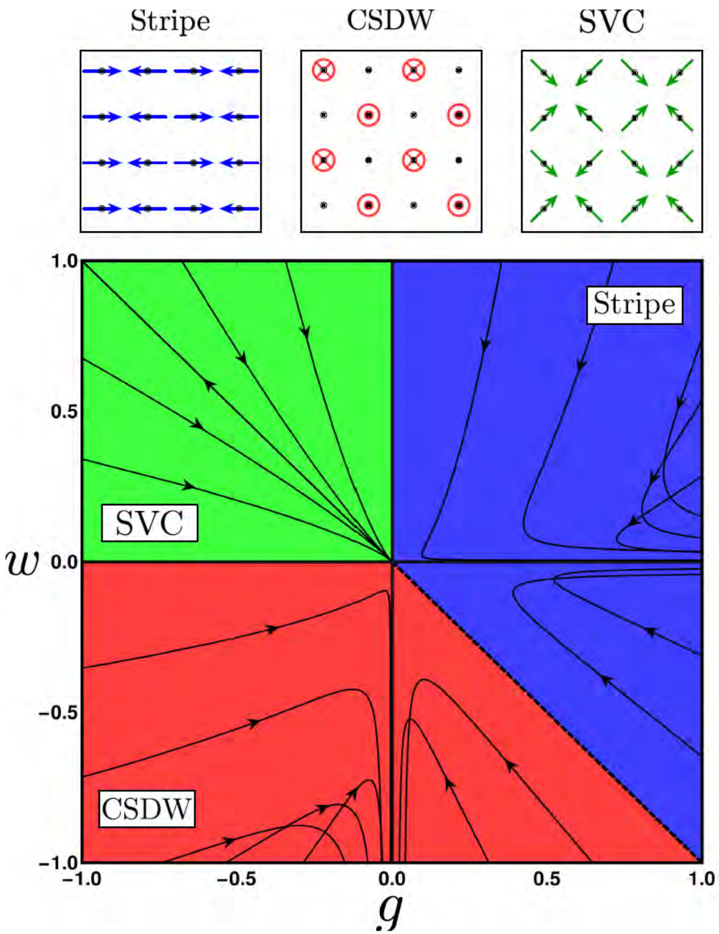


- Stripe spin-density wave (SSDW)
  - M_1 or $M_2 \neq 0$. Breaks
    - $O(3)$ spin rotation symmetry
    - $C_4 \to C_2$ crystal symmetry
- **Charge-spin density wave (CSDW)**
  - M_1 and $M_2 \neq 0$, $M_1 \parallel M_2$. Breaks
    - $O(3)$ spin rotation symmetry
    - Z_2 translational symmetry

[1] R. M. Fernandes *et* al., PRB **93**, 014511 (2016). [2] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Magnetic phases of iron based superconductors

- Three different types of magnetic order

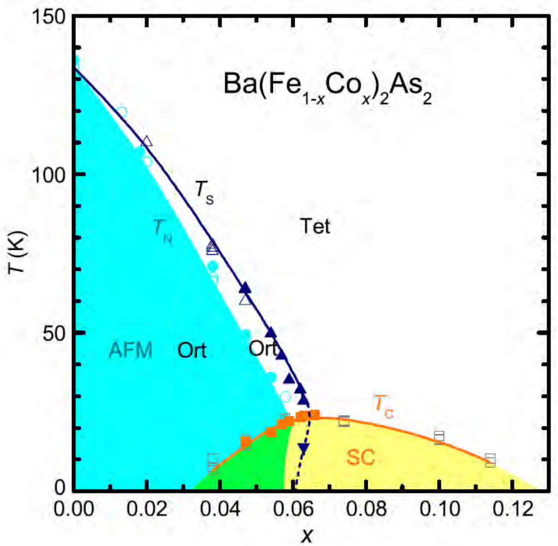


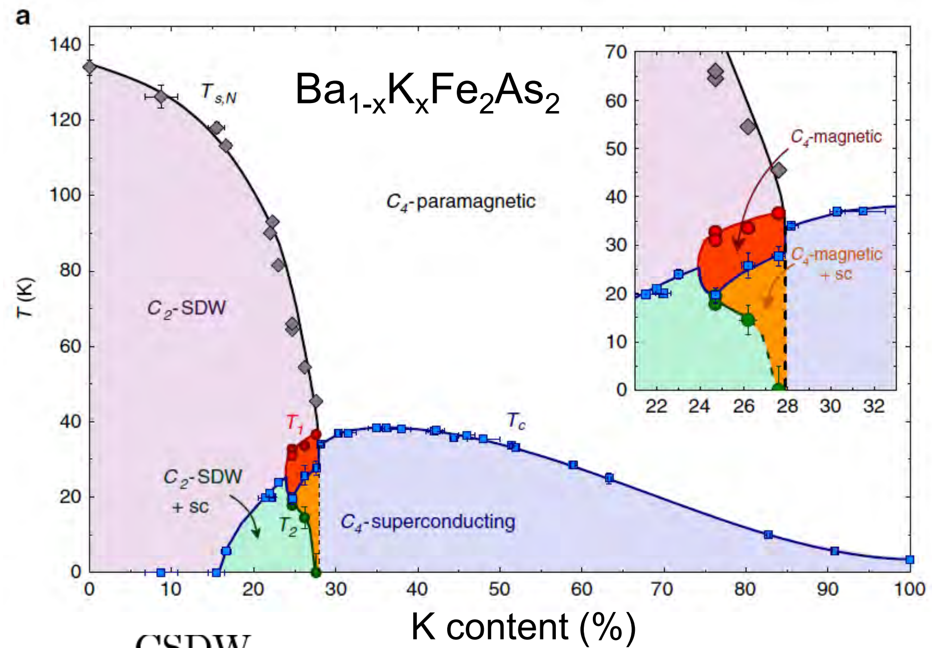
- Stripe spin-density wave (SSDW)
  - M_1 or $M_2 \neq 0$. Breaks
    - $O(3)$ spin rotation symmetry
    - $C_4 \rightarrow C_2$ crystal symmetry
- Charge-spin density wave (CSDW)
  - M_1 and $M_2 \neq 0$, $M_1 \parallel M_2$. Breaks
    - $O(3)$ spin rotation symmetry
    - Z_2 translational symmetry
- **Spin-vortex crystal (SVC)**
  - M_1 and $M_2 \neq 0$, $M_1 \perp M_2$. Breaks
    - $O(3)$ spin rotation symmetry
    - $O(2)$ spin rotation symmetry

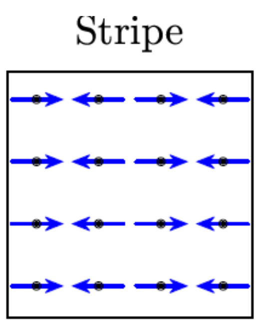
[1] R. M. Fernandes *et* al., PRB **93**, 014511 (2016). [2] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Magnetic phases of iron based superconductors

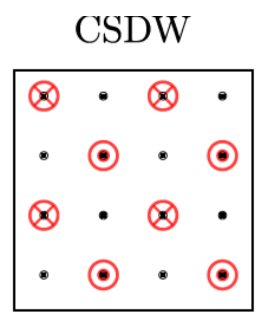
- Different materials described by different parameters u, g, w







- Most common order
- Orthorhombic



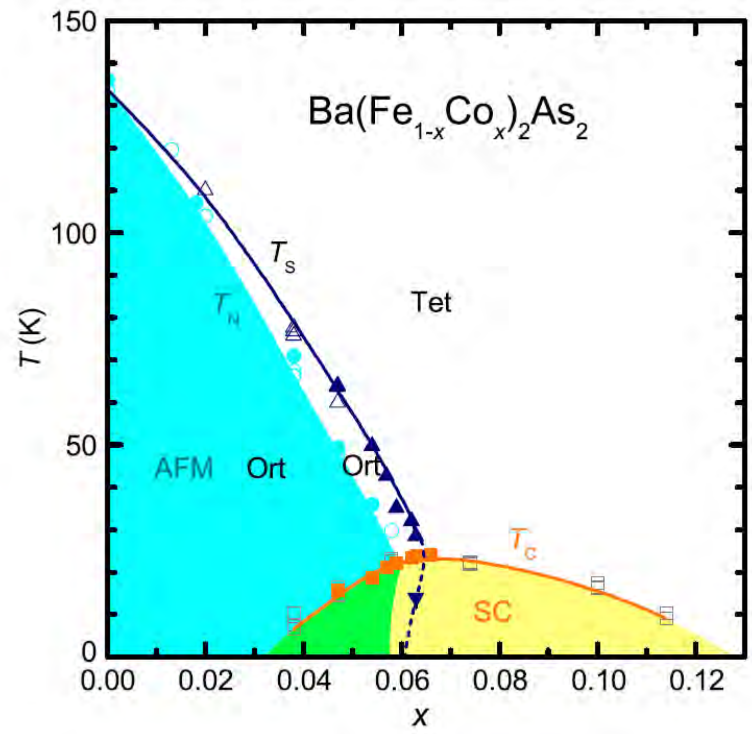
- u, g, w depend on doping
- Stripe magnetism turns into CSDW under doping

[1] S. Nandi *et al.*, PRL **104**, 057006 (2010); [2] A. E. Boehmer *et al.*, Nat. Comm. **6**, 7911 (2015).

Controlling and melting magnetic order

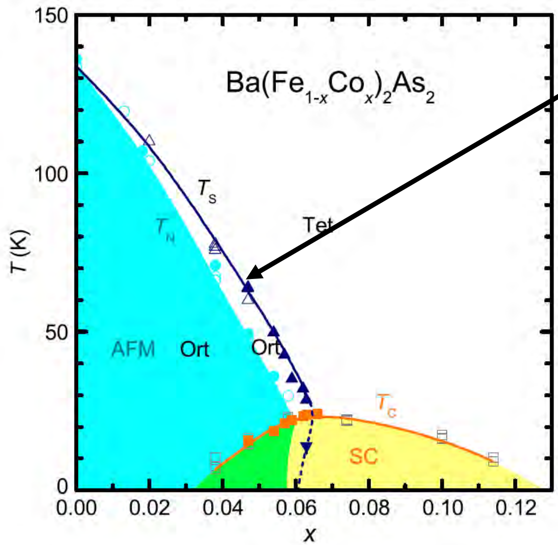
Two important questions arise:

- **How is symmetry restored as magnetic order melts?**
- **How can we control phases?**



Controlling and melting magnetic order

- **How is symmetry restored as magnetic order melts?**
  - One-step or two-step process?



Symmetry restored *via* two-steps: $T_N < T_S$

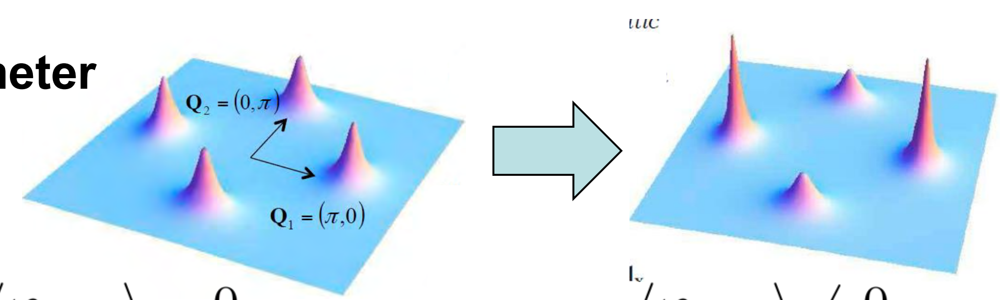
- Spin rotation symmetry $O(3)$ restored at T_N
- Tetragonal symmetry C_4 restored at T_S

Intermediate **nematic** phase

- No magnetic order
- **Only discrete Z_2 order ($C_4 \rightarrow C_2$)**
- Structural change has electronic origin

Composite, emergent Z_2 order parameter

$$\langle \varphi_{\text{nem}} \rangle = g \langle \boldsymbol{M}_X^2 - \boldsymbol{M}_Y^2 \rangle$$



$\langle \varphi_{\text{nem}} \rangle = 0$ $\langle \varphi_{\text{nem}} \rangle \neq 0$

[1] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012);

Emergent orders exist for all three magnetic phases

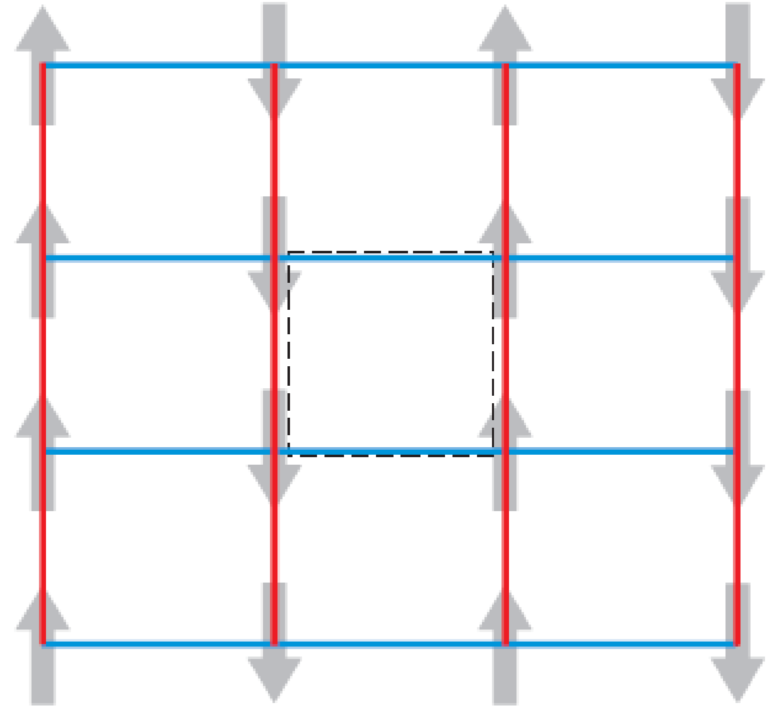
Stripe magnetic order

Melting of $O(3)$

Nematic Z_2 order

$$\phi_{\text{nem}} = \langle \boldsymbol{M}_1^2 - \boldsymbol{M}_2^2 \rangle$$

- x and y-bonds inequivalent
- Orthorhombic: only C_2 rotation symmetry
- Melts via (separate) 1st or 2nd order transition
- Realized in $Ba(Fe_{1-x}Co_x)_2As_2$



[1] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012); [2] R.M. Fernandes *et al.*, PRB **93**, 014511 (2016); [3] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Emergent orders exist for all three magnetic phases

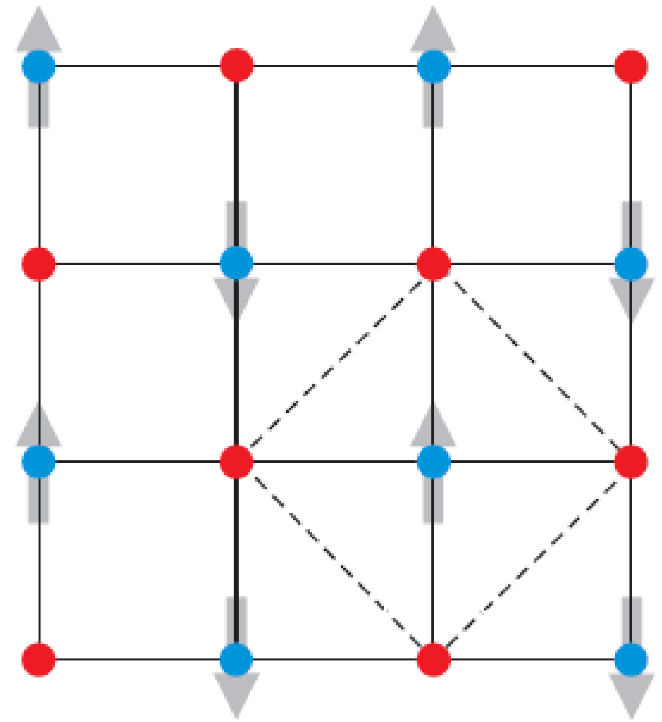
Charge-spin density wave order (CSDW)

Melting of $O(3)$

Charge-density wave Z_2 order

$$\phi_{\text{CDW}} = \langle \boldsymbol{M}_1 \cdot \boldsymbol{M}_2 \rangle$$

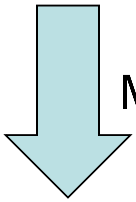
- 2 types of sites inequivalent
- Tetragonal: C_4-symmetric
- Realized in $Ba_{1-x}K_xFe_2As_2$



[1] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012); [2] R.M. Fernandes *et al.*, PRB **93**, 014511 (2016); [3] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Emergent orders exist for all three magnetic phases

Spin-vortex crystal order (SVC)

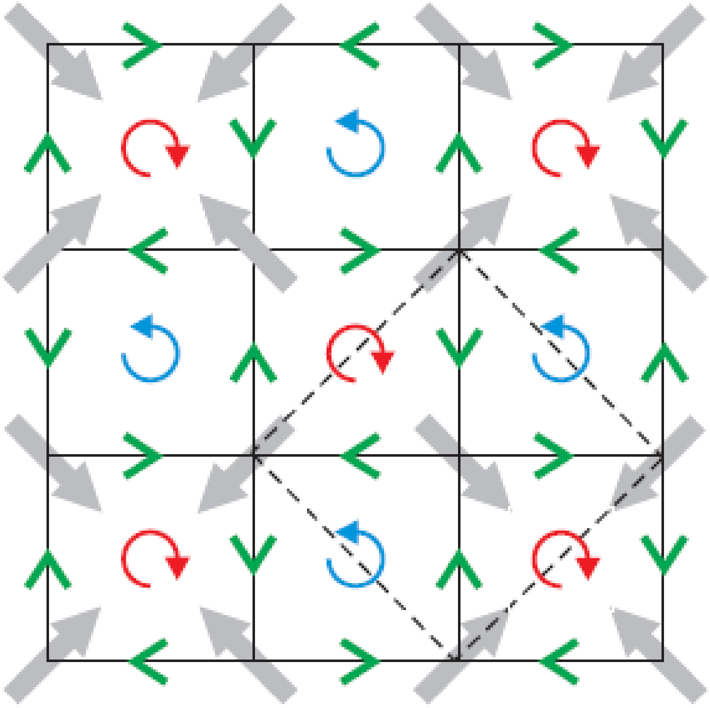


Melting of $O(3)$

Spin-vorticity density wave order (SVDW)

$$\phi_{\mathrm{SVDW}} = \langle \boldsymbol{M}_1 \times \boldsymbol{M}_2 \rangle$$

- Tetragonal: C_4 symmetric
- O_2 order without SOC
- Z_2 order if moments are fixed to certain plane
- Generates staggered electric field on plaquettes



How to realize?

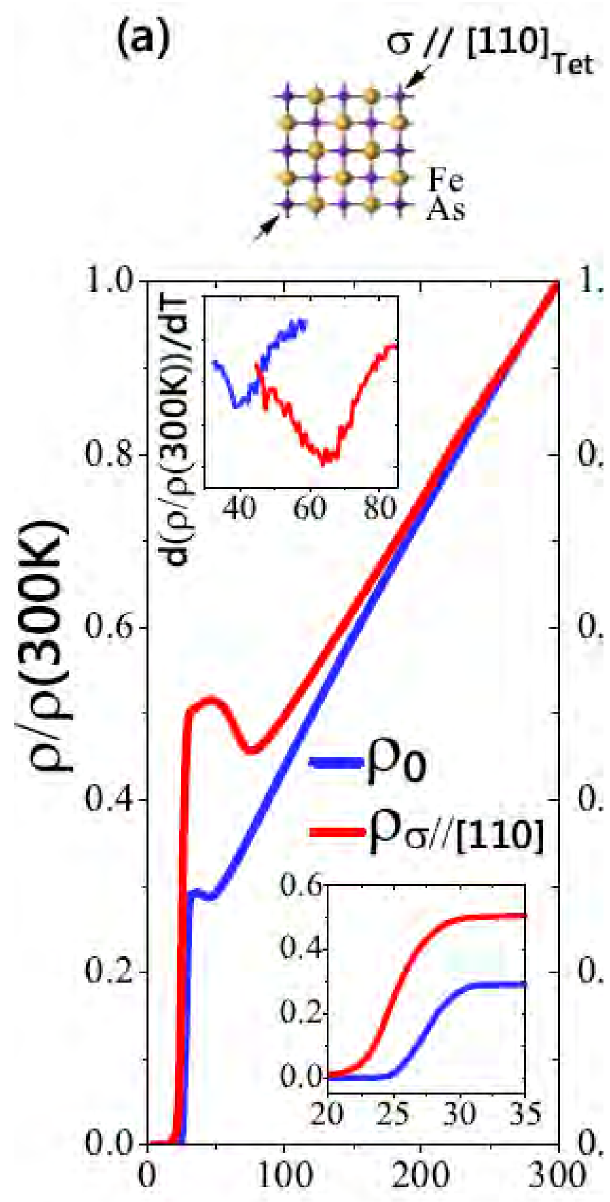
[1] R. M. Fernandes *et al.*, PRB **85**, 024534 (2012); [2] R.M. Fernandes *et al.*, PRB **93**, 014511 (2016); [3] M. H. Christensen, PPO, B. M. Andersen, R. M. Fernandes, (to be submitted, 2017).

Controlling magnetic order

- **How can we control phases?**
  - Doping, pressure, magnetic field
  - Other possibilities

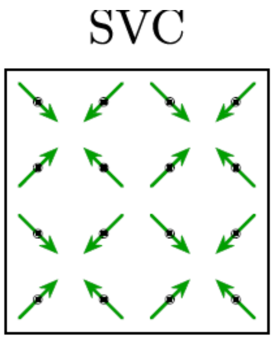
Coupling to emergent order parameter

- Apply external strain σ to cause orthorhombic distortion
- Acts as “conjugate field” for emergent order parameter $\phi_{nematic}$
- $\Delta F = \sigma\phi_{nematic}$
- Transition temperature T_N to stripe order increases (27K in example)

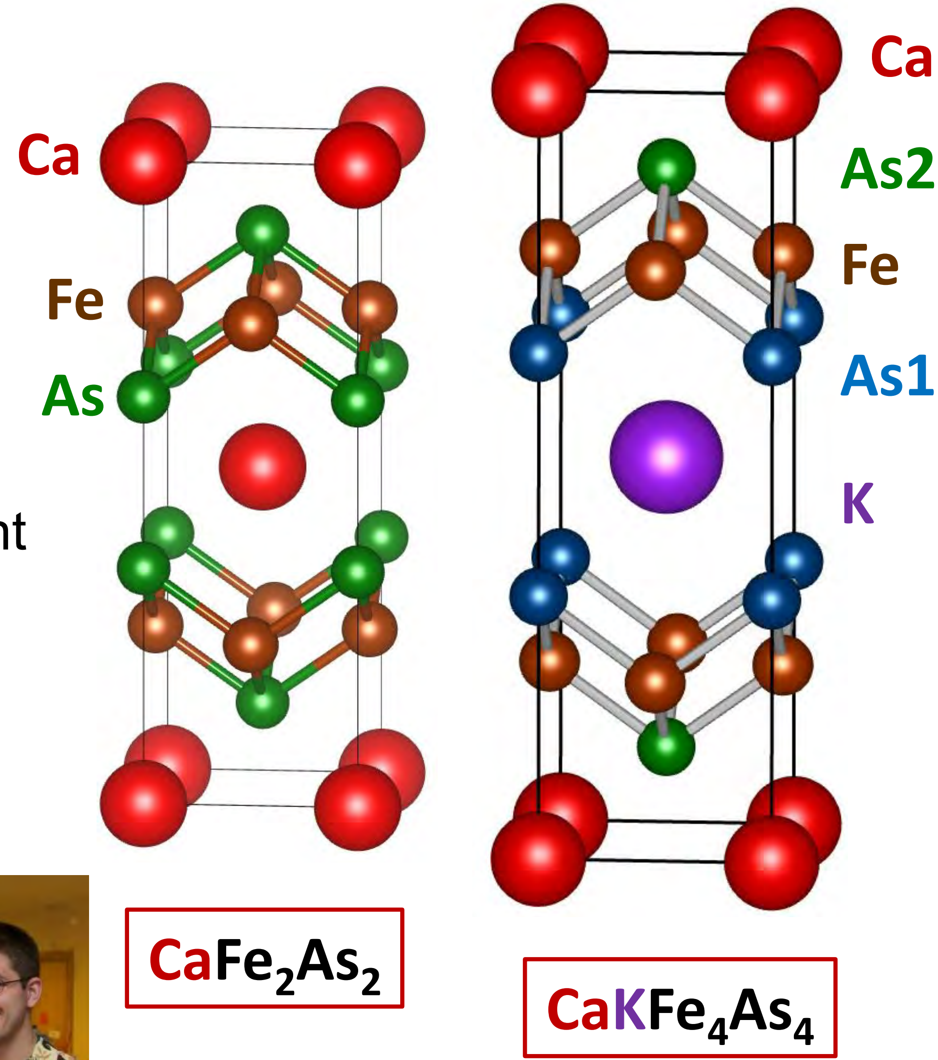


$CaKFe_4As_4$: coupling to SVDW emergent order

Task: **generate Spin Vortex Crystal magnetic order?**



- Realize conjugate field to emergent SVDW order
- Two inequivalent As sites
- Breaks glide plane symmetry
- Lowers SVC magnetic state to be ground state

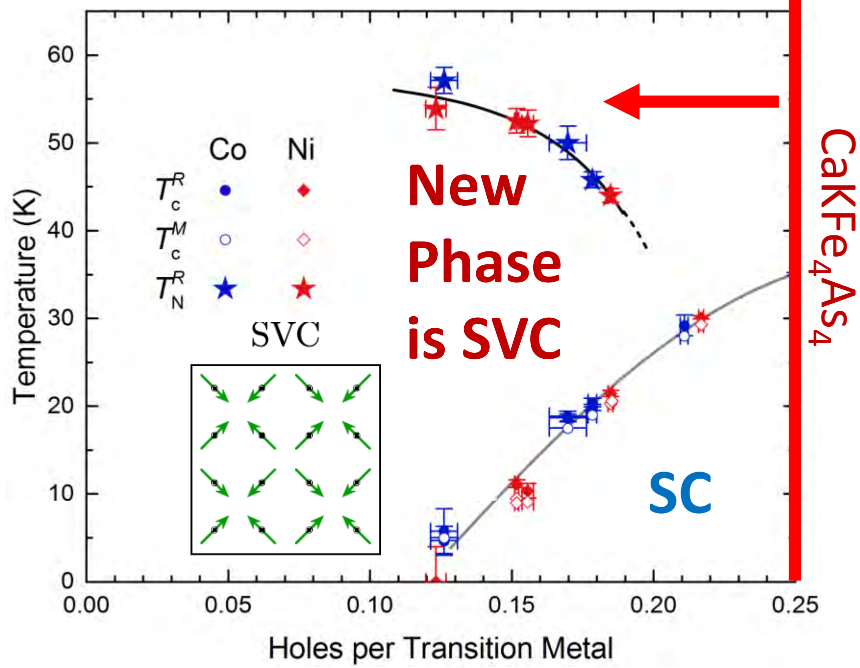


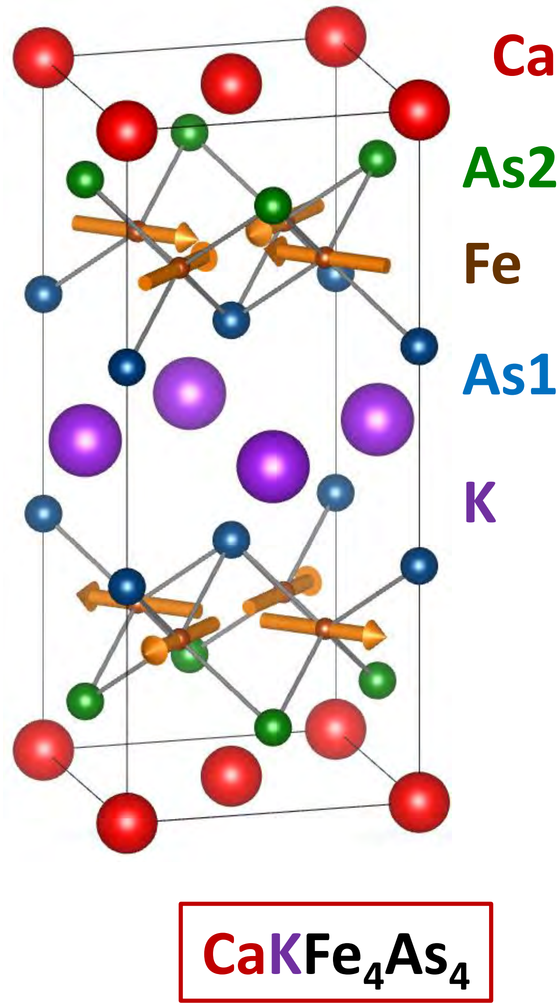
P. C. Canfield

William Meier

$CaKFe_4As_4$: coupling to SVDW emergent order

- Crystal structure generates conjugate field for emergent SVDW
- Lowers **SVC magnetic state**
- **First time SVC phase is experimentally realized!**



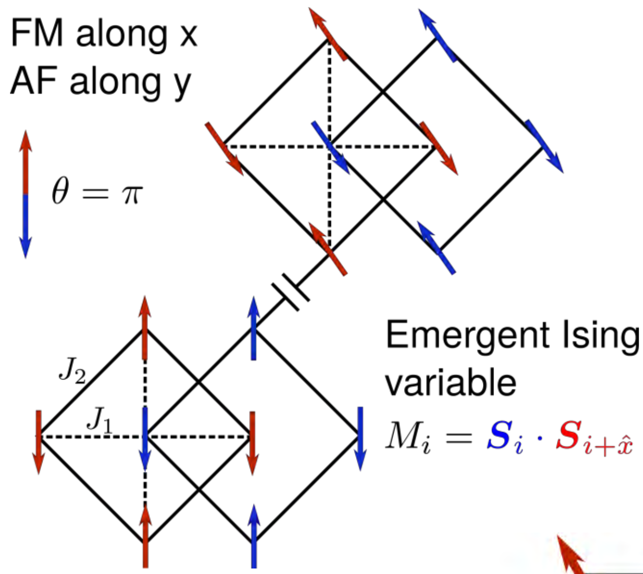


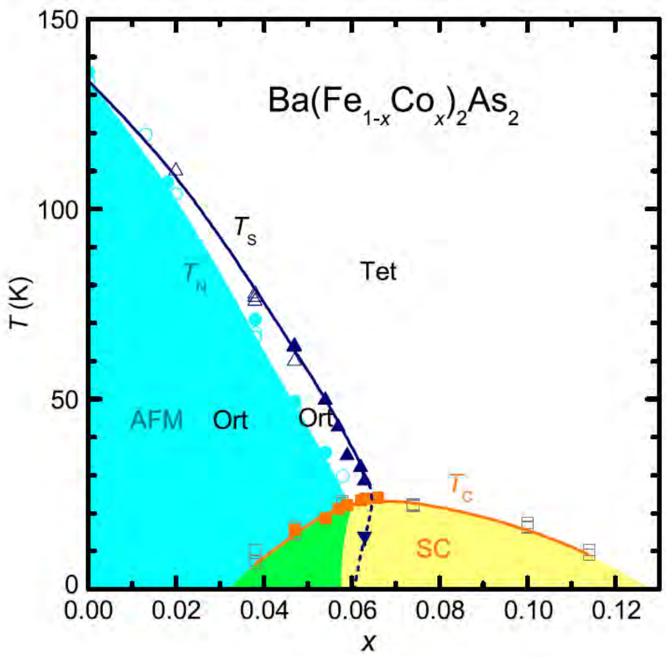
W. R. Meier, …, PPO, ..., P. C. Canfield, arXiv:1706.01067 (2017).

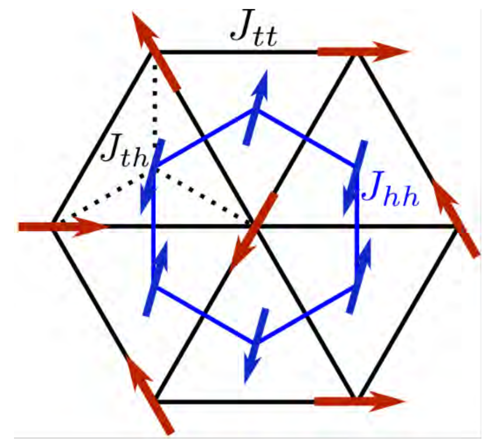
EMERGENT ORDER IN MICROSCOPIC SPIN MODELS

Microscopic spin model: J1-J2 model

- J1-J2 model: application to iron based SC
- Windmill model







Why do thermal phase transitions occur?

- Equilibrium state is minimum of free energy $F = E - TS$
- Competition between internal energy E and entropy S

Example: One-dimensional **Ising model** describing interacting spins

Hamiltonian: J > 0, ferromagnetic nearest-neighbor interaction

$$H = -J \sum_{\langle i,j \rangle} S_i S_j \quad , S_i = \pm 1$$

At T = 0: ground state is all spins aligned (minimal energy E)

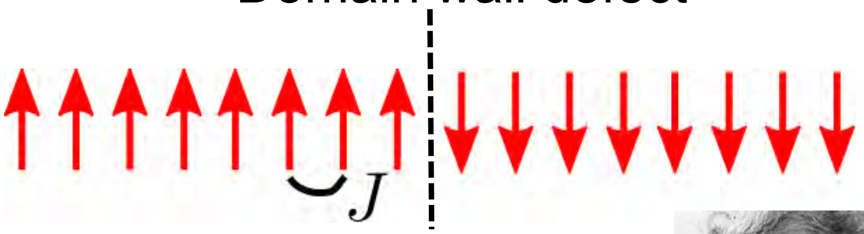
[1] E. Ising, Z. Phys. **31**, 253 (1925).

Why do thermal phase transitions occur?

- Equilibrium state shows minimum of free energy $F = E - TS$
- Competition between internal energy E and entropy S

<u>Example</u>: One-dimensional **Ising model**

$$H = -J \sum_{\langle i,j \rangle} S_i S_j \quad , S_i = \pm 1$$



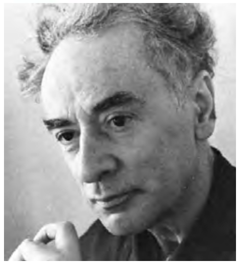
At finite T > 0: **Defects** can be thermally excited.
Question: how many?

Landau-Peierls argument: calculate free energy of free defect

$$E = 2J$$
$$S \simeq k_B \log N$$

$$F \simeq 2J - k_B T \log N \xrightarrow[N\to\infty]{} -\infty$$

Free energy reduced by generating defects.
Proliferation of defects destroys order at T>0.



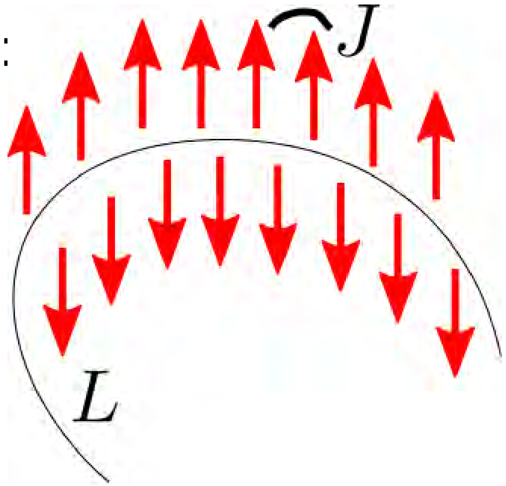
R. Peierls (1907 – 1995)

Order in two-dimensional Ising spin models

- **Two-dimensional** Ising model:

$$E \simeq JL$$
$$S \simeq k_B \log(z-1)^L$$

$$F \simeq L\big[J - k_B T \log(z-1)\big]$$



Domain wall defect of length L

Identical scaling of energy and entropy!

- Low T: Defect raises free energy
- High T: Defect lowers free energy

Defects proliferate above critical T_c: Phase transition (Ising universality).

[1] L. Onsager Phys. Rev. II **85**, 808 (1944).

Continuous spin models in two dimensions

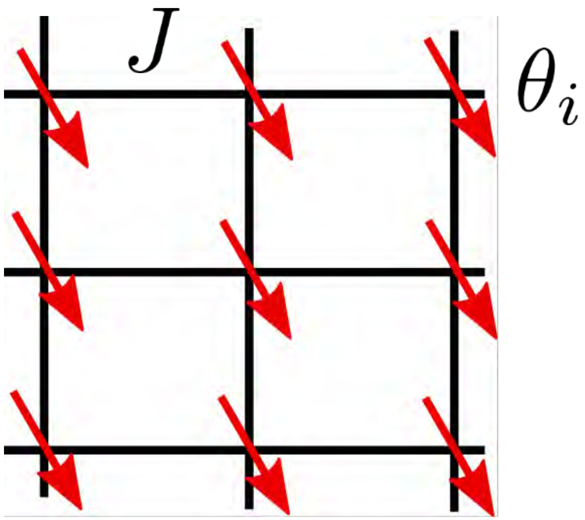
- Continuous spins with N components

$$\boldsymbol{S} = (\cos(\theta_0 + \theta_x), \sin(\theta_0 + \theta_x))$$

N=2: XY (or planar) spin
N=3 = Heisenberg spin

- Hamiltonian for N=2 planar spins

$$H = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$



Ferromagnetic order at zero temperature for both classical and quantum spins (no frustration)

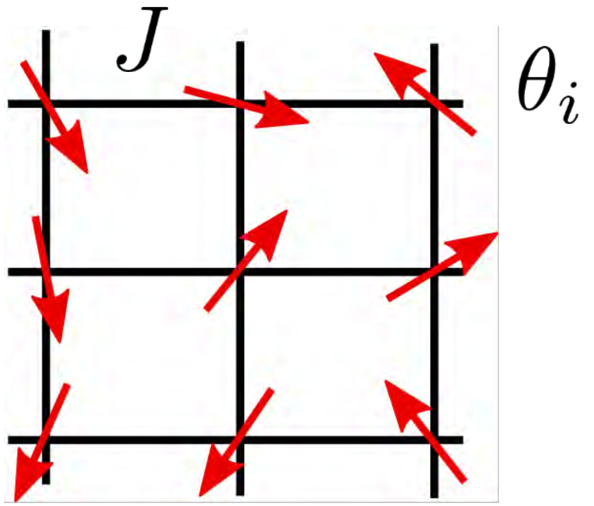
T=0 ground state (FM ordered)

Continuous spin models in two dimensions

Long-wavelength Hamiltonian (gradient energy)

$$H = \frac{J}{2}\int d^2x(\nabla\theta_x)^2 = \frac{J}{2}\int \frac{d^2q}{(2\pi)^2} q^2|\theta_q|^2$$

T > 0 state:



Thermal fluctuations reduce magnetization:

$$\langle\cos\theta_x\rangle = \frac{1}{Z}\mathrm{Re}\int \mathcal{D}\theta_x e^{-\frac{J}{T}\int_x(\nabla\theta)^2 + i\int_x \theta_x} = e^{-\frac{1}{2}\langle\theta_x^2\rangle}$$

Debye-Waller factor infrared divergent:

$$\frac{1}{2}\langle\theta_x^2\rangle = \frac{T}{4\pi J}\int_{1/L}^{1/a}\frac{dq}{q} \propto \ln\frac{L}{a} \to \infty \text{ for } L\to\infty$$

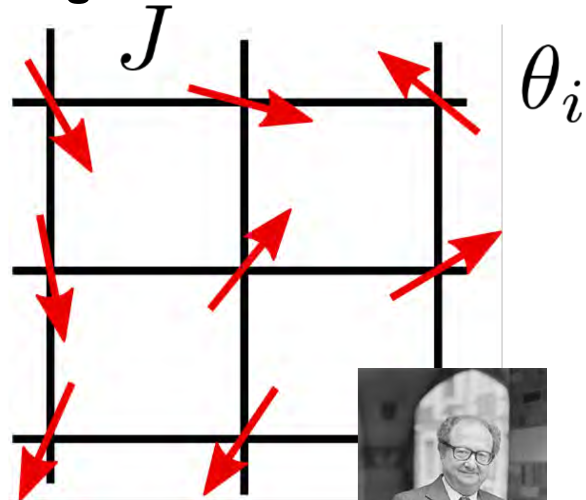
Thermal fluctuations melt order at any finite T!

Continuous spin models in two dimensions

Long-wavelength Hamiltonian (gradient energy)

$$H = \frac{J}{2}\int d^2x(\nabla\theta_x)^2 = \frac{J}{2}\int \frac{d^2q}{(2\pi)^2}q^2|\theta_q|^2$$

T > 0 state: magnetization vanishes



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$$\langle\cos\theta_x\rangle = \frac{1}{Z}\mathrm{Re}\int \mathcal{D}\theta_x e^{-\frac{J}{T}\int_x(\nabla\theta)^2+i\int_x\theta_x} = e^{-\frac{1}{2}\langle\theta_x^2\rangle}$$

Debye-Waller factor infrared divergent:

$$\frac{1}{2}\langle\theta_x^2\rangle = \frac{T}{4\pi J}\int_0^{1/a}\frac{dq}{q} \to \infty$$

Hohenberg-Mermin-Wagner theorem: No symmetry breaking of continuous degrees of freedom in $d \leq 2$ at any finite temperature.

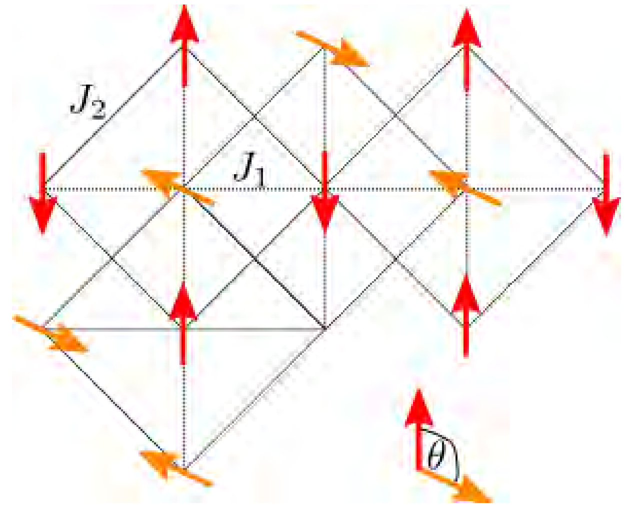
Emergent (discrete) order?

[1] N. D. Mermin, H. Wagner, PRL **17**, 1133 (1966); P. C. Hohenberg, PR **158**, 383 (1967).

J_1-J_2-Heisenberg model on square lattice

- J_1-J_2-Heisenberg model on square lattice

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$J_2 > J_1$

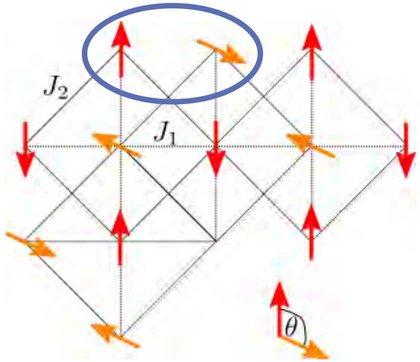
- For $J_2 > J_1$: Neel order parameters on both sublattices only coupled by fluctuations **(order from disorder)**
- At $T > 0$: Finite spin correlation length (Hohenberg-Mermin-Wagner theorem)

$$\xi(T) \sim a_0 e^{2\pi J S^2 / T}$$

[1] J. Villain, J. Phys. Fr **38**, 385 (1977); [2] C. L. Henley, PRL **62**, 2056 (1989); [3] P. Chandra, P. Coleman, A. I. Larkin, PRL **64**, 88 (1990); [4] C. Weber *et al.*, PRL **91**, 177202 (2003);

Order from disorder

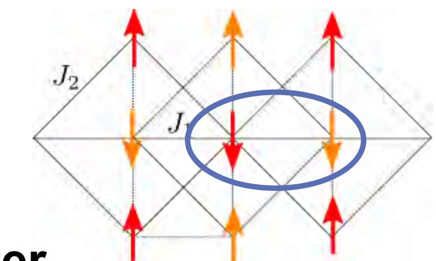
- Fluctuation free energy [1] due to "order from disorder"



$$\delta F = -E(T)[1+\cos^2\theta] \quad \text{minimized for } \theta = 0, \pi$$

with $E(T) = \dfrac{J_1 S^2}{2J_2}\left(\gamma_Q \dfrac{1}{S} + \gamma_T \dfrac{T}{J_2 S^2}\right)$

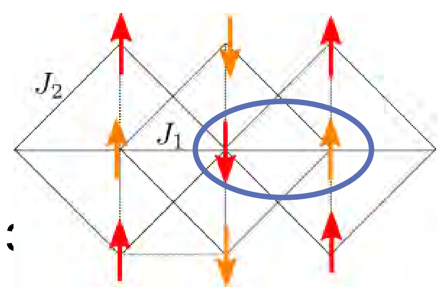
Spins tend to align the fluctuating Weiss' field of the neighbors to their easy plane [3].



$$m_\alpha = +1$$
$$Q = (0, \pi)$$

Emergent discrete Ising $\mathbb{Z}_2$ order parameter

$$m_\alpha \sim S_1 \cdot S_2 = \pm 1$$

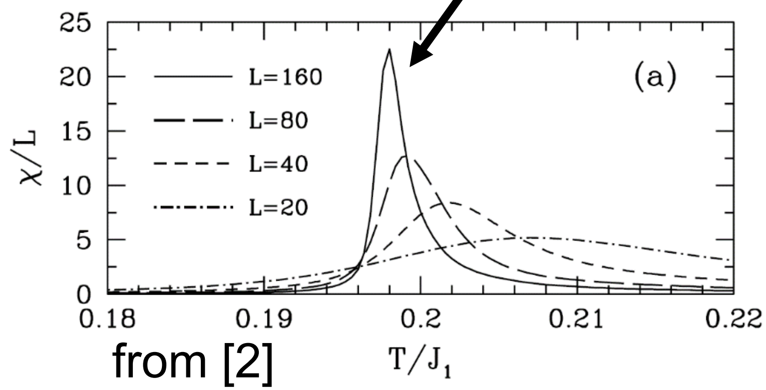


$$m_\alpha = -1$$
$$Q = (\pi, 0)$$

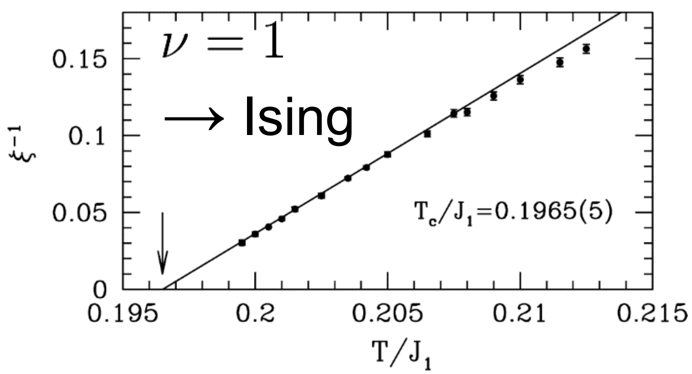
[1] P. Chandra et al., PRL **64**, 88 (1990); [2] J. Villain, J. Phys. Fr
[3] C. L. Henley, PRL **62**, 2056 (1989)

Emergent Ising order parameter in J_1-J_2-model

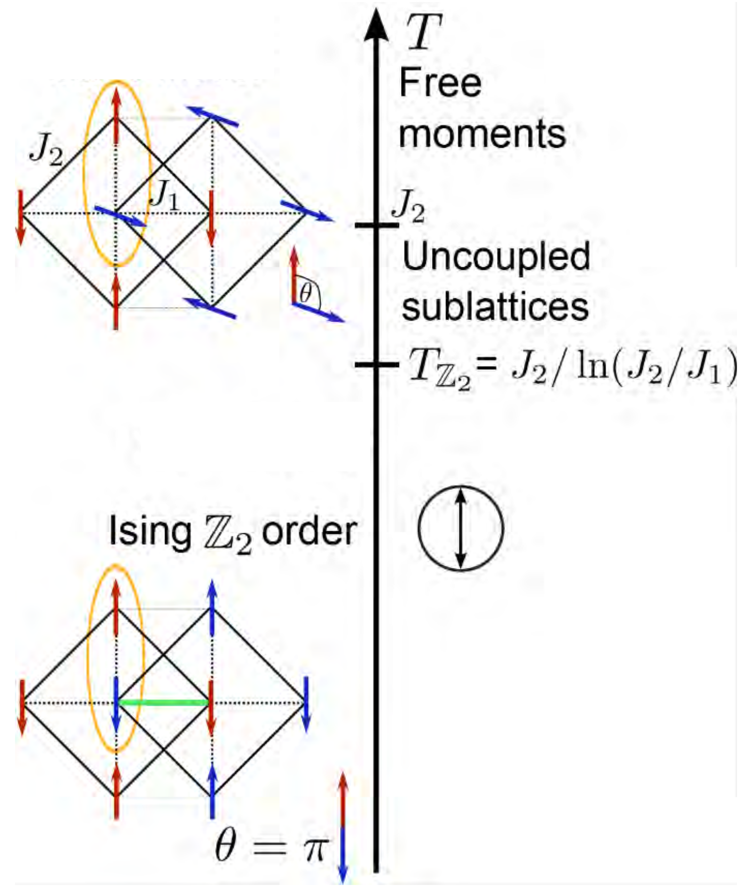
Finite temperature Ising phase transition



from [2]



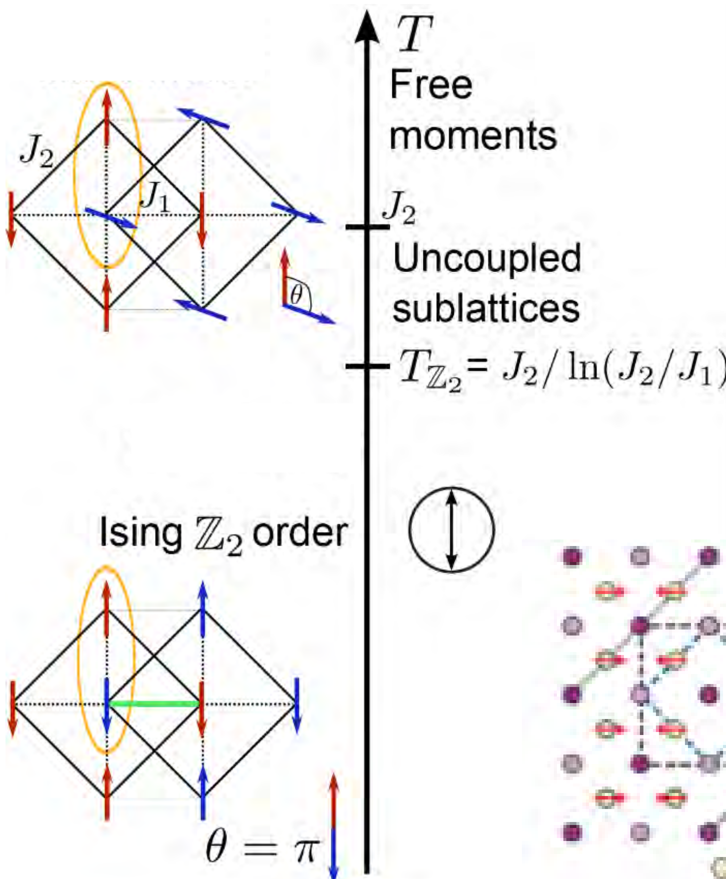
Phase diagram:



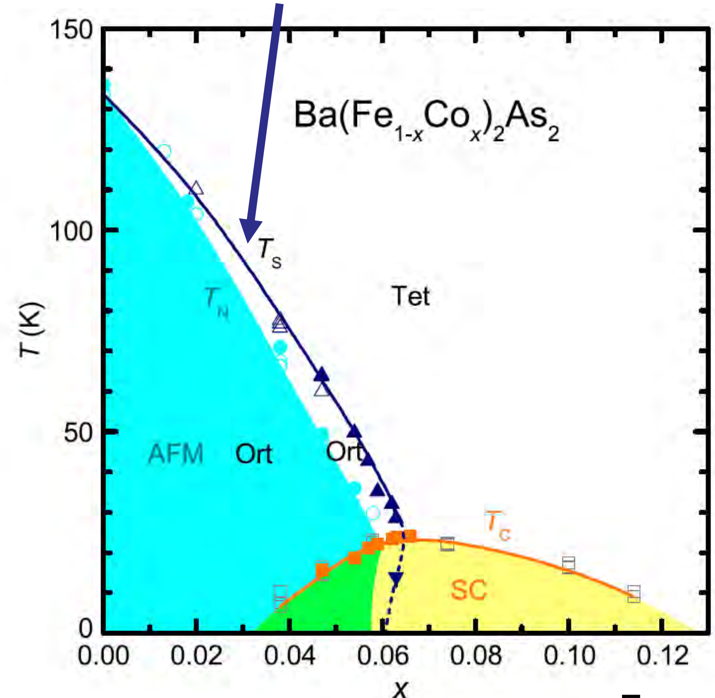
[1] P. Chandra, P. Coleman, A. I. Larkin, PRL **64**, 88 (1990); [2] C. Weber *et al*., PRL **91**, 177202 (2003); [3] R. M. Fernandes *et al*., PRL **105**, 157003 (2010);

Z_2 order drives structural transition

Phase diagram:



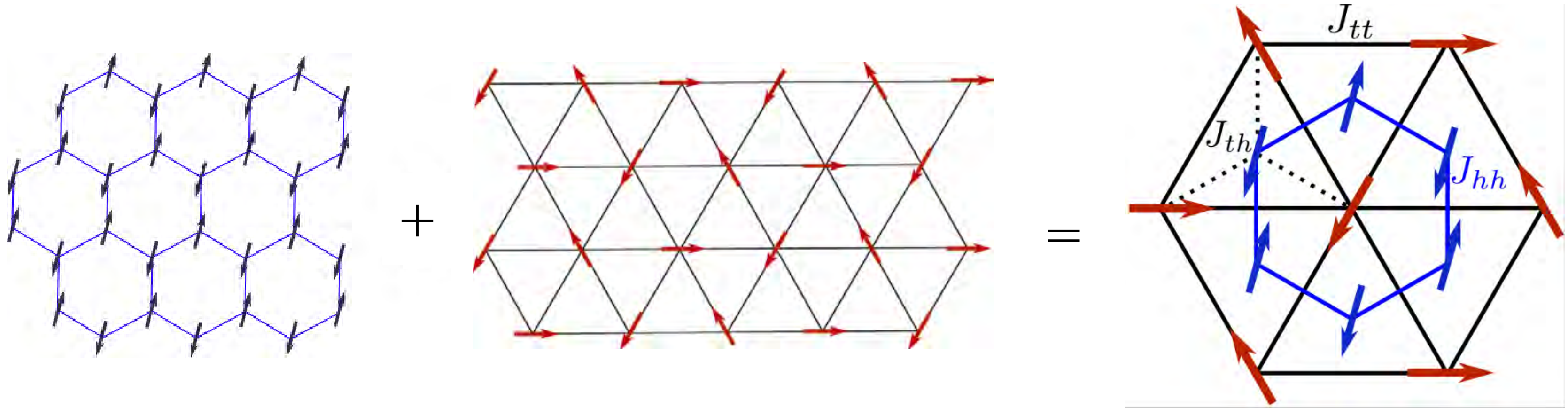
Discrete spin ordering induces structural transition.



From [3]

[1] R. M. Fernandes *et al.*, PRL **105**, 157003 (2010); [2] H. Luetkens, *et al.*, Nat. Mat. **8**, 305 (2009); [3] S. Nandi *et al.*, PRL **104**, 057006 (2010)

2D Heisenberg windmill antiferromagnet



- Honeycomb + triangular lattice sites
- Heisenberg spins $\boldsymbol{S}_t(r_j), \boldsymbol{S}_A(r_j), \boldsymbol{S}_B(r_j)$
- Antiferromagnetic nearest-neighbor coupling

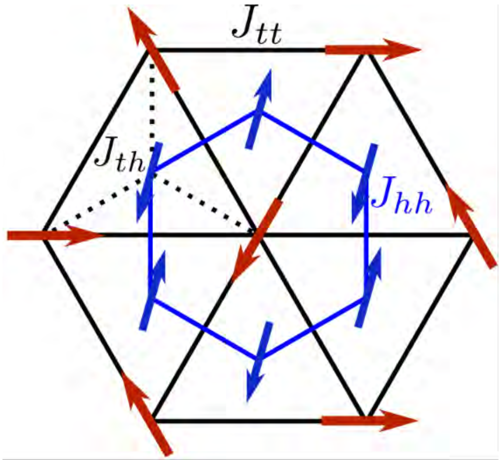
Windmill in Strangnaes (Sweden)

$$H = H_{tt} + H_{AB} + H_{tA} + H_{tB}$$

$$H_{ab} = J_{ab} \sum_{j=1}^{N_L} \sum_{\delta_{ab}} \boldsymbol{S}_a(r_j) \cdot \boldsymbol{S}_b(r_j + \delta_{ab})$$

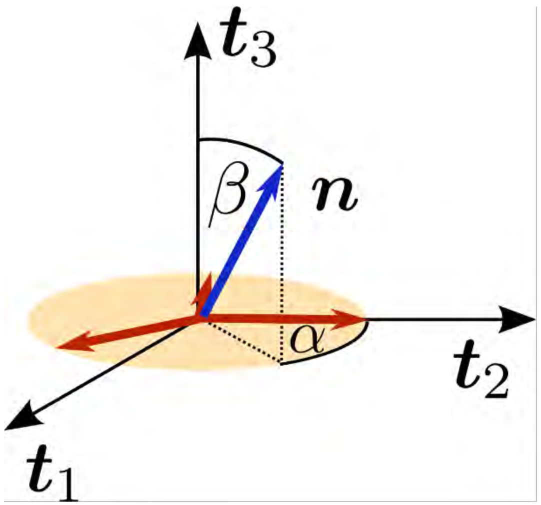
$$a, b \in \{t, A, B\}$$

Ground state of classical spins at small J_{th}



Weak inter-sublattice coupling

$J_{th} \ll J_{tt}, J_{hh}$



Neel order on **honeycomb lattice**

⇨ O(3)/O(2) order parameter $\boldsymbol{n}(\boldsymbol{x})$

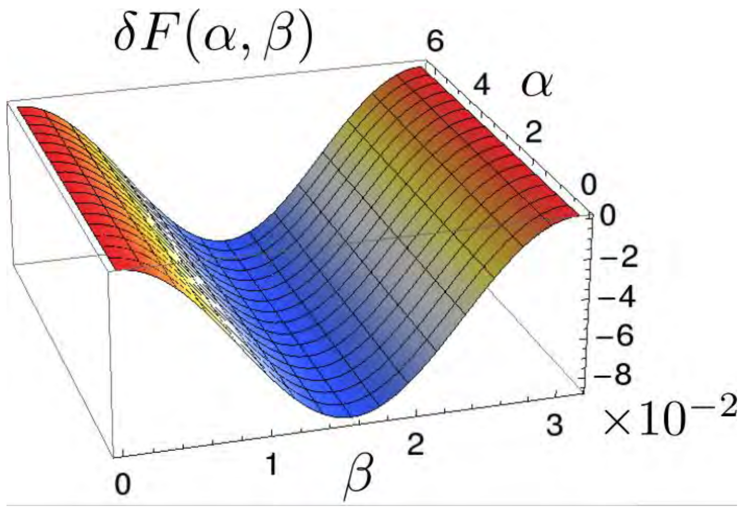
120 degree state on **triangular lattice**

⇨ SO(3) order parameter $t(\boldsymbol{x}) = (\boldsymbol{t}_1, \quad \boldsymbol{t}_2, \quad \boldsymbol{t}_3)$

Classically at T=0 decoupled even for $J_{th} > 0$

[1] B. Jeevanesan, PPO, PRB **90**, 144435 (2014).

Fluctuation coupling "order from disorder"

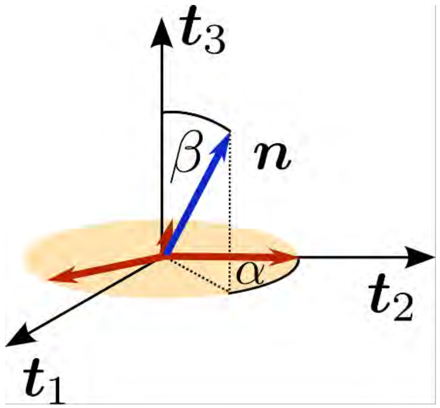


$$J_{th} = 0.4\bar{J}$$
$$\bar{J} = \sqrt{J_{tt}J_{hh}}$$
$$T = 1, S = 1$$

- Fluctuations couple spins on different sublattices
- Spins tend to align perpendicular to fluctuation Weiss field

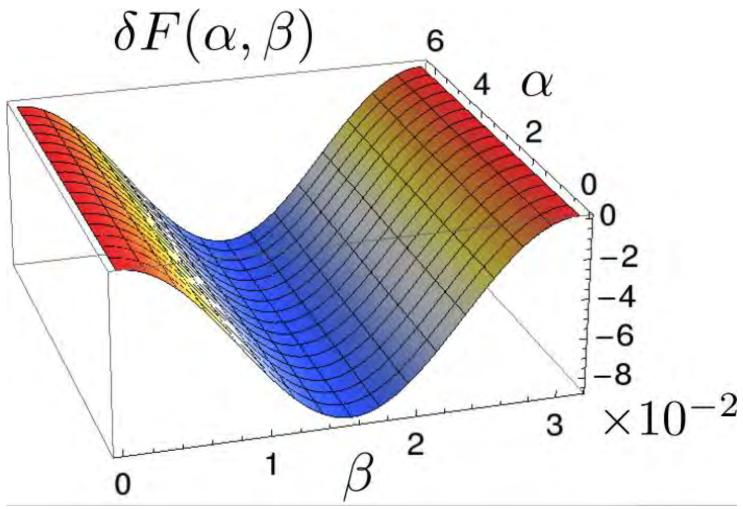
$$S_c = \frac{1}{2}\int d^2x\,(\gamma\cos^2\beta)$$

Coplanar: $\gamma = (J_{th}/\bar{J})^2 A_\gamma\left(J_{tt}/J_{hh}, \bar{J}/T\right)$



[1] C. L. Henley, PRL **62**, 2056 (1989)

Fluctuation coupling "order from disorder"



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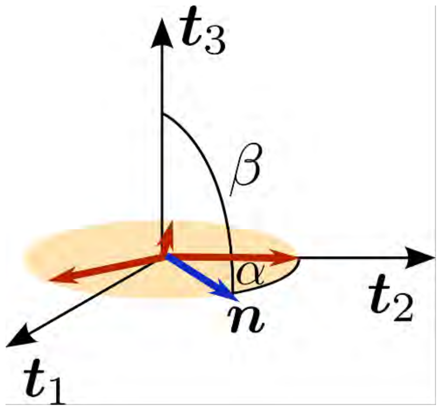

[1] C. L. Henley, PRL **62**, 2056 (1989)

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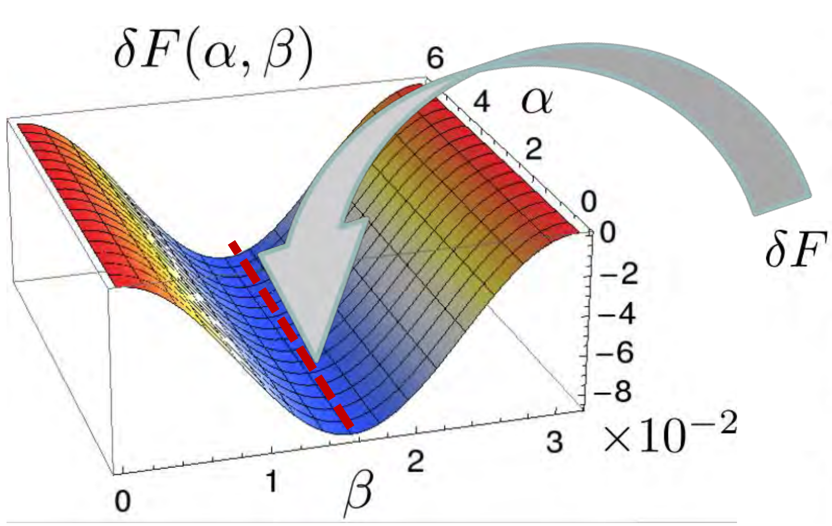
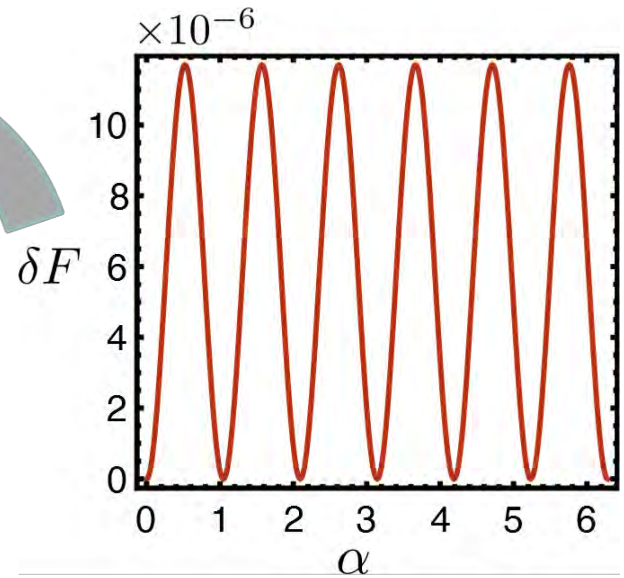


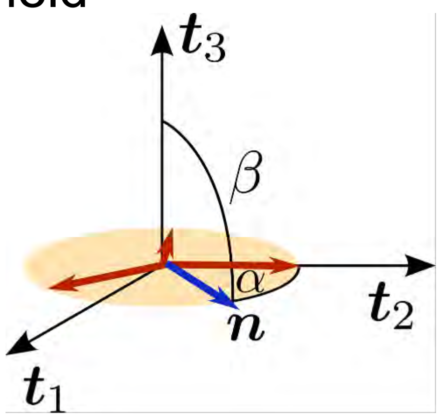


$J_{th} = 0.4\bar{J}$
$\bar{J} = \sqrt{J_{tt} J_{hh}}$
$T = 1, S = 1$

- Fluctuations couple spins on different sublattices
- Spins tend to align perpendicular to fluctuation Weiss field

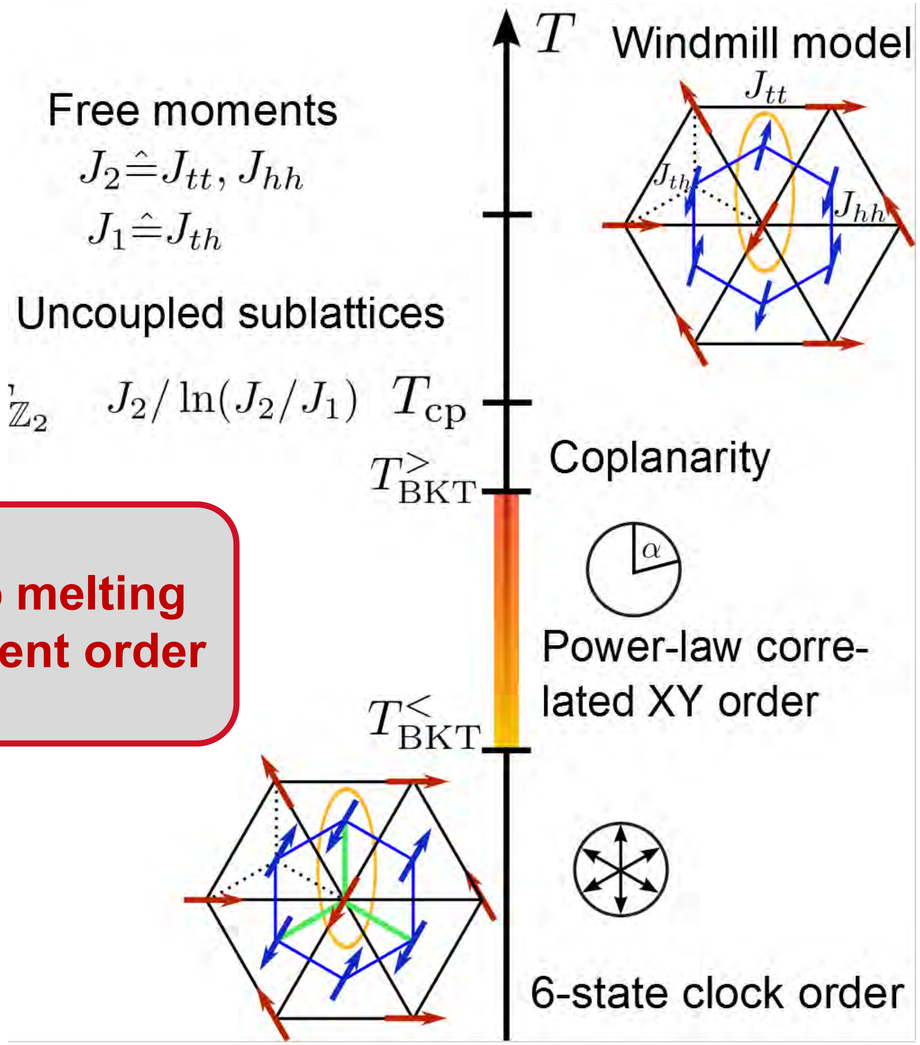
$$S_c = \frac{1}{2} \int d^2x \left(\gamma \cos^2 \beta + \lambda \sin^6 \beta \sin^2 (3\alpha) \right)$$

Coplanar: $\gamma \propto (J_{th}/\bar{J})^2$ Z_6: $\lambda \propto (J_{th}/\bar{J})^6$



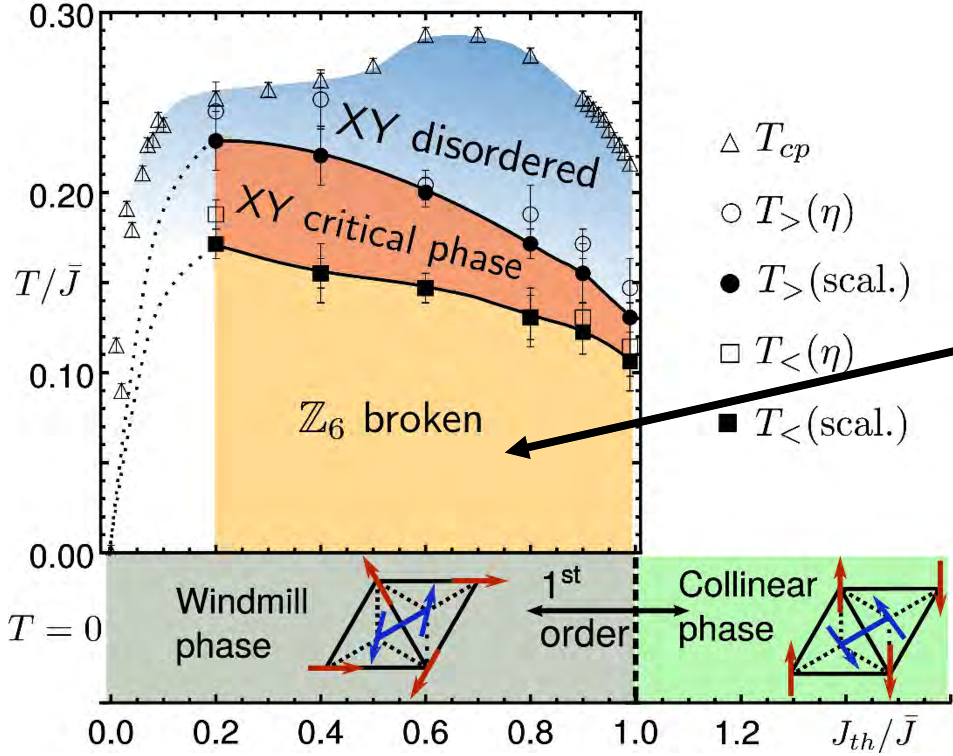
[1] C. L. Henley, PRL **62**, 2056 (1989)

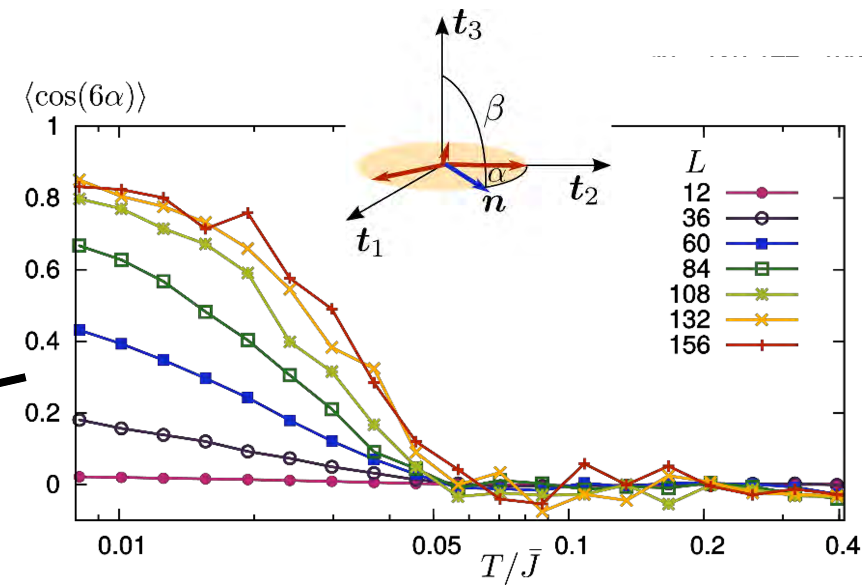
Phase diagram of windmill antiferromagnet



Two-step melting of emergent order

Classical Monte-Carlo simulation: phase diagram





System falls into one of the six minima
Long-range discrete order occurs.

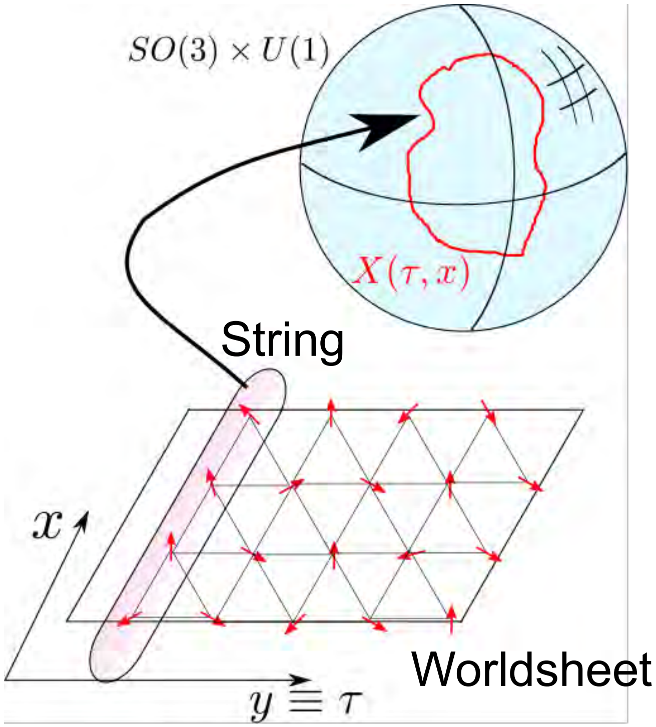
- Proof of Polykov's conjecture that critical phase can exist in Heisenberg system (due to topological vacuum degeneracy).

[1] B. Jeevanesan, P. Chandra, P. Coleman, PPO, Phys. Rev. Lett. **115**, 177201 (2015).

Long-wavelength covariant action

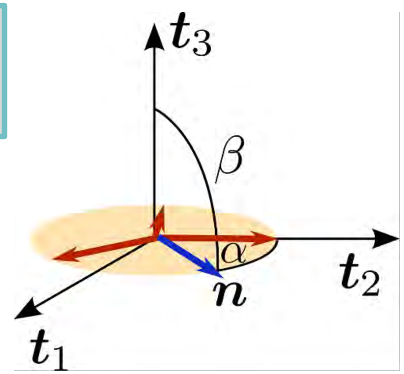
- **Long-wavelength action** of 2d spin system takes form of (Euclidean) **string theory** [1]

$$S = \frac{1}{2}\int d^2x\, g_{ij}[X(x)]\partial_\mu X^i(x)\partial_\mu X^j(x) + \frac{\lambda}{2}\int d^2x \sin^2(3\alpha)$$



3 Euler angles and relative phase

$$X(\tau, x) = (\phi, \theta, \psi, \alpha)$$



Magnetization X = displacement of string in D=4 (compact) dimensions

[1] D. Friedan, PRL **45**, 1057 (1980)

Magnetism as string theory

- Action of 2D spin system takes form of (Euclidean) string theory [1]

$$S = \frac{1}{2}\int d^2x\, g_{ij}[X(x)]\partial_\mu X^i(x)\partial_\mu X^j(x) + \frac{\lambda}{2}\int d^2x \sin^2(3\alpha)$$

- **Spin stiffnesses define metric tensor**

$$X(\tau, x) = (\phi, \theta, \psi, \alpha)$$

$$g = \begin{pmatrix} g^{SO(3)} & \mathcal{K}^T \\ \mathcal{K} & I_\alpha \end{pmatrix}$$

SO(3) stiffnesses I_1, I_2, I_3

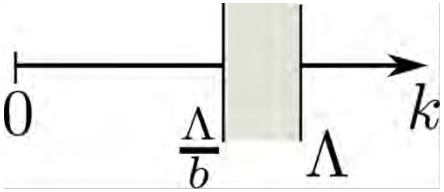
U(1) phase α is coupled to non-Abelian sector

U(1) stiffness

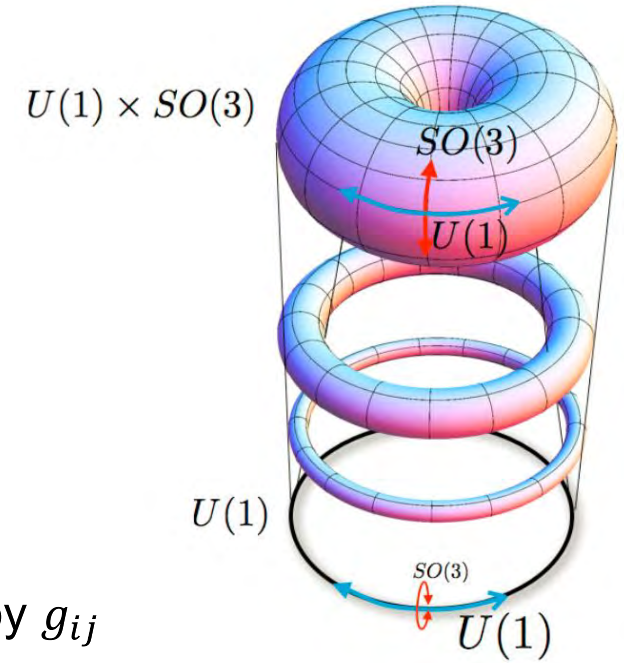
Geometric curvature of manifold (Riemann tensor) determined by spin stiffnesses.

Magnetism and gravity: RG flow = Ricci flow

- Action is covariant with stiffness metric tensor
- Covariance is preserved during RG scaling [1]
- **RG flow of the metric is given by the Ricci flow** [1,2] (two loops)



Integrating out fast momenta



- **RG flow of the metric is given by the Ricci flow** (two loops) [1,2]

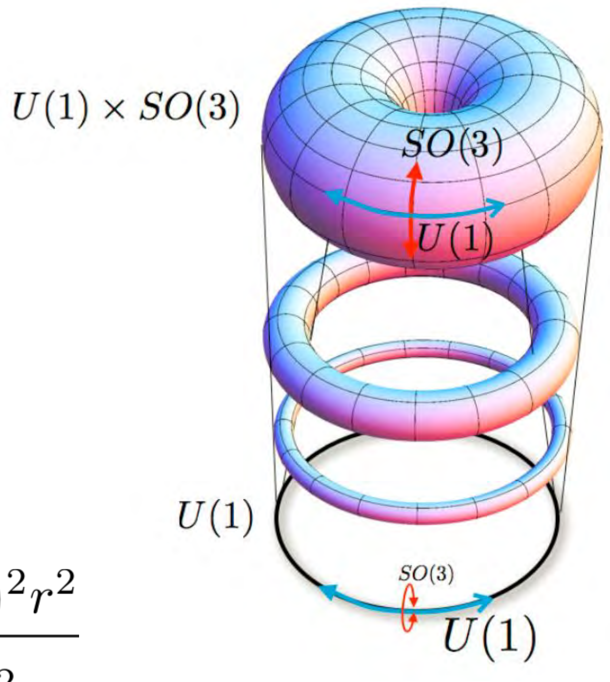
$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij} - \frac{1}{8\pi^2}R_i^{\ klm}R_{jklm}$$

Ricci and Riemann tensor determined by g_{ij}

[1] D. Friedan, PRL **45**, 1057 (1980); [2] R. S. Hamilton, J. Differential Geom. **17**, 255 (1982)

Compactification and magnetism

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij} - \frac{1}{8\pi^2}R_i{}^{klm}R_{jklm}$$



- One-dimensionsal U(1) part of manifold decouples from 3D non-Abelian SO(3) part
- Ricci scalar grows like

$$R = R^{SO(3)} - \frac{1}{2\pi I'_\alpha}\beta_\alpha$$

$$R^{SO(3)} \sim 1/\bar{I}$$

$$\beta_\alpha = \frac{(I_1 - I_2)^2 r^2}{4\pi I_1 I_2}$$

SO(3) part curles up

U(1) becomes flat

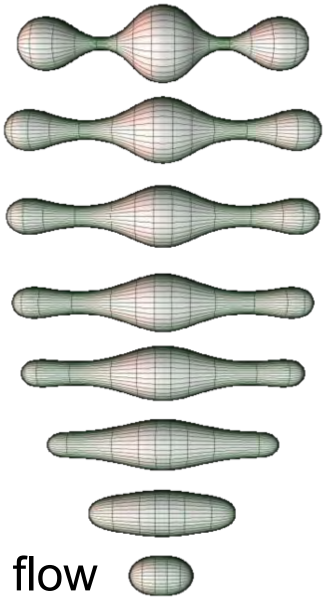
Toy model for compactification

[1] M. Gell-Mann and B. Zwiebach, Phys. Lett. B **141**, 333 (1984);
[2] L. Randall and R. Sundrum, PRL **83**, 4690 (1999)

Experimental proof of Poincare conjecture

- Poincare conjecture (1904), proven by Perelman in 2006
- "Every simply connected, closed 3-manifold is homeomorphic to a 3-sphere"
- Two-loop perturbative Ricci flow experiences singularities (false Landau poles). Not present in exact Ricci flow.
- **Use classical magnet to simulate exact Ricci flow. Experimental proof of Poincare conjecture.**
- Protocol:
  - Suitable magnet realizes given metric
  - Cool system
  - Measure spin correlation functions at various temperatures
  - Extract metric tensor
  - Obtain "surgery-free" generalized Ricci flow of manifold

G. Perelman (2006)
H. Poincare

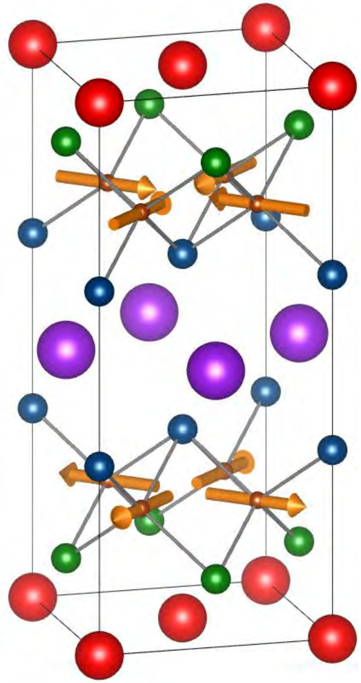


Ricci flow

[1] P. Coleman, A. Tsvelik (private communication)

Summary

- Emergent order leads to rich physical phenomena
- Occurs in various strongly correlated materials
- Iron-based SC
  - Explain origin of structural orthorhombic transition
  - Control magnetic phases via coupling to EO
- Magnetism and gravity: exact Ricci flow
- Magnetic toy model for compactification



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- PPO, P. Chandra, P. Coleman, J. Schmalian Phys. Rev. B **89**, 0994417 (2014).
  Phys. Rev. Lett. **109**, 237205 (2012).