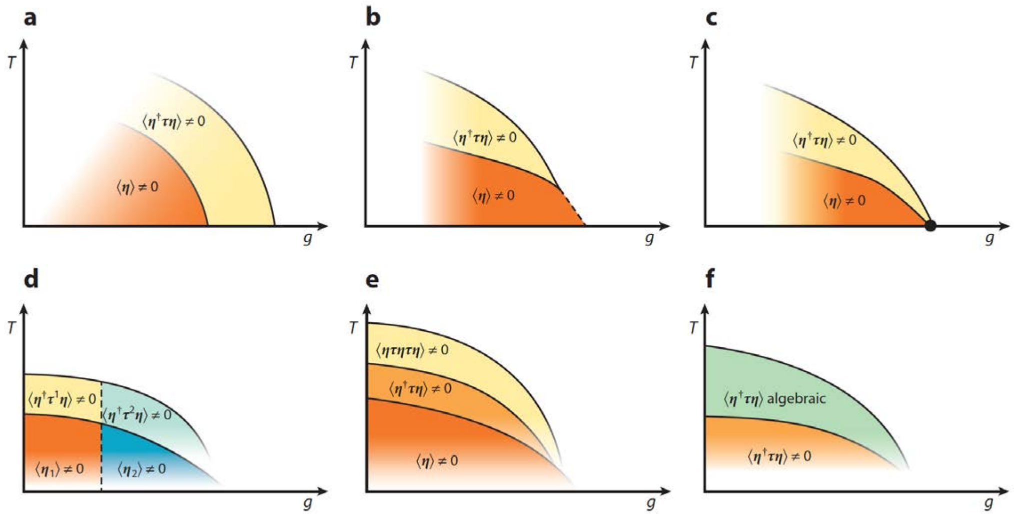
IOWA STATE UNIVERSITY

Department of Physics and Astronomy

Intertwined vestigial order in quantum materials: nematicity and beyond

Peter P. Orth (Iowa State University)

Condensed Matter Seminar, LANL, May 20, 2019



Reference:
R. M. Fernandes, PPO, J. Schmalian
Annu. Rev. Cond. Mat. **10**,133 (2019).

Collaborators

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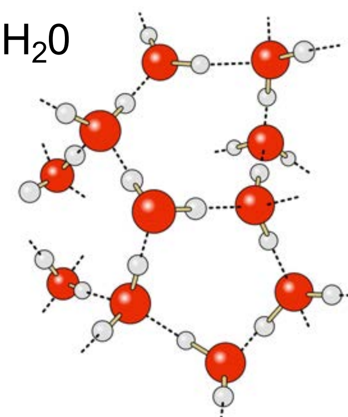
& his group

States of matter: symmetry, order and topology

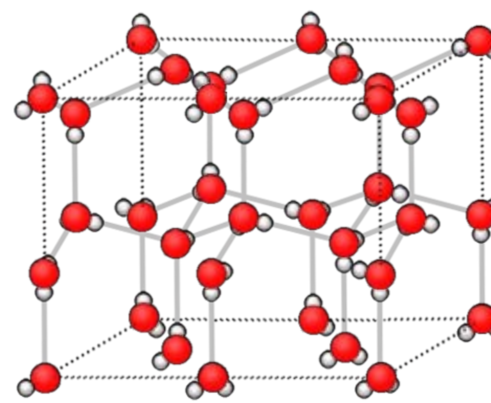
- Condensed matter physics investigates states of matter = phases
  - **Distinguish states by symmetry: Landau paradigm**
  - Distinguish states by topology

Water: H_20



Phase transition

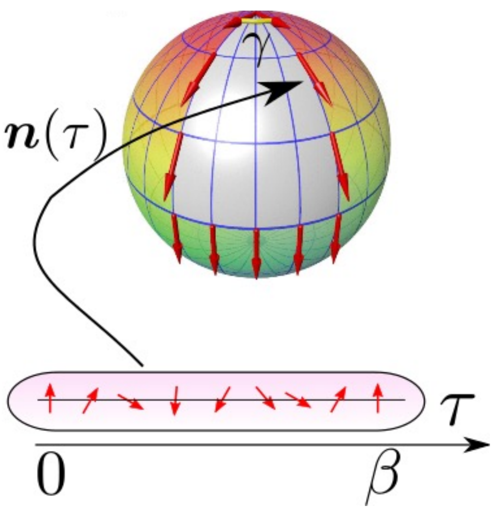
Breaking of continuous translational symmetry



States of matter: symmetry, order and topology

- Condensed matter physics investigates states of matter = phases
  - Distinguish states by symmetry: Landau paradigm
  - **Distinguish states by topology: Topological properties of electronic wavefunction**

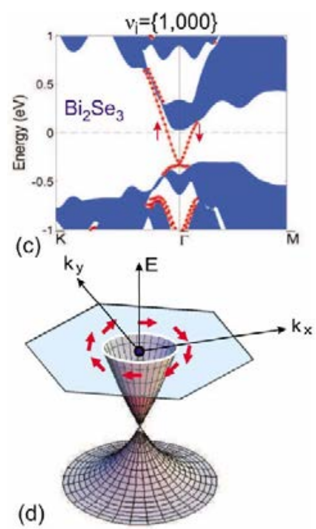
Geometric Berry phase



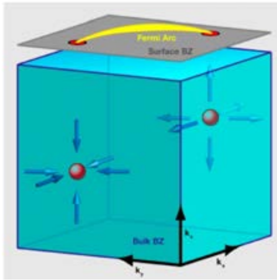
Topological insulators and metals

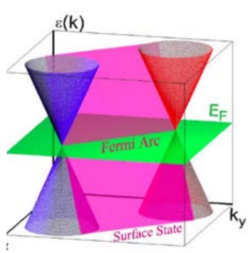
TI

Bi_2Se_3



Weyl SM

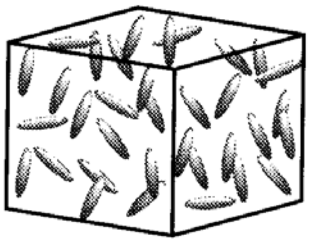




[1] M. Z. Hasan, C. L. Kane, RMP **82** 3045 (2010); N. P. Armitage *et al.*, arXiv:1705:01111 (2017).

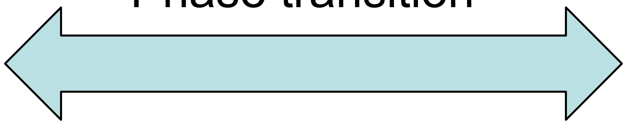
States of matter: symmetry and order

- Condensed matter physics investigates states of matter = phases
  - Liquids → Solids: translational symmetry breaking by crystalline order
  - Liquids → Nematics, Smectics: rotational symmetry breaking

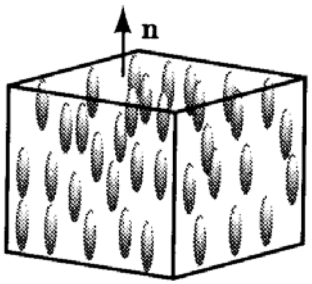


Liquid phase

Phase transition

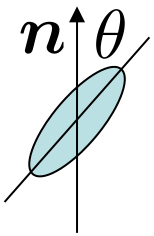


Liquid crystals



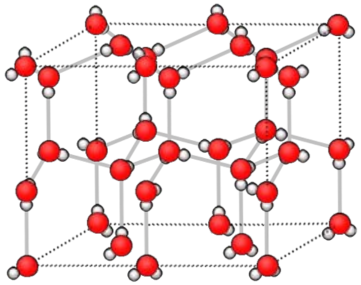
Nematic phase

Order parameter:

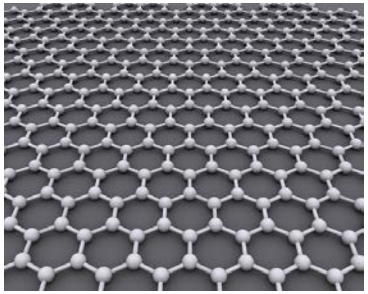


$$S = \langle \cos^2\theta - \frac{1}{3} \rangle \neq 0$$

- Solids break both continuous translational and rotational symmetries:
  - Discrete symmetries remain: 230 space groups, 32 point groups (in 3D)
  - Rigidity to shear
  - Goldstone modes
  - Quasi-crystals



Water ice: 3D crystal



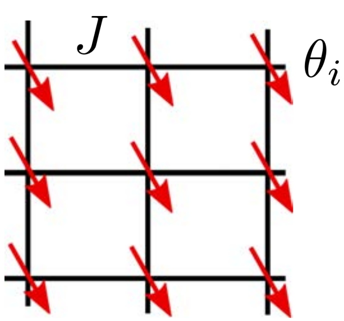
Graphene: 2D crystal

[1] Wikipedia; [2] P. M. Chaikin, T. C. Lubensky "Principles of Condensed Matter Physics

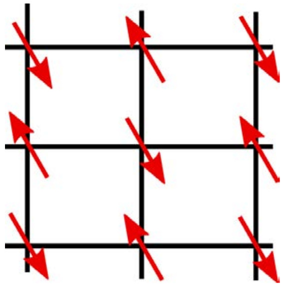
States of matter: symmetry and order

- Condensed matter physics investigates states of matter = phases
  - Spatial symmetry-breaking: crystals, amorphous solids, nematics
  - Internal symmetry-breaking:
    - Spin space $SU(2)$: magnetic order

Ferromagnet



Antiferromagnet



Chromium

Order parameters:

FM Magnetization: $\langle \boldsymbol{S} \rangle = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{S}_i$

AFM Magnetization: $\langle \boldsymbol{S} \rangle_{\boldsymbol{Q}=(\pi,\pi)} = \frac{1}{N} \sum_{i=1}^{N} e^{i\boldsymbol{Q}\cdot\boldsymbol{r}_i} \boldsymbol{S}_i$

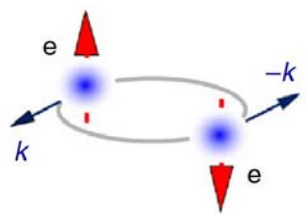
Skyrmion spin structure

[1] Wikipedia;

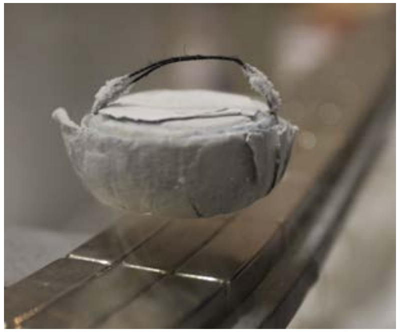
States of matter: symmetry and order

- Condensed matter physics investigates states of matter = phases
  - Spatial symmetry-breaking: crystals, amorphous solids, nematics
  - Internal symmetry-breaking:
    - Spin space $SU(2)$: magnetic order
    - Particle conservation global $U(1)$: superconductivity

Superconductivity



Cooper pair

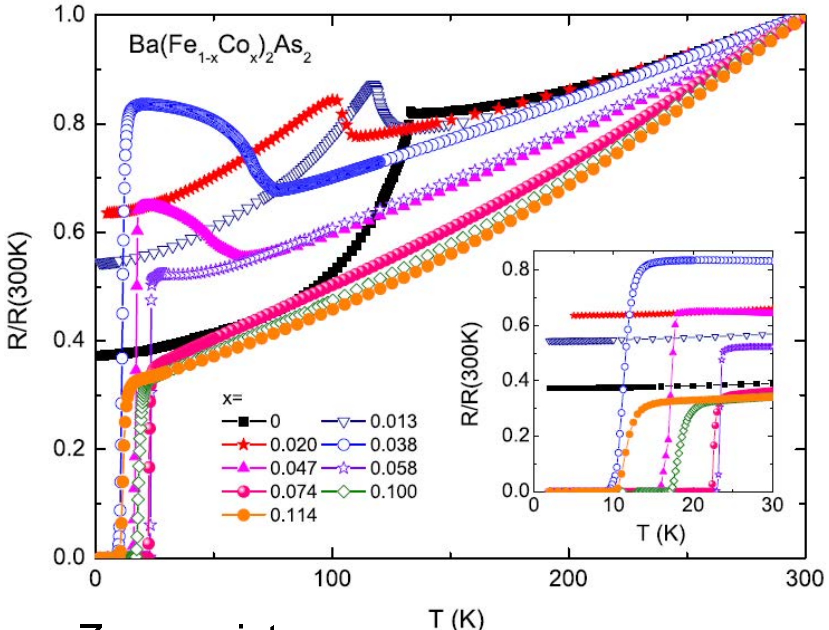


Meissner effect

Order parameter

$$\Delta = -V_0 \sum_{\boldsymbol{k}} \langle c_{-\boldsymbol{k},\downarrow} c_{\boldsymbol{k},\uparrow} \rangle$$

Superconducting gap function (s-wave singlet pairing state)



Zero resistance

[1] Wikipedia; [2] N. Ni *et al.*, PRB **78**, 214515 (2008).

States of matter: symmetry and order

- Condensed matter physics investigates states of matter = phases
  - Spatial symmetry-breaking: crystals, amorphous solids, nematics
  - Internal symmetry-breaking: magnets, superconductors

In all examples discussed so far:
Order occurs in elementary degrees of freedom: charge, spin.

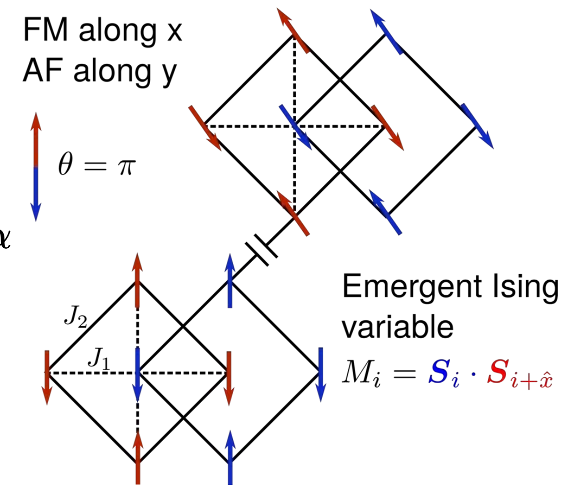
Order can also occur in composite objects such as higher order correlation functions (of charge and spin):

Composite order

$$\boxed{\langle \eta_\alpha^* \eta_\beta \rangle \neq 0}$$

For example, primary magnetic order

$$\eta_\alpha = \left(\langle \boldsymbol{S}_{\boldsymbol{Q}_1} \rangle, \langle \boldsymbol{S}_{\boldsymbol{Q}_2} \rangle \right)_\alpha$$



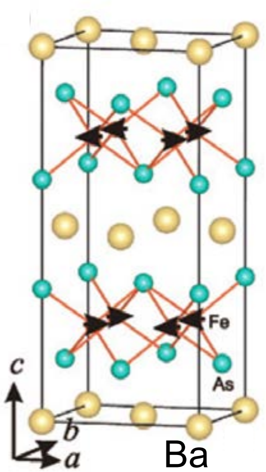
- ➢ Composite order then relative spin order
- ➢ But not all combinations break a symmetry

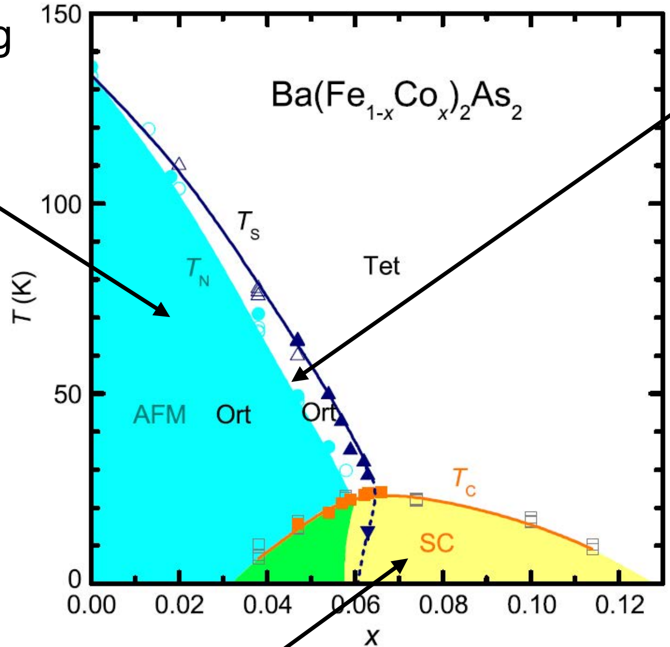
[1] R. M. Fernandes, PPO, J. Schmalian, Annu. Rev. Cond. Mat. **10**,133 (2019).

Example: iron-based superconductors

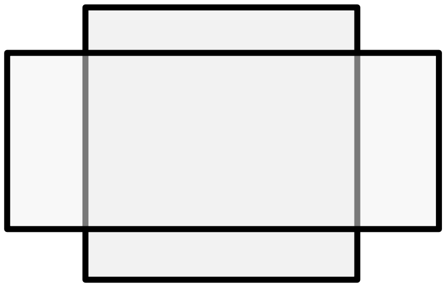
- Primary orders: magnetic and superconducting
- Vestigial order: nematic (122) and spin-vorticity density wave (1144) orders

Magnetic order (breaking of **spin rotational symmetry**)





Nematic order (**rotational symmetry** breaking from tetragonal C_4 to orthorhombic C_2)



Superconductivity (breaking of **global U(1) symmetry**)

Definition of vestigial order

- In the ordered phase, we always find composite order

$$\langle \eta_\alpha \rangle \neq 0 \quad \Longrightarrow \quad \langle \eta_\alpha \eta_\beta \rangle \neq 0$$ (trivial)

- Vestigial phase: composite order without primary order

$\langle \eta_\alpha \rangle = 0$ yet $\langle \eta_\alpha \eta_\beta \rangle \neq 0$

vestige noun

ves·tige | \ ˈve-stij \

Definition of *vestige*

1 a (1) : a trace, mark, or visible sign left by something (such as an ancient city or a condition or practice) vanished or lost

(2) : the smallest quantity or trace

Vestigial order

- In the ordered phase, we always find composite order

$$\langle \eta_\alpha \rangle \neq 0 \quad \Longrightarrow \quad \langle \eta_\alpha \eta_\beta \rangle \neq 0 \quad \text{(trivial)}$$

- Vestigial phase: composite order without primary order

$$\langle \eta_\alpha \rangle = 0 \ \text{ yet } \ \langle \eta_\alpha \eta_\beta \rangle \neq 0$$

General idea: a “mother phase” from which several different vestigial orders with similar energy scales emerge.

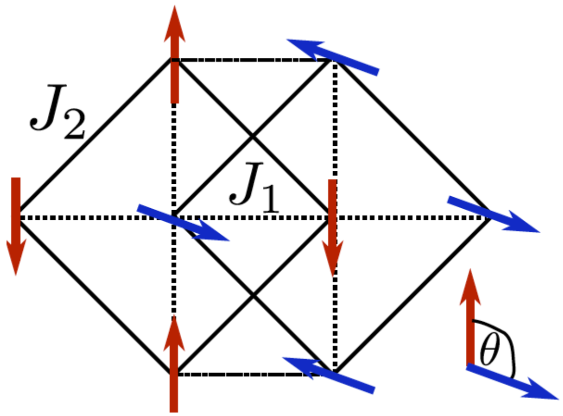
Primary order (degeneracy) $\xrightarrow{\text{Partial melting}}$ Vestigial order $\xleftarrow{\text{Symmetry-breaking fluctuations}}$ Disordered phase

➢ Similar to the phenomenon of “order by disorder” in frustrated magnets

[1] Nie et al, PNAS (2014); [2] Villain, J. Phys. France (1977); Henley, PRL (1989); Chandra, Coleman, Larkin, PRL (1990).

Example of vestigial order in antiferromagnet

- Frustrated J1-J2 antiferromagnet on square lattice



$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Consider $J_2 > J_1$:

- Interpenetrating square lattices with dominant Neel-order fluctuations
- At $T > 0$, magnetic correlation length finite (Mermin-Wagner): $\langle \eta_\alpha \rangle = 0$
- Sublattices only coupled by fluctuations $\boldsymbol{S}_i = \langle \boldsymbol{S}_i \rangle + \delta \boldsymbol{S}_i$

[1] Villain, J. Phys. France (1977); Henley, PRL (1989); Chandra, Coleman, Larkin, PRL (1990).

Example of vestigial order in antiferromagnet

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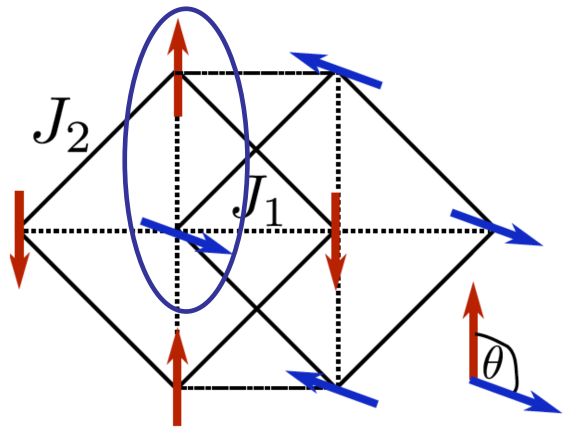
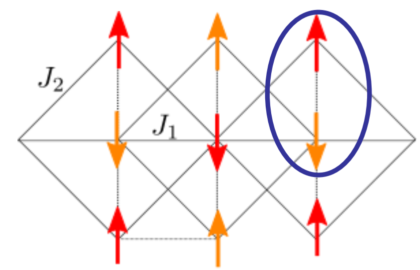


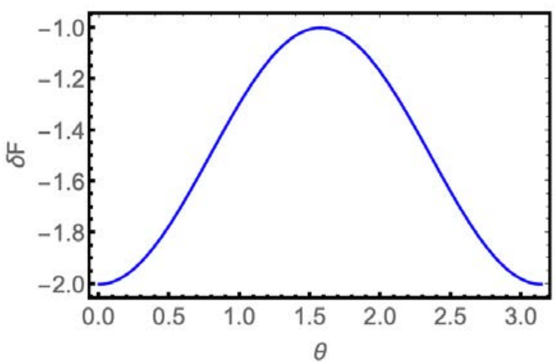
Composite order parameter:

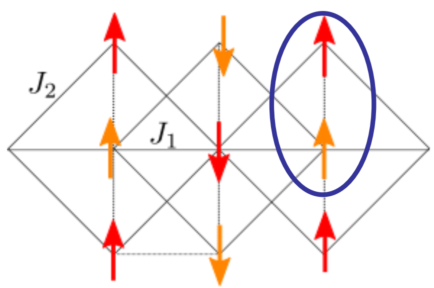
$$\boxed{\langle \phi \rangle \equiv \langle \eta_\alpha \eta_\beta \rangle = \langle \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \rangle}$$



$$\langle \phi \rangle = -1$$

Fluctuation free-energy mimina at $\theta = 0, \pi$





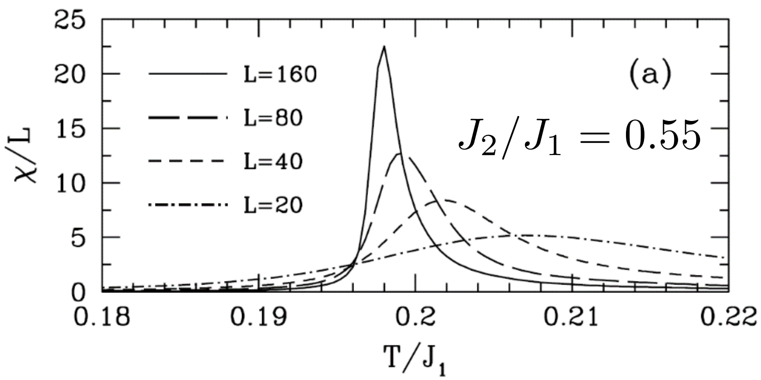
$$\langle \phi \rangle = +1$$

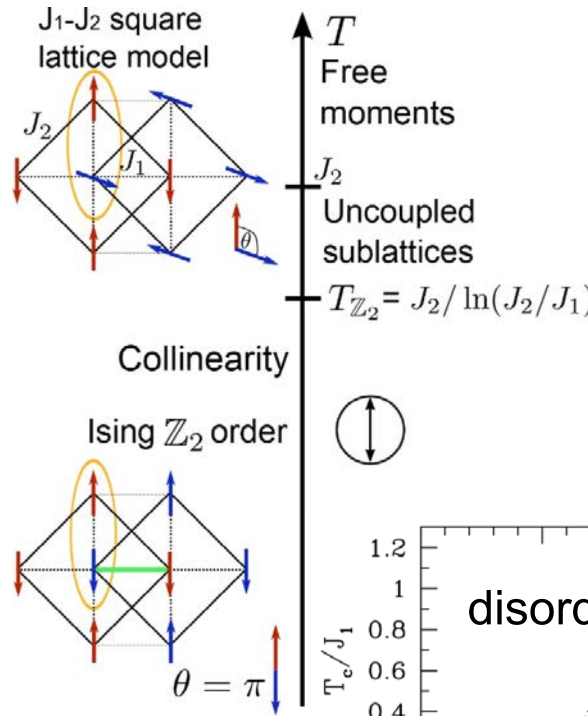
From spin-wave analysis. Entropy maximized ⇢ “Order by Disorder”.

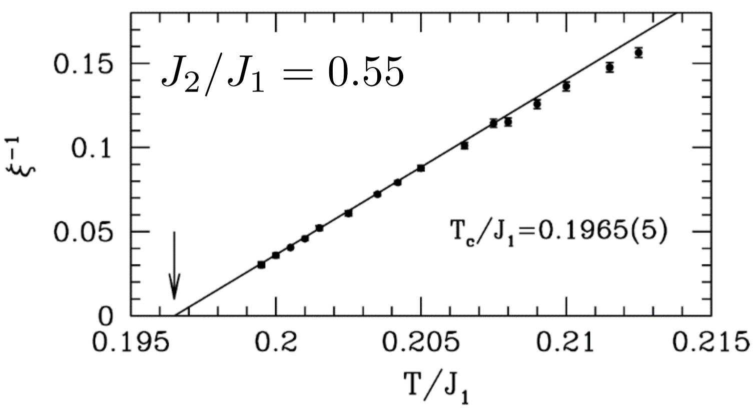
[1] Chandra, Coleman, Larkin, PRL (1990).

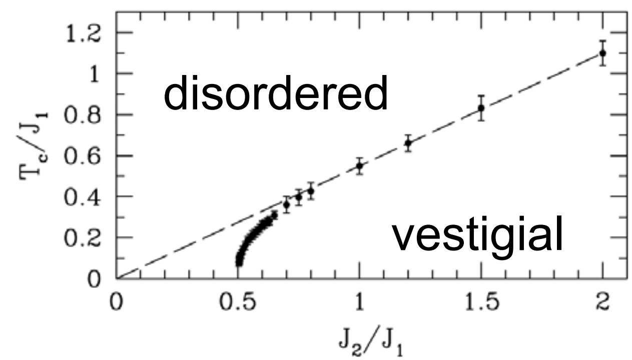
Ising phase transition at finite temperature

- Finite T phase transition into phase with finite vestigial order
- Classical Monte-Carlo results



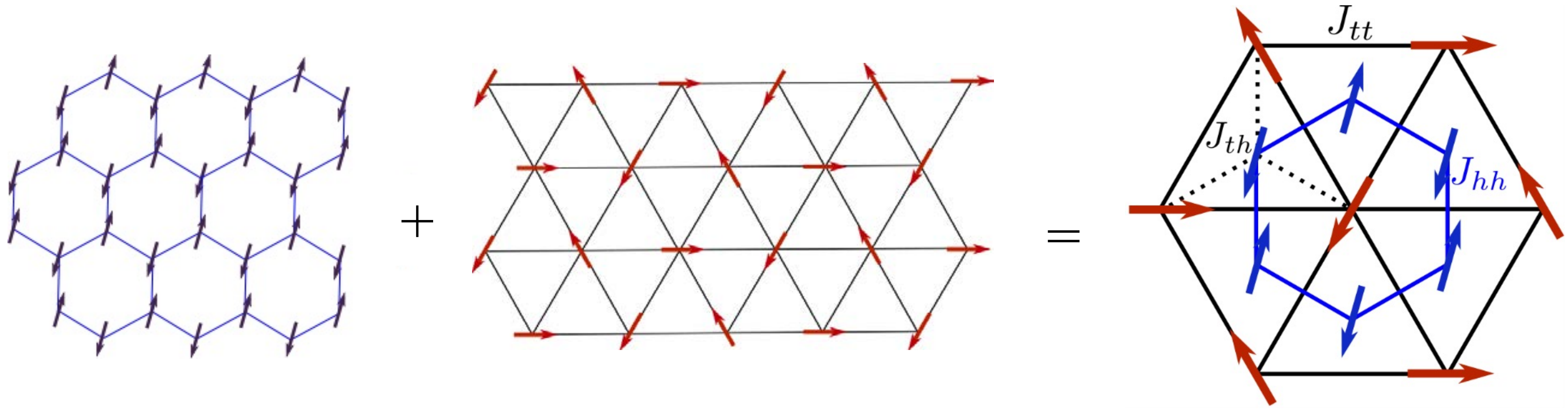






[1] P. Chandra, P. Coleman, A. I. Larkin, PRL **64**, 88 (1990); [2] C. Weber et al., PRL **91**, 177202 (2003);

2D Heisenberg windmill antiferromagnet



- Honeycomb + triangular lattice sites
- Heisenberg spins $\boldsymbol{S}_t(r_j), \boldsymbol{S}_A(r_j), \boldsymbol{S}_B(r_j)$
- Antiferromagnetic nearest-neighbor coupling

$$H = H_{tt} + H_{AB} + H_{tA} + H_{tB}$$

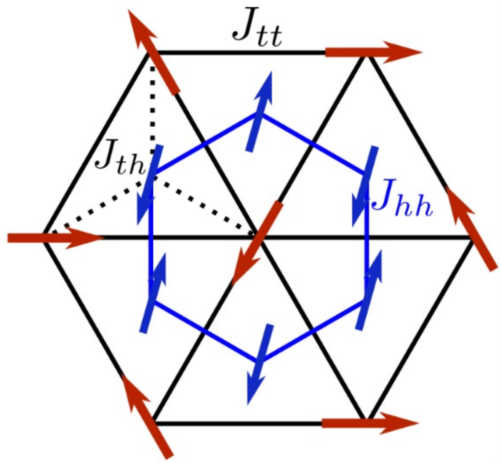
$$H_{ab} = J_{ab} \sum_{j=1}^{N_L} \sum_{\delta_{ab}} \boldsymbol{S}_a(r_j) \cdot \boldsymbol{S}_b(r_j + \delta_{ab})$$

$$a, b \in \{t, A, B\}$$

Windmill in Strangnaes (Sweden)

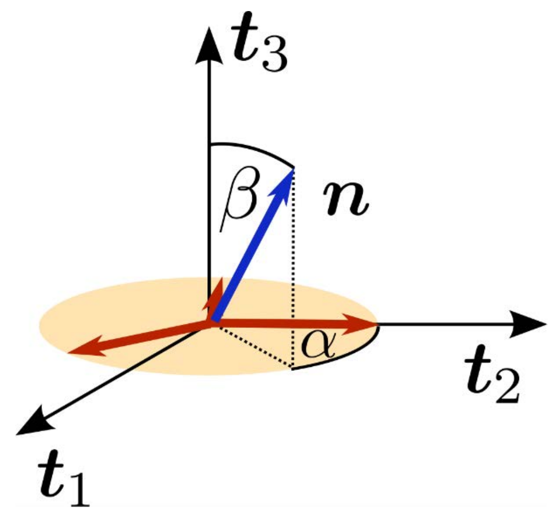
[1,2] PPO, P. Chandra, P. Coleman, J. Schmalian, PRL (2012); PRB (2014); [3] PPO, B. Jeevanesan, PRB (2014); B. Jeevanesan, P. Chandra, P. Coleman, PPO, PRL (2015).

Ground state of classical spins at small J_{th}



Weak inter-sublattice coupling

$$J_{th} \ll J_{tt}, J_{hh}$$



Neel order on **honeycomb lattice**

⇨ O(3)/O(2) order parameter $\boldsymbol{n}(\boldsymbol{x})$

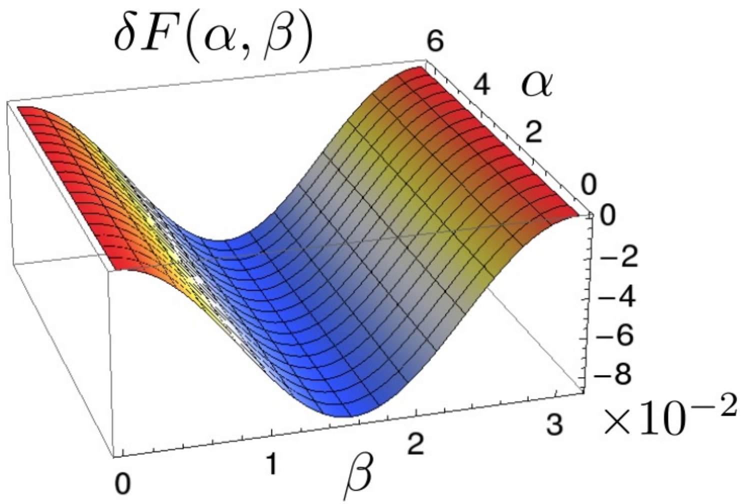
120 degree state on **triangular lattice**

⇨ SO(3) order parameter $t(\boldsymbol{x}) = (\boldsymbol{t}_1, \quad \boldsymbol{t}_2, \quad \boldsymbol{t}_3)$

Classically at T=0 decoupled even for $J_{th} > 0$

[1] B. Jeevanesan, PPO, PRB **90**, 144435 (2014).

Fluctuation coupling "order from disorder"

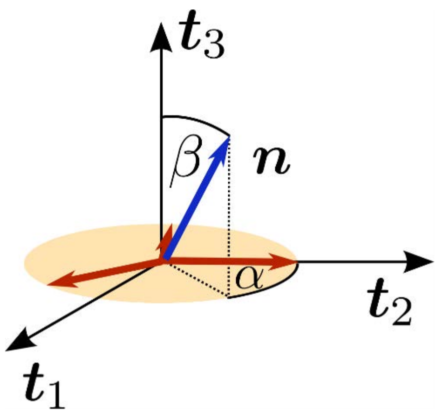


$$J_{th} = 0.4\bar{J}$$
$$\bar{J} = \sqrt{J_{tt}J_{hh}}$$
$$T = 1, S = 1$$

- Fluctuations couple spins on different sublattices
- Spins tend to align perpendicular to fluctuation Weiss field

$$\boxed{S_c = \frac{1}{2}\int d^2x\,(\gamma\cos^2\beta)}$$

Coplanar: $\gamma = (J_{th}/\bar{J})^2 A_\gamma\left(J_{tt}/J_{hh}, \bar{J}/T\right)$



[1] C. L. Henley, PRL **62**, 2056 (1989)

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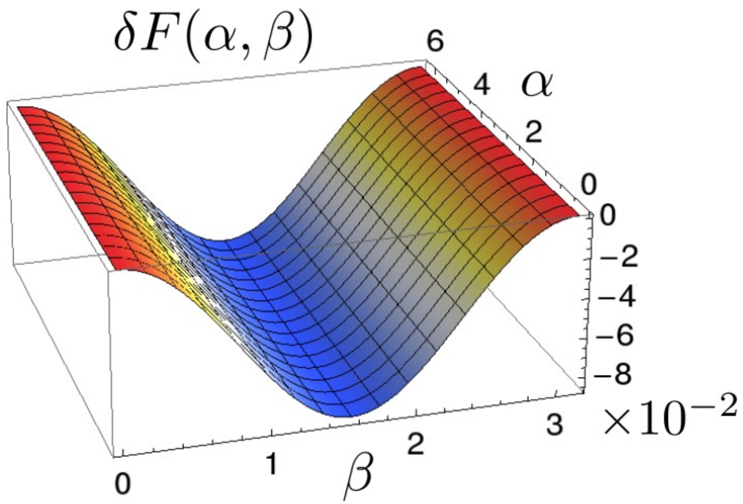

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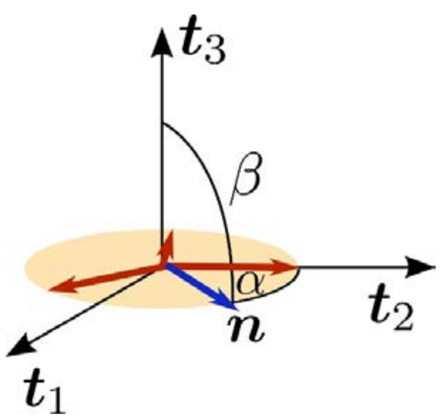

[1] C. L. Henley, PRL **62**, 2056 (1989)

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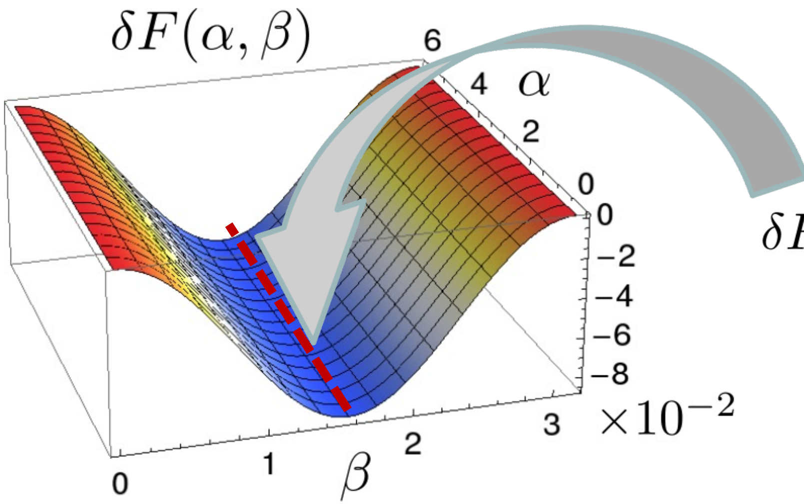
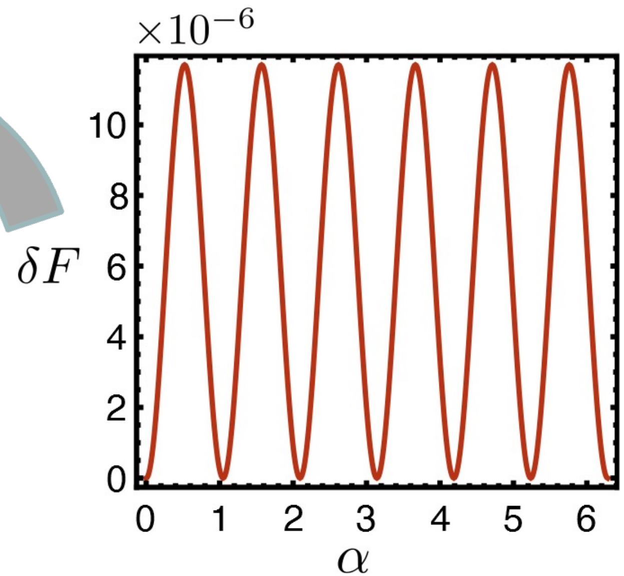


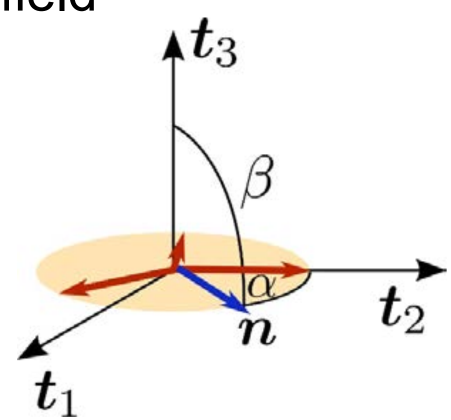


$J_{th} = 0.4\bar{J}$
$\bar{J} = \sqrt{J_{tt}J_{hh}}$
$T = 1, S = 1$

- Fluctuations couple spins on different sublattices
- Spins tend to align perpendicular to fluctuation Weiss field

$$S_c = \frac{1}{2}\int d^2x \left(\gamma \cos^2\beta + \lambda \sin^6\beta \sin^2(3\alpha)\right)$$

Coplanar: $\gamma \propto (J_{th}/\bar{J})^2$ Z$_6$: $\lambda \propto (J_{th}/\bar{J})^6$

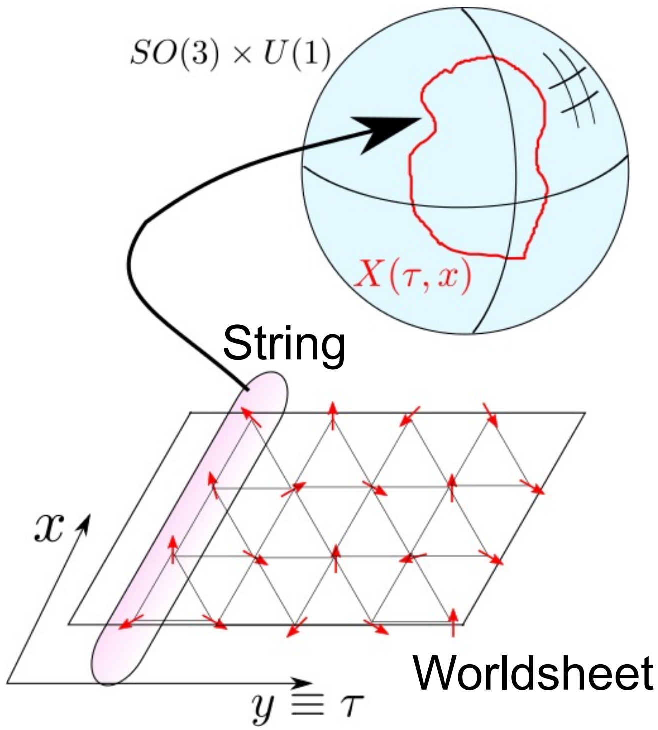


[1] C. L. Henley, PRL **62**, 2056 (1989)

Long-wavelength covariant action

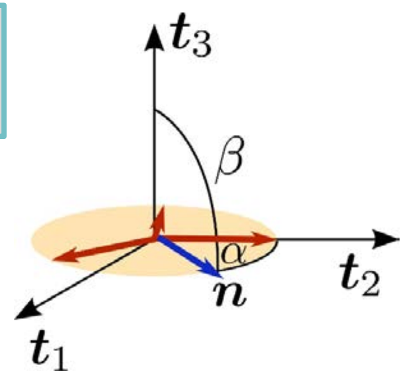
- **Long-wavelength action** of 2d spin system takes form of (Euclidean) **string theory** [1]

$$S = \frac{1}{2}\int d^2x\, g_{ij}[X(x)]\partial_\mu X^i(x)\partial_\mu X^j(x) + \frac{\lambda}{2}\int d^2x \sin^2(3\alpha)$$



3 Euler angles and relative phase

$$X(\tau,x) = (\phi,\theta,\psi,\alpha)$$



Magnetization X = displacement of string in D=4 (compact) dimensions

[1] D. Friedan, PRL **45**, 1057 (1980)

Magnetism as string theory

- Action of 2D spin system takes form of (Euclidean) string theory [1]

$$S = \frac{1}{2}\int d^2x \; g_{ij}[X(x)]\partial_\mu X^i(x)\partial_\mu X^j(x) + \frac{\lambda}{2}\int d^2x \sin^2(3\alpha)$$

- **Spin stiffnesses define metric tensor**

$$X(\tau, x) = (\phi, \theta, \psi, \alpha)$$

$$g = \begin{pmatrix} g^{SO(3)} & \mathcal{K}^T \\ \mathcal{K} & I_\alpha \end{pmatrix}$$

SO(3) stiffnesses I_1, I_2, I_3

U(1) stiffness

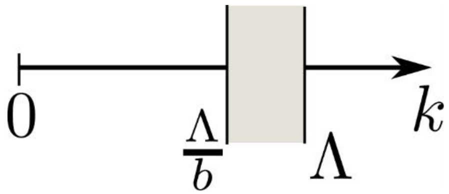
U(1) phase α is coupled to non-Abelian sector

Geometric curvature of manifold (Riemann tensor) determined by spin stiffnesses.

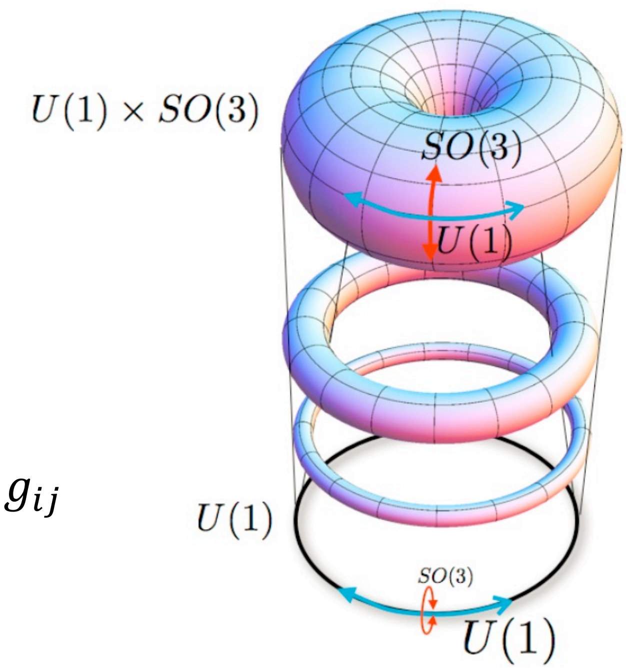
[1] D. Friedan, PRL **45**, 1057 (1980);
[2, 3] PPO, P. Chandra, P. Coleman, J. Schmalian, PRL (2012); PRB (2014)

Magnetism and gravity: RG flow = Ricci flow

- Action is covariant with stiffness metric tensor
- Covariance is preserved during RG scaling [1]
- **RG flow of the metric is given by the Ricci flow** [1,2] (two loops)



Integrating out fast momenta



$$\boxed{\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij} - \frac{1}{8\pi^2}R_i{}^{klm}R_{jklm}}$$

Ricci and Riemann tensor determined by g_{ij}

[1] D. Friedan, PRL **45**, 1057 (1980); [2] R. S. Hamilton, J. Differential Geom. **17**, 255 (1982)

Compactification and magnetism

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij} - \frac{1}{8\pi^2} R_i{}^{klm} R_{jklm}$$

- One-dimensionsal U(1) part of manifold decouples from 3D non-Abelian SO(3) part
- Ricci scalar grows like

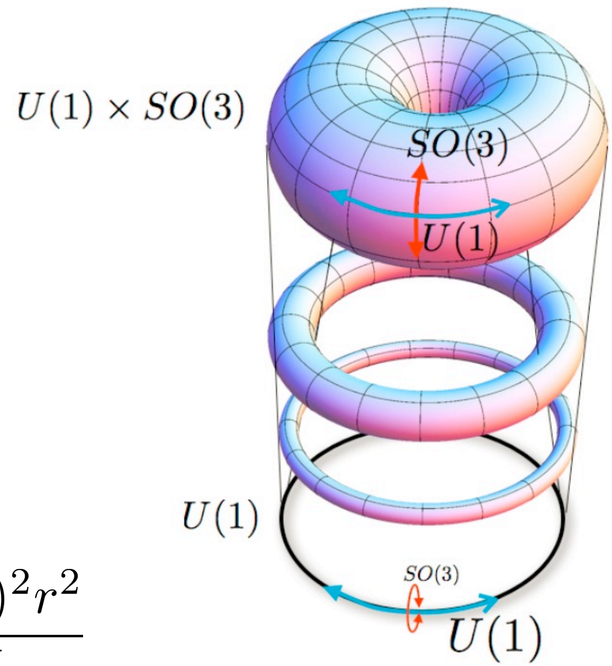
$$R = R^{SO(3)} - \frac{1}{2\pi I'_\alpha} \beta_\alpha$$

$$R^{SO(3)} \sim 1/\bar{I}$$

$$\beta_\alpha = \frac{(I_1 - I_2)^2 r^2}{4\pi I_1 I_2}$$

SO(3) part curles up

U(1) becomes flat



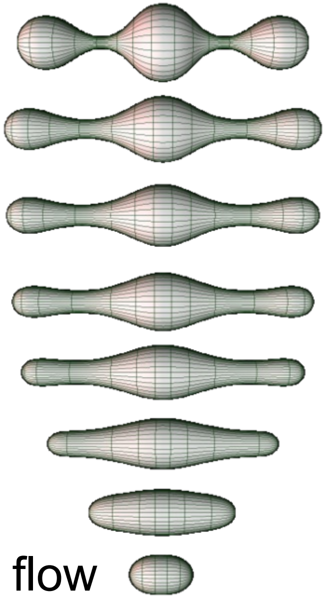
Toy model for compactification

[1] M. Gell-Mann and B. Zwiebach, Phys. Lett. B **141**, 333 (1984);
[2] L. Randall and R. Sundrum, PRL **83**, 4690 (1999)
[3, 4] PPO, P. Chandra, P. Coleman, J. Schmalian, PRL (2012); PRB (2014);

Experimental proof of Poincare conjecture

- Poincare conjecture (1904), proven by Perelman in 2006
- "Every simply connected, closed 3-manifold is homeomorphic to a 3-sphere"
- Two-loop perturbative Ricci flow experiences singularities (false Landau poles). Not present in exact Ricci flow.
- **Use classical magnet to simulate exact Ricci flow. Experimental proof of Poincare conjecture.**
- Protocol:
  - Suitable magnet realizes given metric
  - Cool system
  - Measure spin correlation functions at various temperatures
  - Extract metric tensor
  - Obtain "surgery-free" generalized Ricci flow of manifold

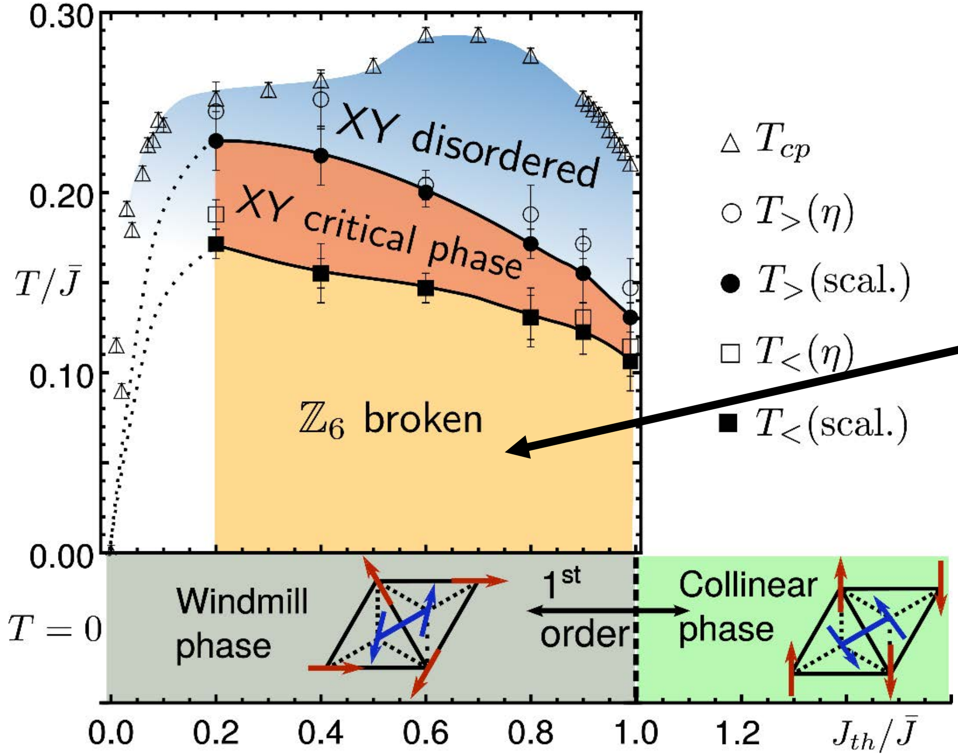
G. Perelman (2006)
H. Poincare

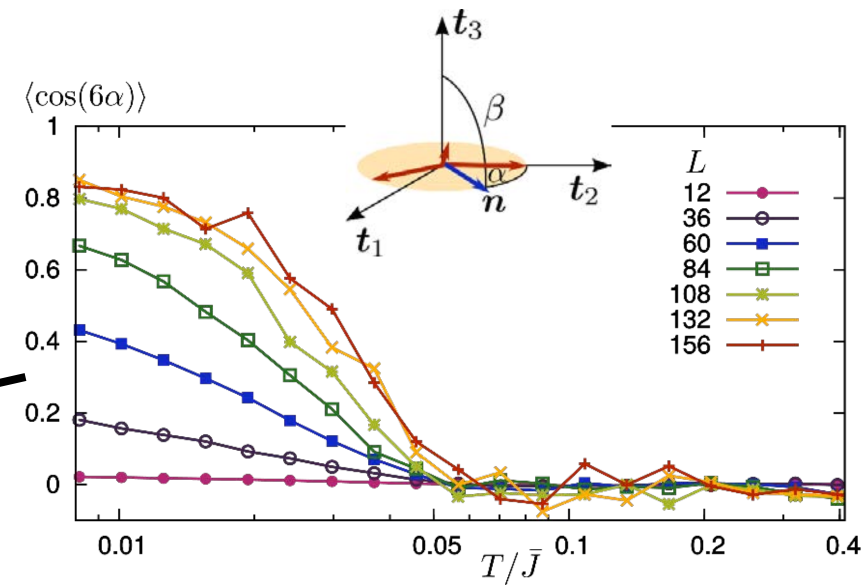


Ricci flow

[1] P. Coleman, A. Tsvelik (private communication)

Classical Monte-Carlo simulation: phase diagram



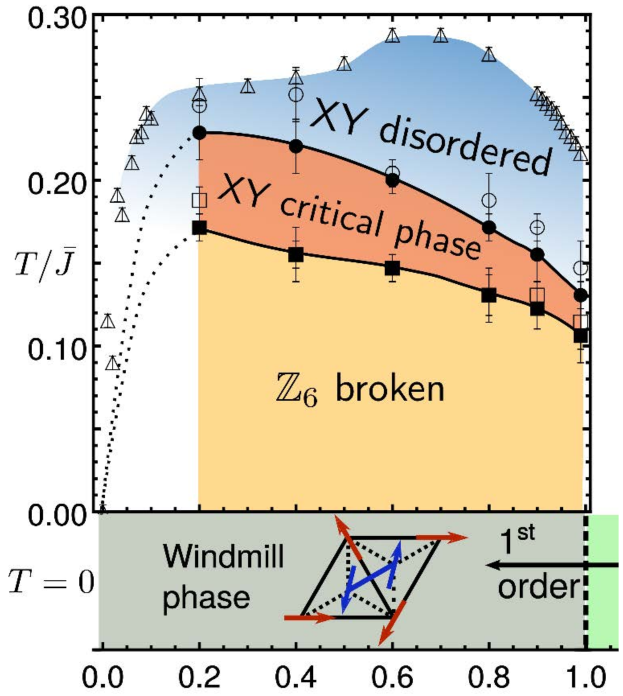


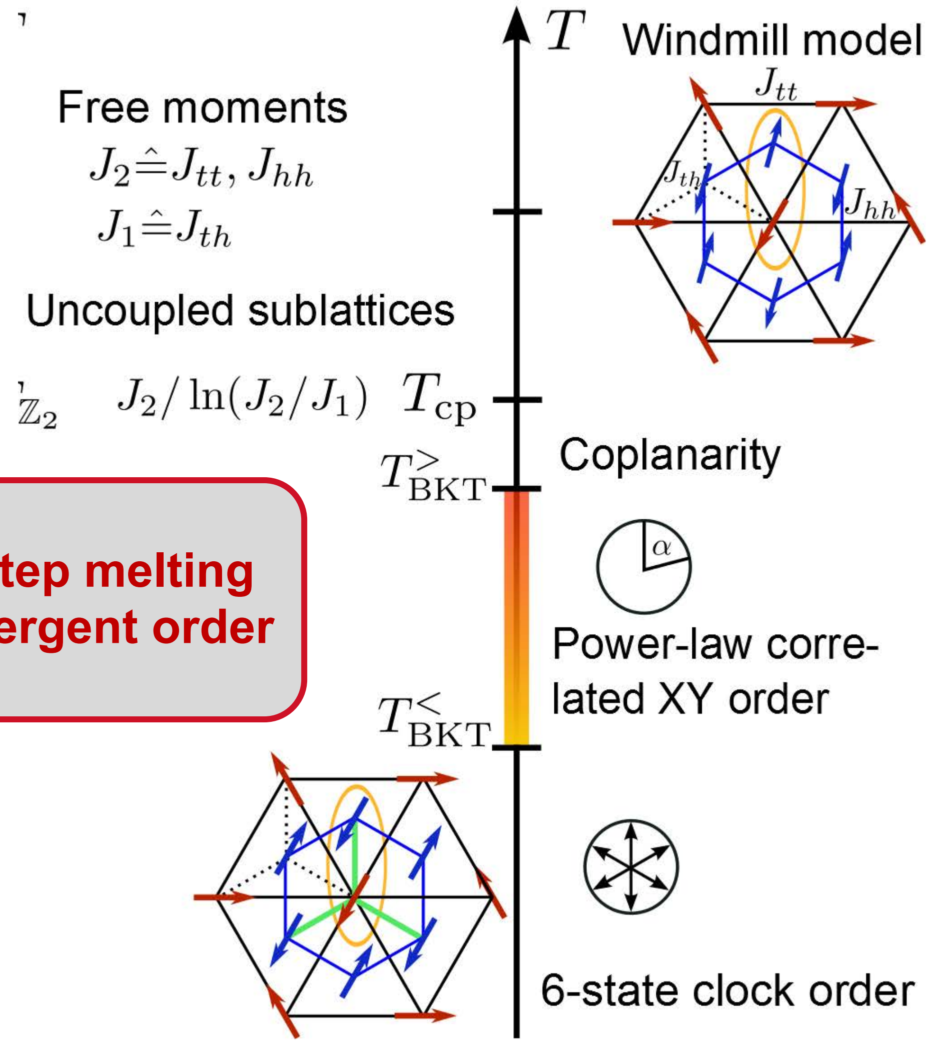
System falls into one of the six minima
Long-range discrete order occurs.

- Large-scale parallel-tempering classical Monte-Carlo simulations
- Proof of Polykov's conjecture that critical phase can exist in Heisenberg system (due to topological vacuum degeneracy).

[1] B. Jeevanesan, P. Chandra, P. Coleman, PPO, PRL (2015).

Phase diagram of windmill antiferromagnet



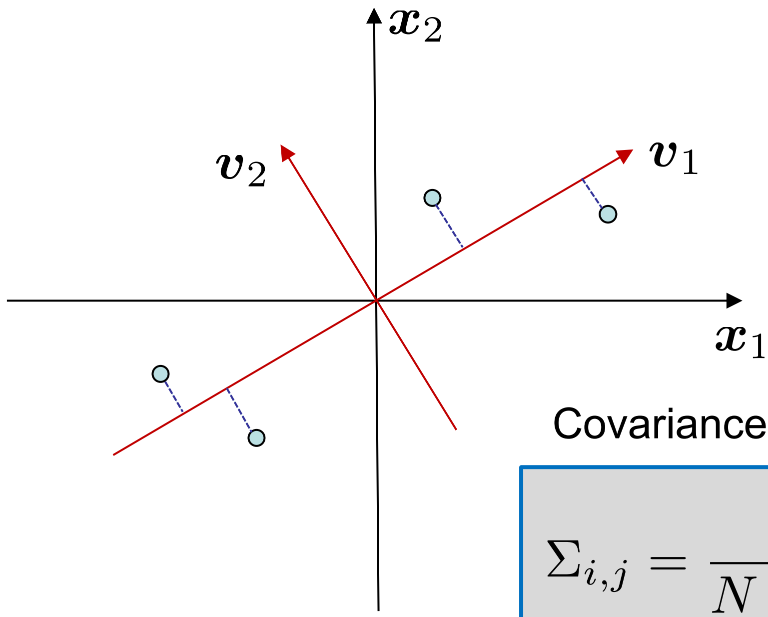


Two-step melting of emergent order

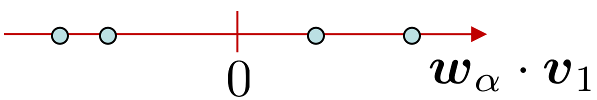
[1,2] PPO, P. Chandra, P. Coleman, J. Schmalian PRL (2012); PRB (2014);
[3] B. Jeevanesan, P. Chandra, P. Coleman, PPO PRL (2015).

Interlude: Can machine-learning find vestigial order

- Unsupervised machine learning algorithm
- Principal Component Analysis (PCA)
  - Linear orthogonal transformation to basis with decreasing data variance (maximal projections)



v_1 corresponds to maximal eigenvalue λ_1 of covariance matrix



Projection of data along v_1 maximal

➢ Strongest correlations in the data

➢ Minimal data loss

Covariance matrix:

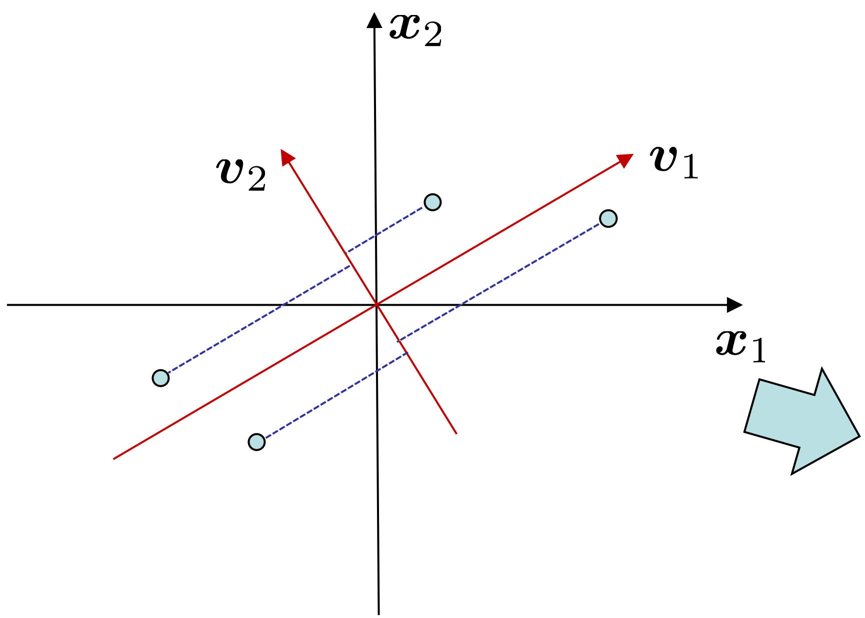
$$\boxed{\Sigma_{i,j} = \frac{1}{N-1} \sum_{\alpha=1}^{N} (w_\alpha)_i (w_\alpha)_j}$$

Rescaled data to zero mean and equal standard-deviations along axes.

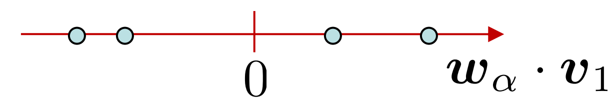
Work in progress with Mathias S. Scheurer (Harvard).

Interlude: Can machine-learning find vestigial order

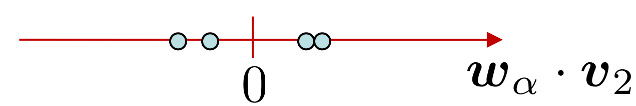
- Unsupervised machine learning algorithm
- Principal Component Analysis (PCA)
  - Linear orthogonal transformation to basis with decreasing data variance (maximal projections)



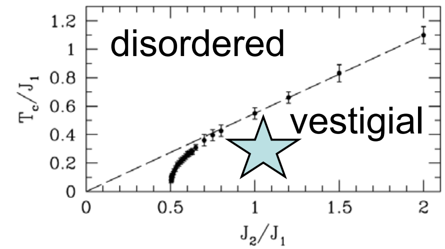
v_1 corresponds to maximal eigenvalue λ_1 of covariance matrix



v_2 corresponds to next largest eigenvalue λ_2 of covariance matrix

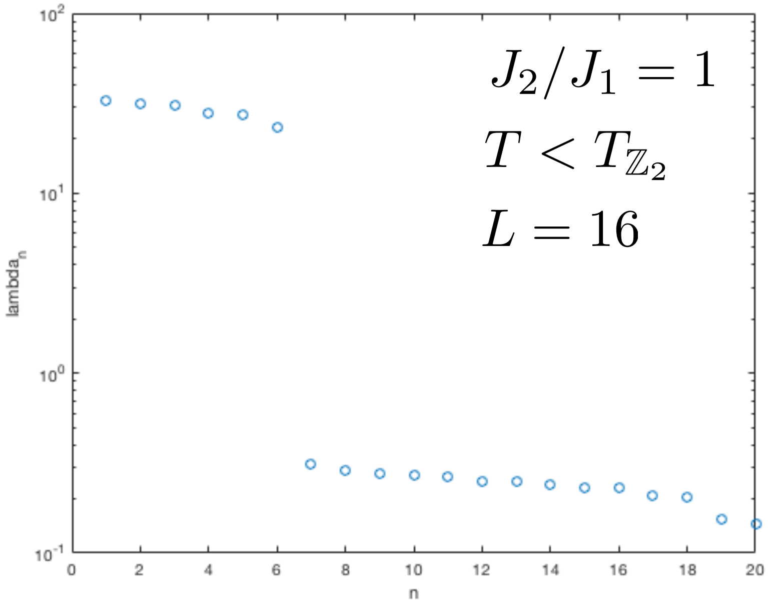


Interlude: Can machine-learning find vestigial order



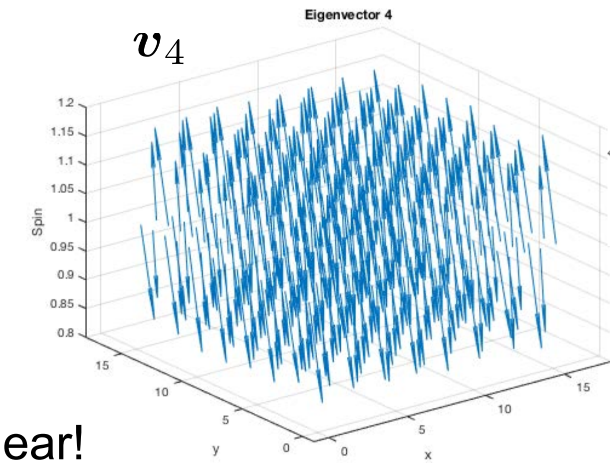
- Unsupervised machine learning algorithm
- Principal Component Analysis (PCA)
  - Linear orthogonal transformation to basis with decreasing data variance
  - Feed spin snapshot data obtained in classical Monte-Carlo simulation

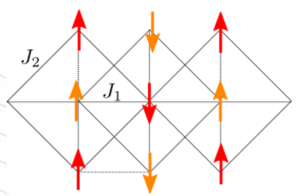
$$\boldsymbol{x} = (S_1^x, S_1^y, S_1^z, \ldots, S_N^z)$$



No sign of vestigial order as PCA is linear!

- Six principal modes $\lambda_1, \ldots, \lambda_6$
- Correspond to
  - 3 spin polarizations at
  - 2 wavevectors $(\pi, 0)$ and $(0, \pi)$



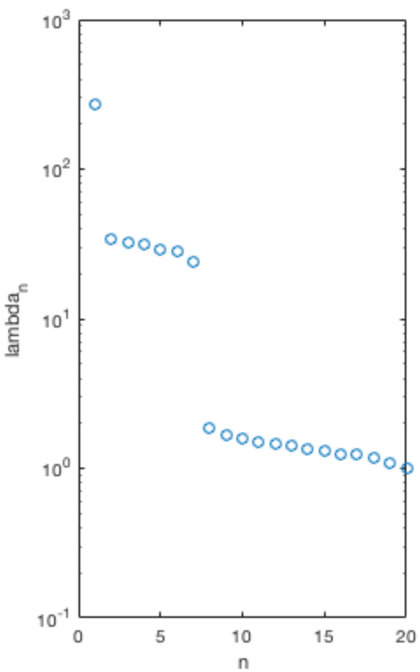


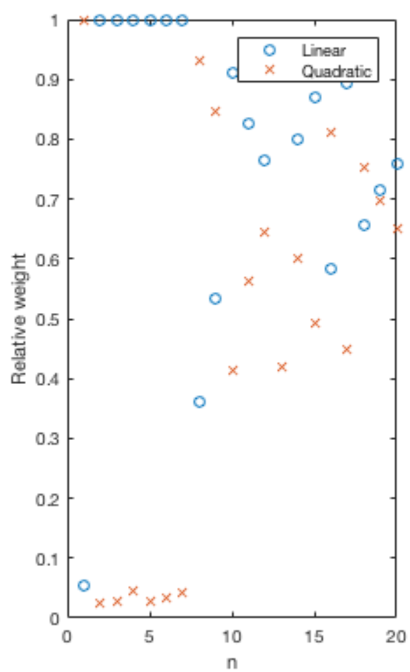
$$\boldsymbol{q} = (\pi, 0)$$

Interlude: Can machine-learning find vestigial order

- Feed in addition bond expectation values of spin snapshot data

$$\boldsymbol{x} = (\underbrace{S_1^x, S_1^y, S_1^z, \ldots, S_N^z}_{\text{Linear “spin” sector}}, \underbrace{\boldsymbol{S}_1 \cdot \boldsymbol{S}_2, \boldsymbol{S}_1 \cdot \boldsymbol{S}_{1+L}, \ldots, \boldsymbol{S}_L \cdot \boldsymbol{S}_1}_{\text{Quadratic “bond” sector}})$$



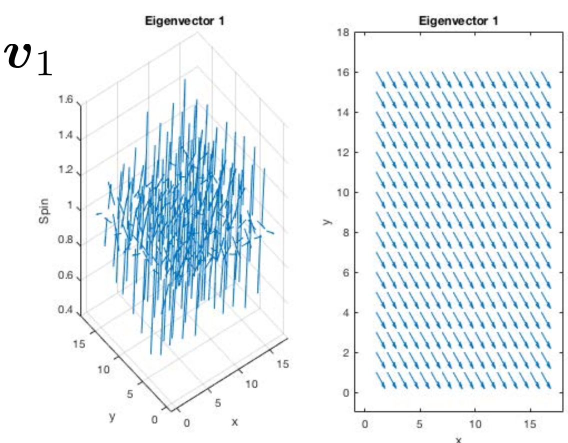


$$J_2/J_1 = 1$$

$$T < T_{\mathbb{Z}_2} \qquad L = 16$$

- One dominant mode λ_1 appears now
  - Mode lies entirely in bond sector
- 6 sub-leading modes $\lambda_2, \ldots, \lambda_7$ as before

$\boldsymbol{v}_1$



“Spin” sector: no correlations

“Bond” sector: vestigial order

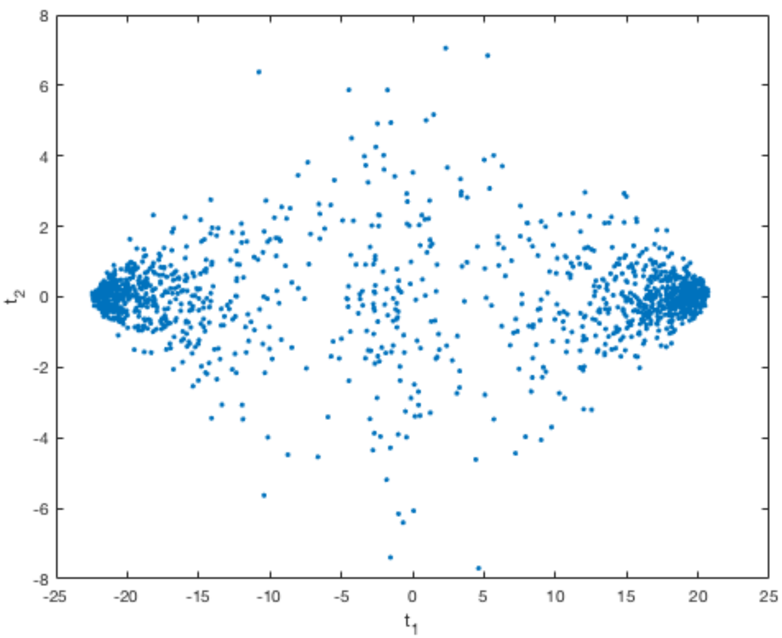
$$\boxed{\boldsymbol{v}_1 = \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} - \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+L}}$$

Interlude: Can machine-learning find vestigial order

- Feed in addition bond expectation values of spin snapshot data

$$x = (S_1^x, S_1^y, S_1^z, \ldots, S_N^z, \boldsymbol{S}_1 \cdot \boldsymbol{S}_2, \boldsymbol{S}_1 \cdot \boldsymbol{S}_{1+L}, \ldots, \boldsymbol{S}_L \cdot \boldsymbol{S}_1)$$

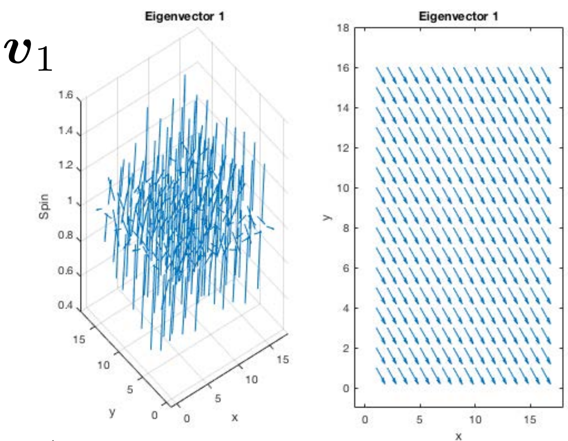
Linear "spin" sector   Quadratic "bond" sector



Two clusters along v_1 direction
→ Z_2 (Ising) vestigial order

- One dominant mode λ_1 appears now
  - Mode lies entirely in bond sector
- 6 sub-leading modes $\lambda_2, \ldots, \lambda_7$ as before

$\boldsymbol{v}_1$



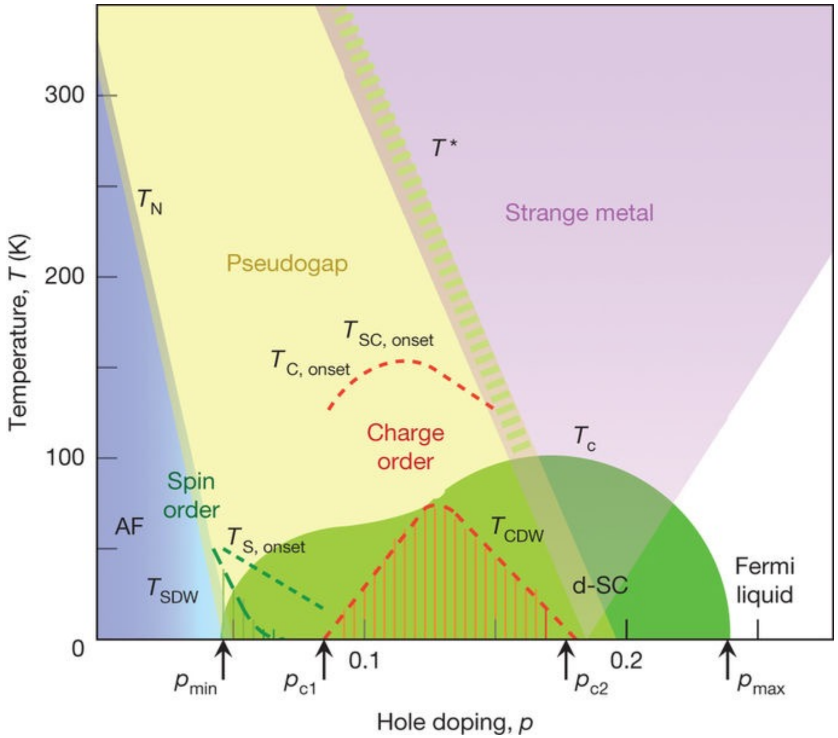
"Spin" sector:
no correlations

"Bond" sector: vestigial order

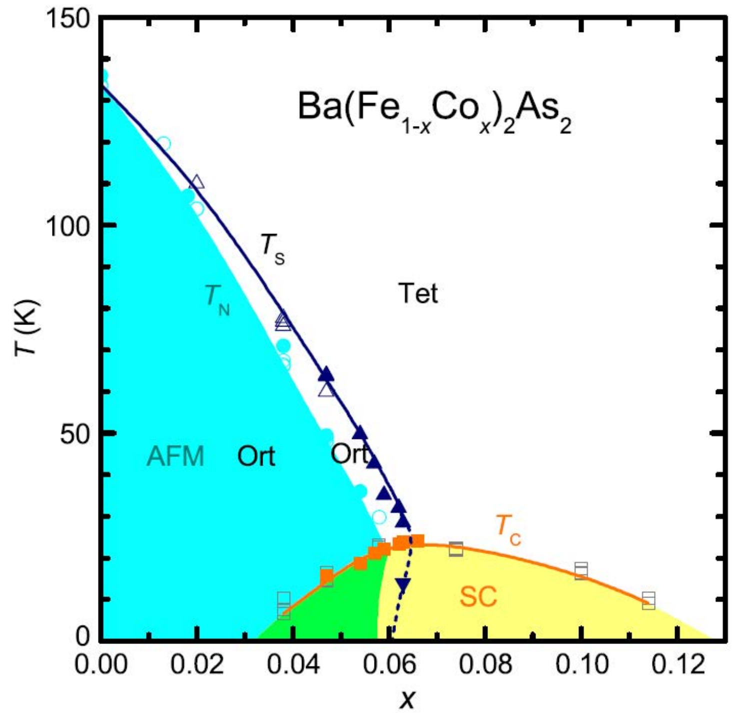
$$\boxed{\boldsymbol{v}_1 = \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} - \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+L}}$$

Vestigial order in complex materials

- Naturally gives rise to complexity observed in phase diagrams across correlated quantum materials



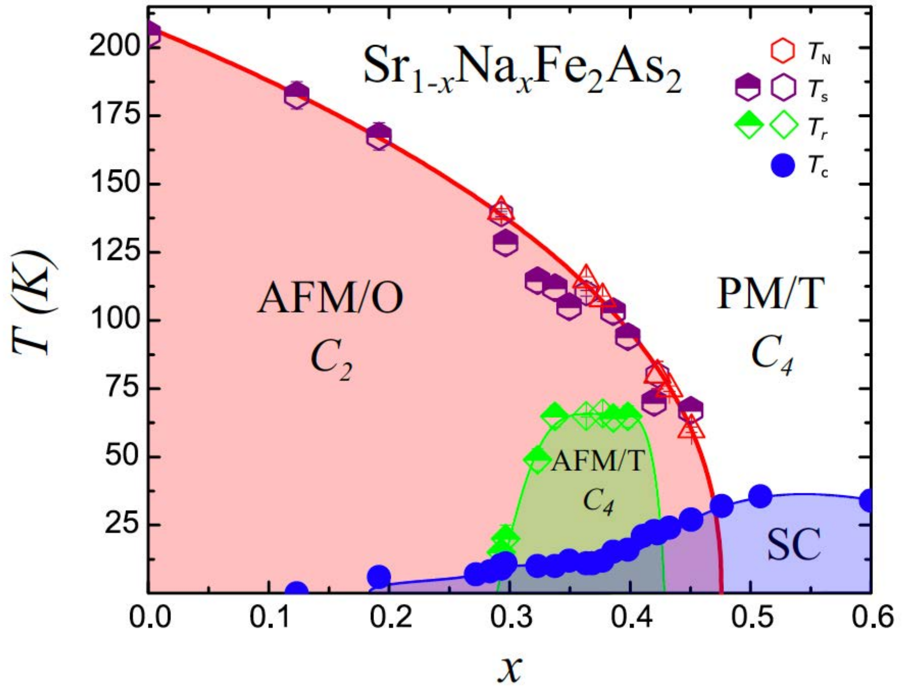
Cuprate Supercondutor (YBCO)



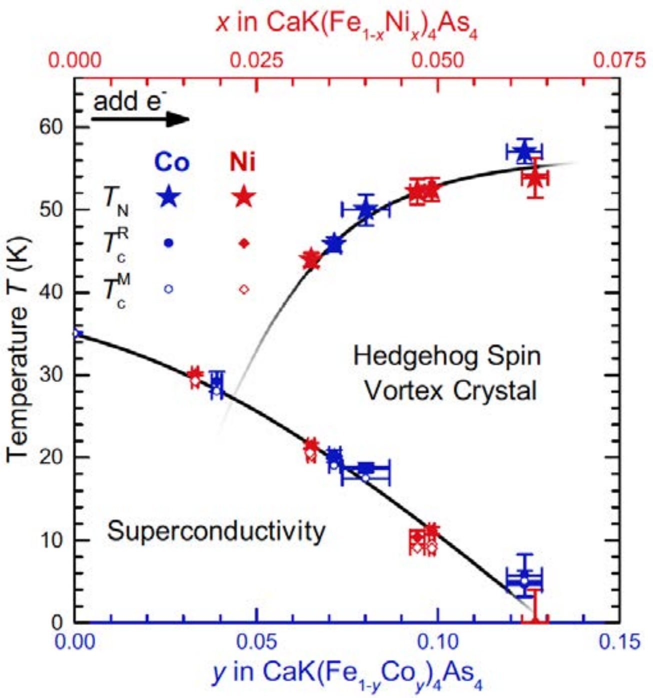
Iron-Based Superconductor
(hole doped Ba-122)

[1] Keimer et al., Nature (2015); [2] S. Nandi *et al.*, PRL **104**, 057006 (2010); [2] P. C. Canfield, S. L. Budko, Annu. Rev. Cond. Mat. **1**, 27 (2010).

Complexity of phase diagrams in correlated materials



Iron-Based Superconductor
(electron doped Sr-122)



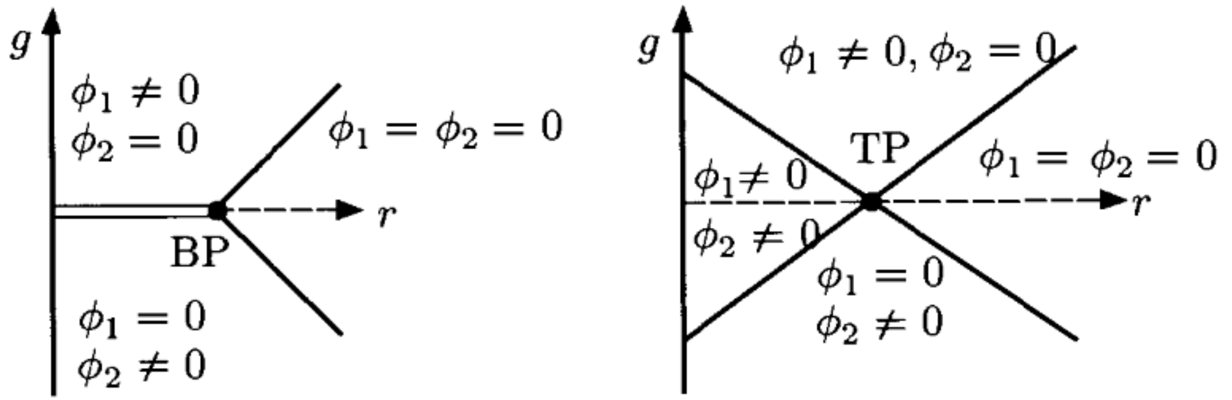
Iron-Based Superconductor
(electron doped 1144)

[1] Taddei et al., PRB (2016); [2] W. R. Meier, …, PPO, …P.C. Canfield, npj Quantum Materials (2018).

Complex phase diagrams

- Multiple ordered states that break different symmetries but exhibit comparable energy scales
- Landau theory of competing orders: ϕ_1, ϕ_2

$$f = \frac{1}{2}r(\phi_1^2 + \phi_2^2) - \frac{1}{2}g(\phi_1^2 - \phi_2^2) + u_1\phi_1^4 + u_2\phi_2^4 + 2u_{12}\phi_1^2\phi_2^2.$$

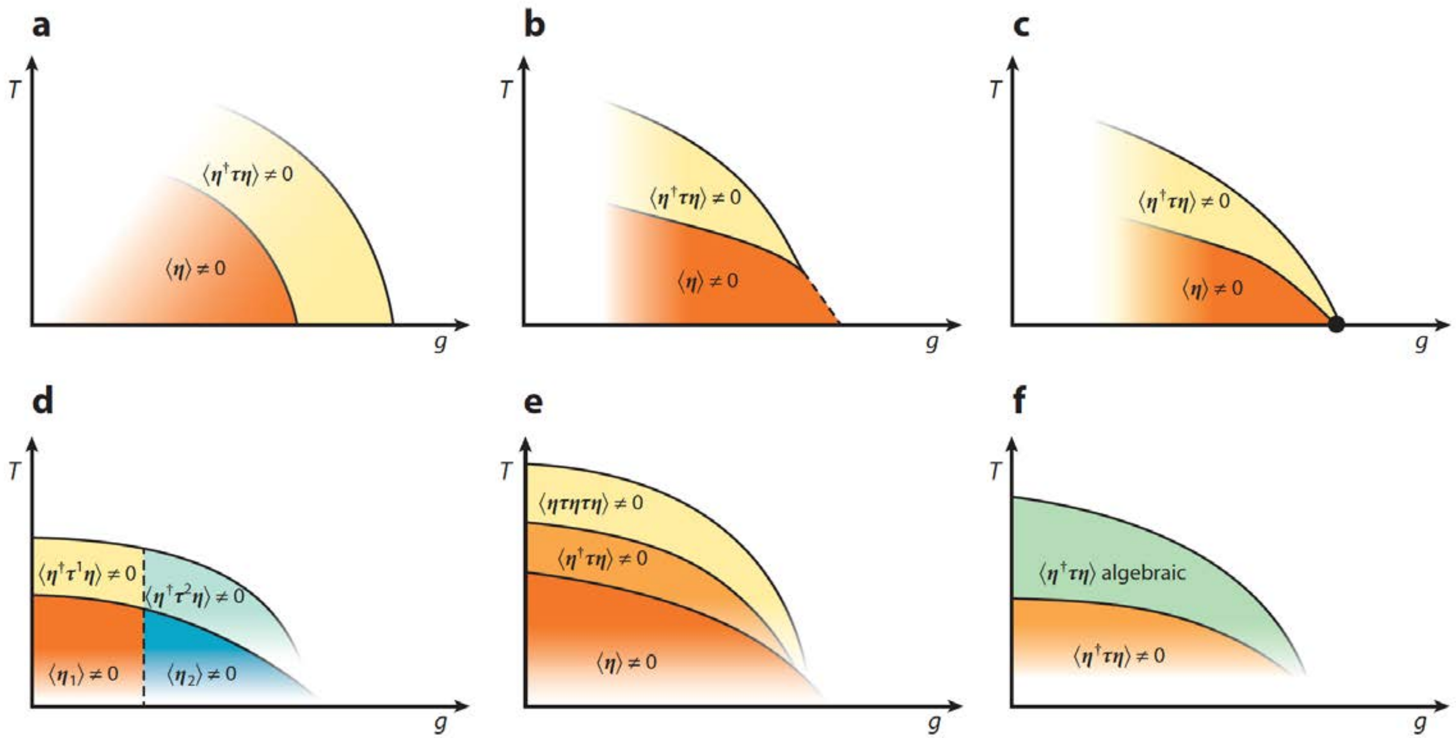


➢ Fine-tuning of multiple coupling constants required to explain complexity (several multicritical points).

[1] P. M. Chaikin, T. C. Lubensky "Principles of Condensed Matter Physics; [2] Kosterlitz, Nelson, Fisher, PRB (1976).

Vestigial order: a natural explanation of complexity

- Fluctuation-driven vestigial orders: powerful framework to describe interplay between multiple phases with comparable transition temperatures
- Based on symmetry alone, no fine tuning of parameters



[1] R. M. Fernandes, PPO, J. Schmalian, Annu. Rev. Cond. Mat. **10**, 133 (2019); [2] see also Fradkin et al, RMP (2015).

Group theory definition of vestigial order

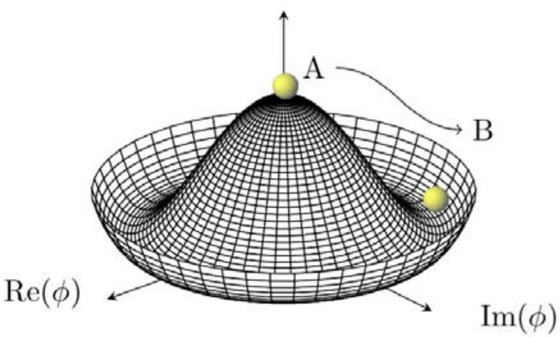
- Symmetry group of the system: $\mathcal{G}$ = (spatial) x (internal) symm.
- Complex primary order parameter η_α transforms according to an irreducible representation (irrep) Γ of $\mathcal{G}$
- Components $\alpha = 1, \ldots, d_\Gamma$, where d_Γ = dimensionality of irrep
- Example: singlet s-wave superconductivity (l=0)

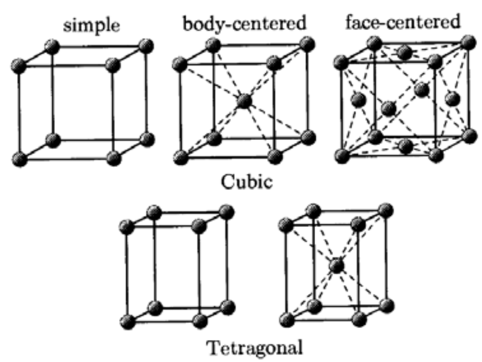
η
Complex order parameter

$\mathcal{G} = G \otimes \mathrm{U}(1)$
Point group of lattice

$\Gamma = A_1 \otimes e^{ik\varphi}$
Trivial irrep

Y_{00}
Basis function





D_{4h} (h=16)	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$
A_{1g}	1	1	1	1	1	1	1	1	1	1
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1
B_{1g}	1	−1	1	1	−1	1	−1	1	1	−1
B_{2g}	1	−1	1	−1	1	1	−1	1	−1	1
E_g	2	0	−2	0	0	2	0	−2	0	0
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1
B_{1u}	1	−1	1	1	−1	−1	1	−1	−1	1
B_{2u}	1	−1	1	−1	1	−1	1	−1	1	−1
E_u	2	0	−2	0	0	−2	0	2	0	0

[1] P. M. Chaikin, T. C. Lubensky “Principles of Condensed Matter Physics; [2] Gernot-katzers-spice-pages.com

Primary and vestigial phase

$$\eta \qquad \mathcal{G} = G \otimes \mathrm{U}(1) \qquad \Gamma = A_1 \otimes e^{ik\varphi} \qquad Y_{00}$$

Complex order parameter

Point group of lattice

Trivial irrep

Basis function

- Primary phase: $\langle \eta \rangle \neq 0$

  Here: Breaking of U(1) symmetry, as trivially under point group operations.

- Composite order parameter:

$$\boxed{\phi_m = \sum_{\alpha,\beta=1}^{d_\Gamma} \eta^*_\alpha \Lambda^m_{\alpha\beta} \eta_\beta}$$

$d_\Gamma \times d_\Gamma$ matrix

➢ Transforms under one of the m irreps Γ^m of the product $\Gamma^* \otimes \Gamma$

➢ Note that $\Gamma^* \otimes \Gamma = A_{1g}$, if $d_\Gamma = 1$ as $(-1)^2 = 1$.

➢ Composite object can transform non-trivially only for $d_\Gamma > 1$

[1] P. M. Chaikin, T. C. Lubensky “Principles of Condensed Matter Physics; [2] Gernot-katzers-spice-pages.com

Primary and vestigial phase

η

Complex order parameter

$\mathcal{G} = G \otimes \mathrm{U}(1)$

Point group of lattice

$\Gamma = A_1 \otimes e^{ik\varphi}$

Trivial irrep

Y_{00}

Basis function

- Primary phase: $\langle \eta \rangle \neq 0$

  Breaking of U(1) symmetry. Transforms trivially under point group operations.

- Composite order parameter:

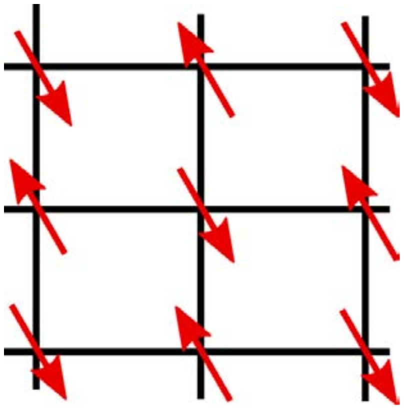
$$\boxed{\phi_1 = \langle |\eta|^2 \rangle \neq 0}$$

➢ Transforms trivially as A_{1g}
➢ Always non-zero: gap amplitude fluctuations: No vestigial order!
➢ Not all composite objects break a symmetry

[1] P. M. Chaikin, T. C. Lubensky “Principles of Condensed Matter Physics; [2] Gernot-katzers-spice-pages.com

Vestigial order from magnetic primary order

- Neel magnetic order with $Q = (\pi, \pi)$



$$\mathcal{G} = G \otimes \mathrm{SO}(3)$$

G is, for example, tetragonal point group like D_{4h} or C_{4v}.

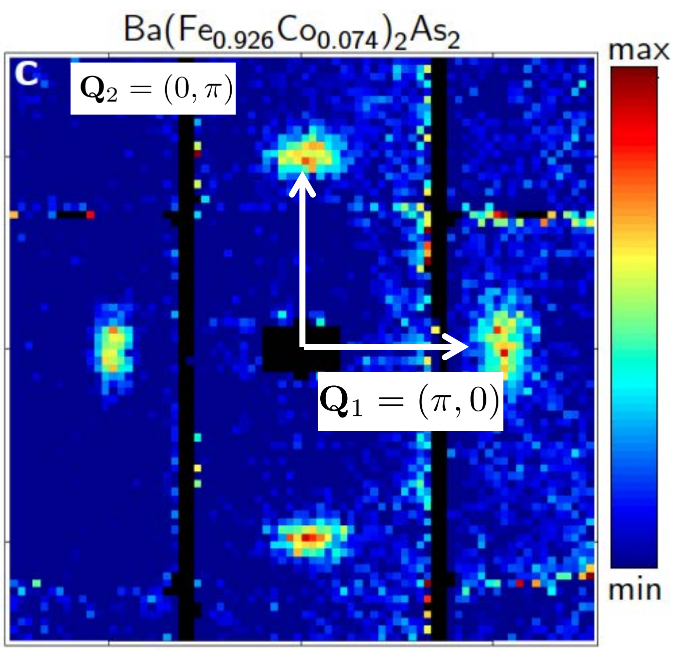
- Local spin $\boldsymbol{S}(\boldsymbol{r}) = \boldsymbol{m}_{\boldsymbol{Q}=(\pi,\pi)} \cos(\boldsymbol{Q} \cdot \boldsymbol{r})$
- Vestigial order transforms as A_{1g} as $d_\Gamma = 1$ in spatial part G: m_Q.

  $\rightarrow$ no vestigial order that breaks lattice symmetries (from G part)
- Need multiple wave-vectors related by lattice symmetry.

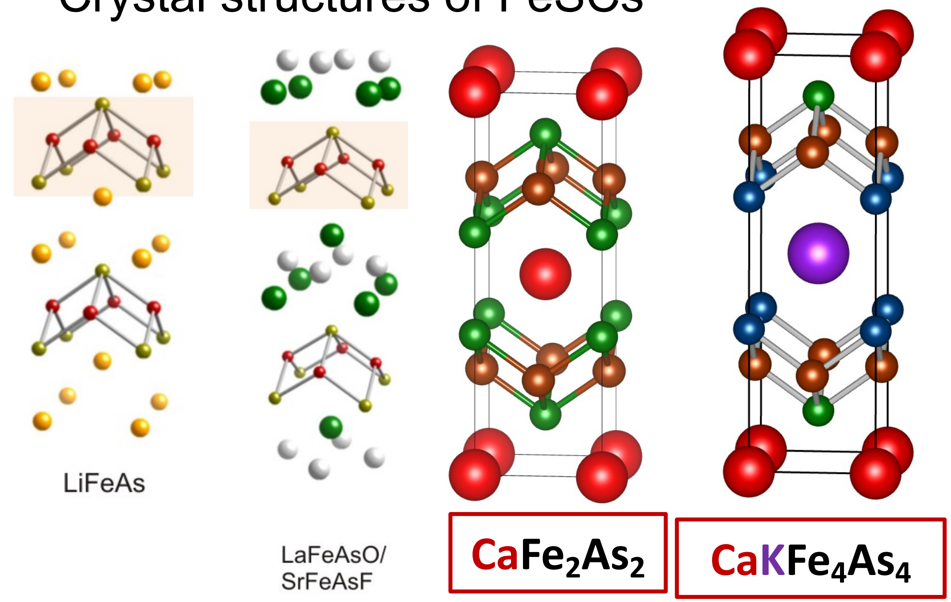
Magnetism in iron-based superconductors

- Magnetic fluctuations peaked at two inequivalent wave-vectors $Q_1 = (\pi, 0)$ and $Q_1 = (0, \pi)$

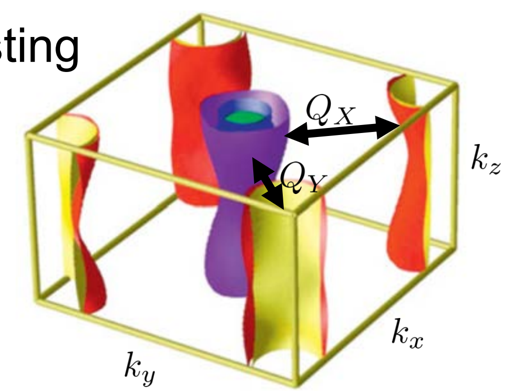
Neutron scattering



Crystal structures of FeSCs



Bandstructure with nesting wavevectors



$$\boldsymbol{S}(\boldsymbol{r}) = \boldsymbol{m}_1 \cos(\boldsymbol{Q}_1 \cdot \boldsymbol{r}) + \boldsymbol{m}_2 \cos(\boldsymbol{Q}_2 \cdot \boldsymbol{r})$$

[1] Tucker et al, PRB (2012);] [2] J. Paglione and R. Greene Nature Physics 6, 645 (2010); [3] R. M. Fernandes et al., PRB 85, 024534 (2012); [4] Chubukov et al, PRB (2009); [5] W. Meier

Spin-density waves on the square lattice

- Free-energy in tetragonal system and including time-reversal (without spin-orbit coupling)

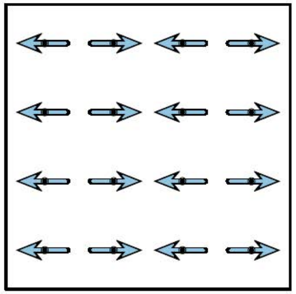
$$S = \int_k r_0(k)(\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2) + \frac{u}{2}\int_r (\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2)^2 - \frac{g}{2}\int_r (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)^2 + 2w\int_r (\boldsymbol{m}_1 \cdot \boldsymbol{m}_2)^2$$

Second-order coefficient:

$$r_0 = r_0 + k^2 + \gamma|\omega_n|$$

- Three-types of magnetic order minimize free-energy

SSDW



$g > 0,\ g > -w$

Stripe spin-density wave (SSDW)

$$\langle \mathbf{m}_1 \rangle \neq 0$$

$$\langle \mathbf{m}_2 \rangle = 0$$

$$\boldsymbol{Q}_1 = (\pi, 0)$$

[1] Lorenzana et al, PRL (2008); [2] Eremin et al, PRB (2010); [3] Brydon et al, PRB (2011); [4] Giovannetti et al, Nat. Commun. (2011
Wang et al., PRB (2015), M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018).

Spin-density waves on the square lattice

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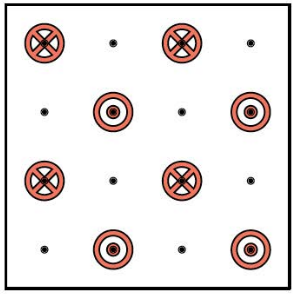
$$S = \int_k r_0(k)(\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2) + \frac{u}{2}\int_r (\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2)^2 - \frac{g}{2}\int_r (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)^2 + 2w\int_r (\boldsymbol{m}_1 \cdot \boldsymbol{m}_2)^2$$

Second-order coefficient:

$$r_0 = r_0 + k^2 + \gamma|\omega_n|$$

- Three-types of magnetic order minimize free-energy

CSDW



$w < 0,\ |w| > g$

Charge-spin density wave (CSDW)

$$\langle m_1 \rangle = \langle m_2 \rangle \neq 0$$

$$\langle \mathbf{m}_1 \rangle \parallel \langle \mathbf{m}_2 \rangle$$

Collinear double-Q

[1] Lorenzana et al, PRL (2008); [2] Eremin et al, PRB (2010); [3] Brydon et al, PRB (2011); [4] Giovannetti et al, Nat. Commun. (2011
Wang et al., PRB (2015), M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018).

Spin-density waves on the square lattice

- Free-energy in tetragonal system and including time-reversal (without spin-orbit coupling)

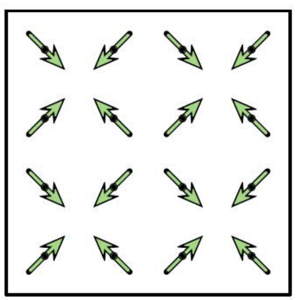
$$S = \int_k r_0(k)(\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2) + \frac{u}{2}\int_r (\boldsymbol{m}_1^2 + \boldsymbol{m}_2^2)^2 - \frac{g}{2}\int_r (\boldsymbol{m}_1^2 - \boldsymbol{m}_2^2)^2 + 2w\int_r (\boldsymbol{m}_1 \cdot \boldsymbol{m}_2)^2$$

Second-order coefficient:

$$r_0 = r_0 + k^2 + \gamma|\omega_n|$$

- Three-types of magnetic order minimize free-energy

SVC



$w > 0,\ g < 0$

Spin-vortex crystal (SVC)

$\langle m_1 \rangle = \langle m_2 \rangle \neq 0$

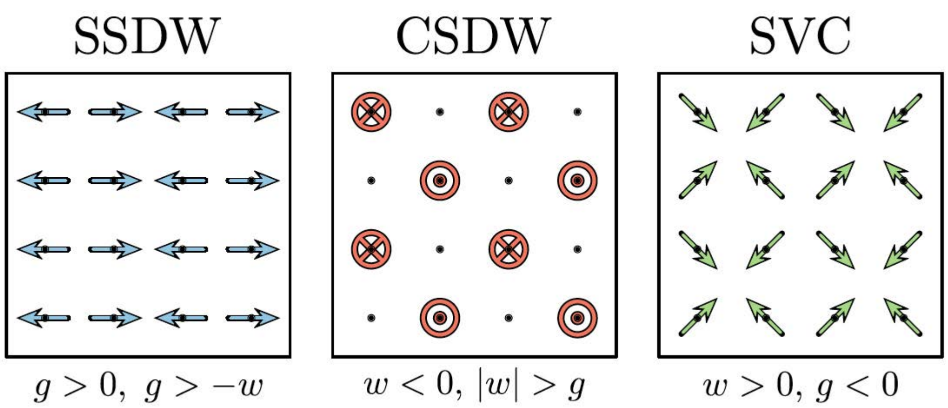
$\langle \mathbf{m}_1 \rangle \perp \langle \mathbf{m}_2 \rangle$

Non-collinear double-Q

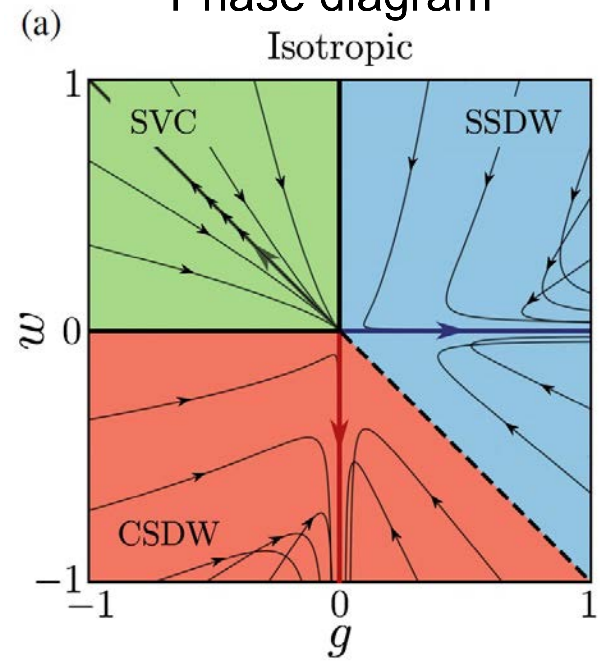
[1] Lorenzana et al, PRL (2008); [2] Eremin et al, PRB (2010); [3] Brydon et al, PRB (2011); [4] Giovannetti et al, Nat. Commun. (2011
Wang et al., PRB (2015), M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018).

Spin-density waves on the square lattice (primary orders)

- Three-types of magnetic order minimize free-energy



Phase diagram

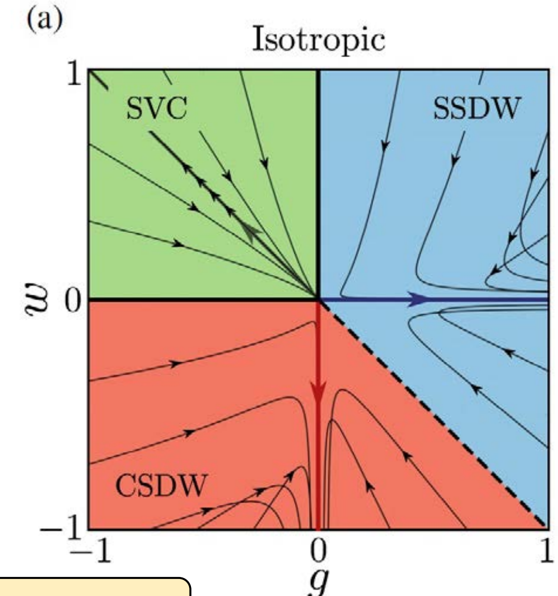


- All states break spatial symmetries in addition to SO(3)

[1] Lorenzana et al, PRL (2008); [2] Eremin et al, PRB (2010); [3] Brydon et al, PRB (2011); [4] Giovannetti et al, Nat. Commun. (2011
Wang et al., PRB (2015), M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018).

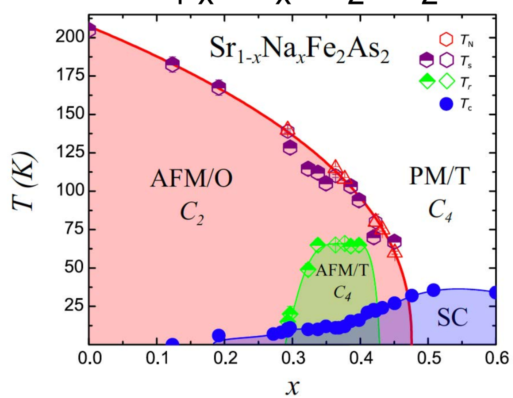
Interlude: complexity from universality

- Mean-field analysis so far
- Interactions are relevant as system is at upper critical dimension (d=2, z=2)
- Perform renormalization group analysis
- Find mean-field phases are stable with respect to interactions
- Transitions become first-order

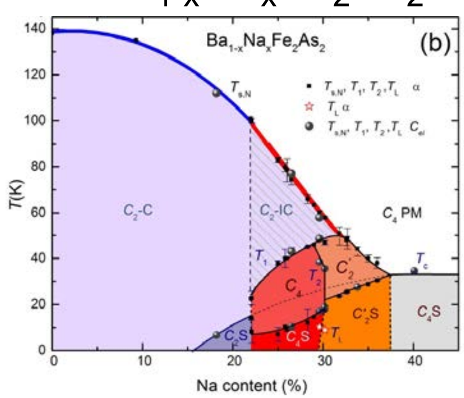


Complexity of magnetic phases close to SC dome fine tuned?

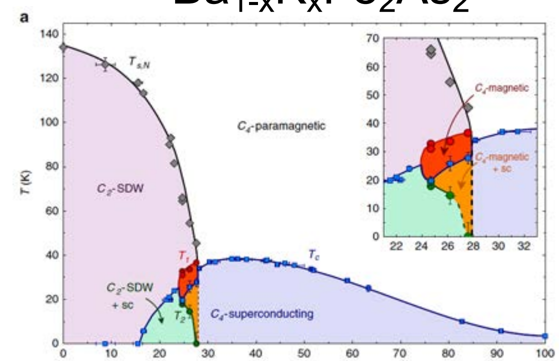
$Sr_{1-x}Na_xFe_2As_2$



$Ba_{1-x}Na_xFe_2As_2$



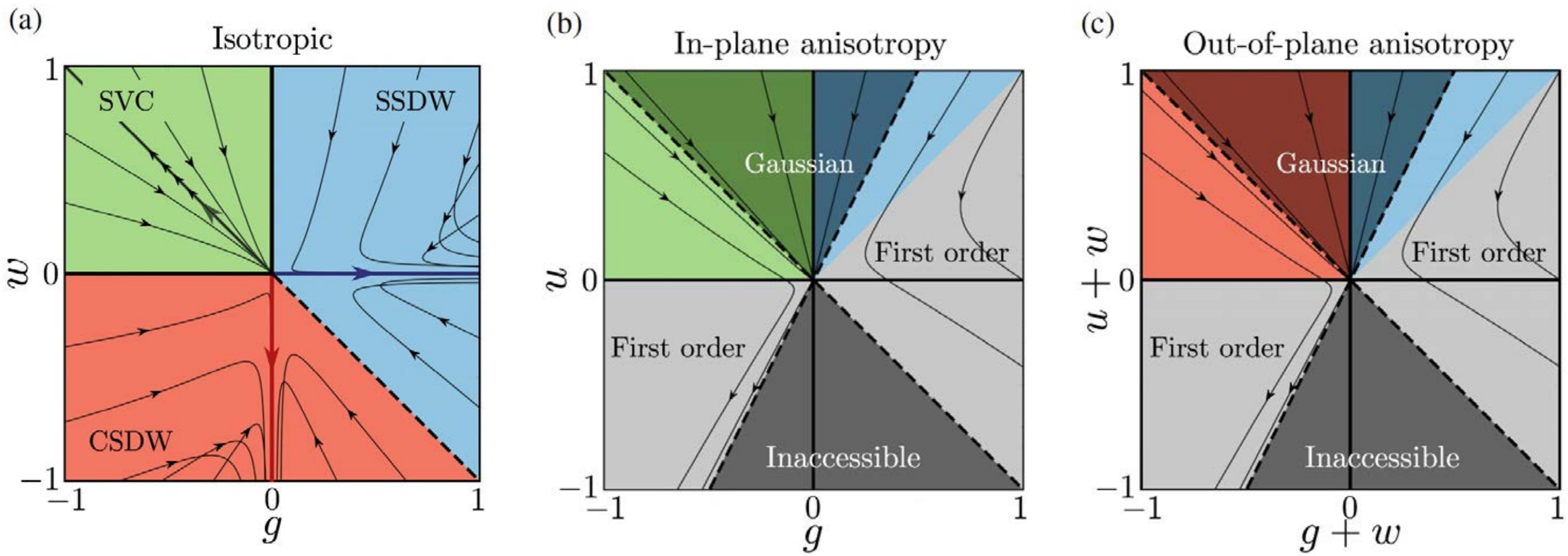
$Ba_{1-x}K_xFe_2As_2$



K content (%)

Böhmer et al., Nat. Commun. (2015); Tadei et al, PRB (2015).

Emergent magnetic degeneracy from spin-orbit



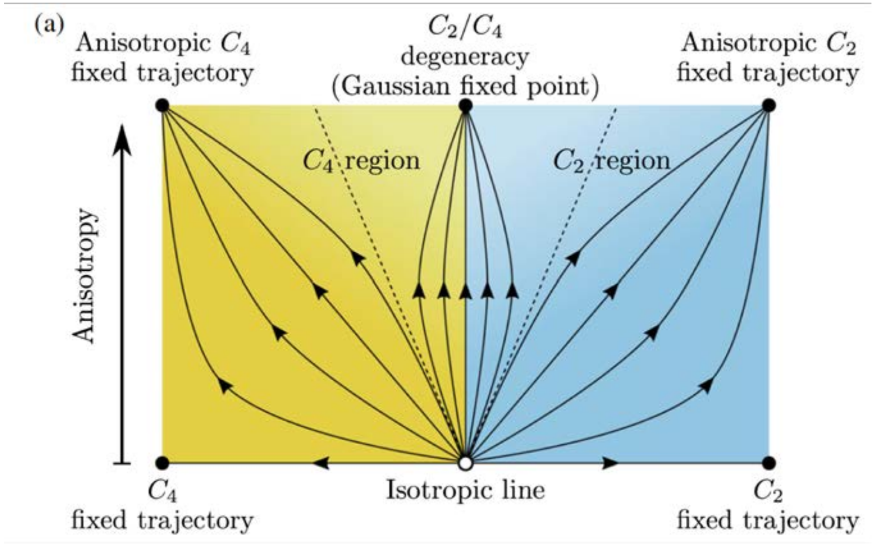
Spin-orbit coupling amplified by interactions (in RG flow):

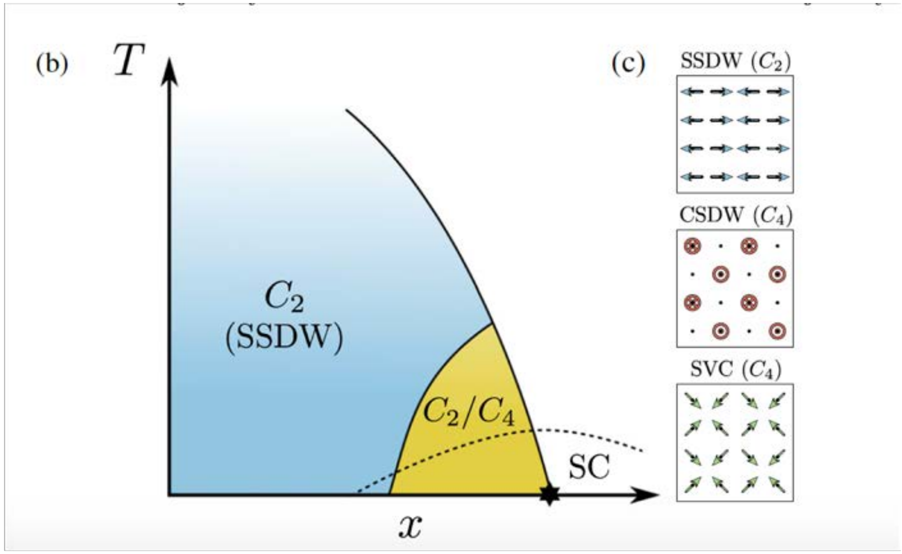
$$S^{(2)}_{\mathrm{SOC}} = \int_k r_0(k)(\mathbf{M}_1^2 + \mathbf{M}_2^2) + \alpha_1 \int_k (M_{x,1}^2 + M_{y,2}^2) + \alpha_2 \int_k (M_{x,2}^2 + M_{y,1}^2) + \alpha_3 \int_k (M_{z,1}^2 + M_{z,2}^2)$$

- New Gaussian fixed point emerges at finite spin-orbit coupling
- Magnetic phases become degenerate at low energies
- Proliferation of new magnetic phases expected at QCP

M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018), PRB (2018).

Universality + SOC leads to magnetic degeneracy

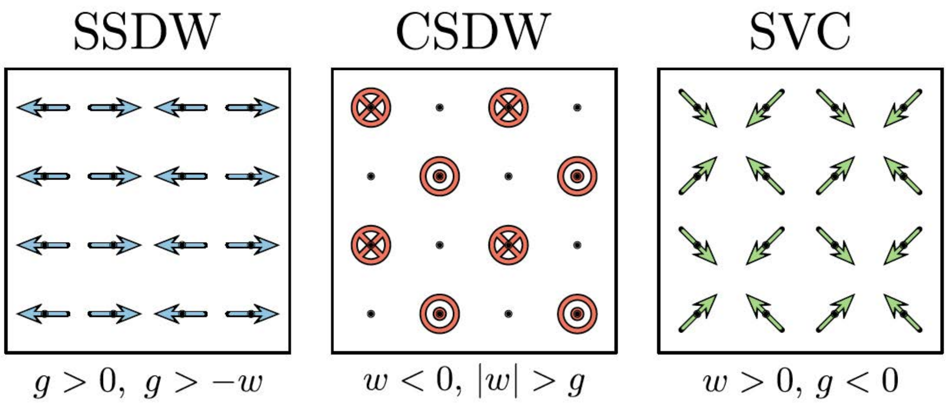




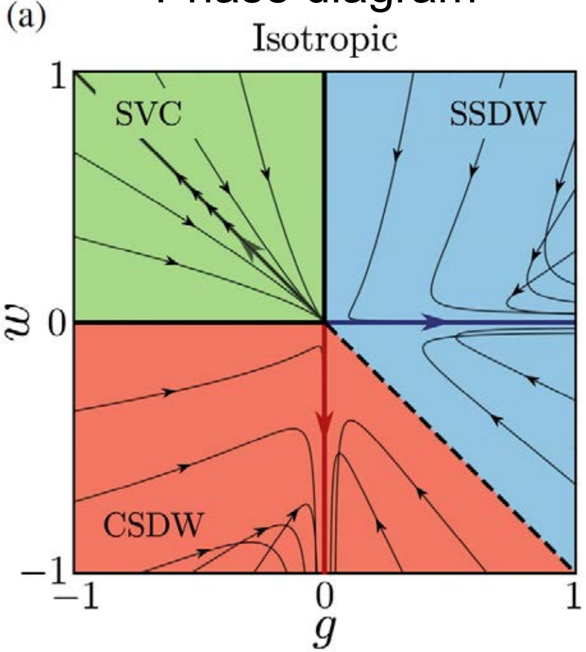
- RG flow continues on longer scales close to QCP
- Phases become more and more degenerate
- Basin of attraction of Gaussian fixed point increases with anisotropy

Vestigial orders from spin-density waves on the square lattice

- Three-types of magnetic order minimize free-energy



Phase diagram



- All states break spatial symmetries in addition to SO(3)
- Construct composite order parameters

$$\boxed{\phi_m = \sum_{\alpha,\beta=1}^{d_\Gamma} \eta_\alpha^* \Lambda_{\alpha\beta}^m \eta_\beta}$$

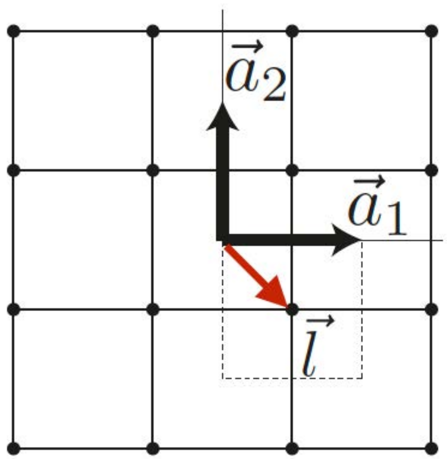
Symmetry group and irrep of primary order?

- ➢ Primary order breaks translational symmetry
- ➢ Use extended point groups (treat as q=0 order)

[1] Lorenzana et al, PRL (2008); [2] Eremin et al, PRB (2010); [3] Brydon et al, PRB (2011); [4] Giovannetti et al, Nat. Commun. (2011); Wang et al., PRB (2015), M. H. Christensen, PPO, B. Andersen, R. M. Fernandes, PRL (2018).

Extended point group C_{4v}'''

- Extended point group: add translations $t_1=a_1$, $t_2=a_2$, a_1+a_2 to point group elements
- Spin-density wave with $Q_{1,2}$ then becomes $q = 0$ order
- Can analyze irreps of extended point group (instead of space group)



C_{4v} ($h=8$)	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$
A_1	1	1	1	1	1
A_2	1	1	1	–1	–1
B_1	1	–1	1	1	–1
B_2	1	–1	1	–1	1
E	2	0	–2	0	0

$$\otimes\{t_1, t_2, t_1 + t_2\}$$

[1] Basko, PRB (2012); [2] Serbyn, Lee, PRB (2013); [3] Venderbos, PRB (2016).

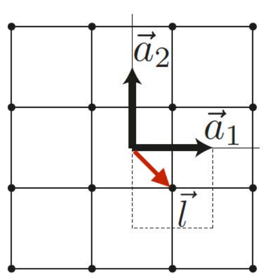
Vestigial order of $Q_{1,2}$ SDW on square lattice

TABLE VIII. Character table of the point group C'''_{4v}. Translations t_1 and t_2 correspond to $T(\vec{a}_1)$ and $T(\vec{a}_2)$, respectively, and $t_3 = T(\vec{a}_1 + \vec{a}_2)$. The conjugacy classes consist of the elements $C_1''' = \{I\}$, $C_2''' = \{t_1, t_2\}$, $C_3''' = \{t_3\}$, $C_4''' = \{C_2\}$, $C_5''' = \{t_1C_2, t_2C_2\}$, $C_6''' = \{t_3C_2\}$, $C_7''' = \{C_4, C_4^{-1}, t_3C_4, t_3C_4^{-1}\}$, $C_8''' = \{t_1C_4, t_1C_4^{-1}, t_2C_4, t_2C_4^{-1}\}$, $C_9''' = \{\sigma_{v1}, \sigma_{v2}\}$, $C_{10}''' = \{t_1\sigma_{v1}, t_2\sigma_{v2}\}$, $C_{11}''' = \{t_2\sigma_{v1}, t_1\sigma_{v2}\}$, $C_{12}''' = \{t_3\sigma_{v1}, t_3\sigma_{v2}\}$, $C_{13}''' = \{\sigma_{d1}, \sigma_{d2}, t_3\sigma_{d1}, t_3\sigma_{d2}\}$, and $C_{14}''' = \{t_1\sigma_{d1}, t_1\sigma_{d2}, t_2\sigma_{d1}, t_2\sigma_{d2}\}$. The character table is taken from Ref. [21]. Notation is altered with respect to Ref. [21] to be consistent with the notation and definitions of this work.

Conjugacy class		t_1, t_2					C_4		σ_v				σ_d	
Point group C'''_{4v}	C_1'''	C_2'''	C_3'''	C_4'''	C_5'''	C_6'''	C_7'''	C_8'''	C_9'''	C_{10}'''	C_{11}'''	C_{12}'''	C_{13}'''	C_{14}'''
A_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A_2	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1
B_1	1	1	1	1	1	1	−1	−1	1	1	1	1	−1	−1
B_2	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1
E_1	2	2	2	−2	−2	−2	0	0	0	0	0	0	0	0
A_1'	1	−1	1	1	−1	1	1	−1	1	−1	−1	1	1	−1
A_2'	1	−1	1	1	−1	1	1	−1	−1	1	1	−1	−1	1
B_1'	1	−1	1	1	−1	1	−1	1	1	−1	−1	1	−1	1
B_2'	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1
E_1'	2	−2	2	−2	2	−2	0	0	0	0	0	0	0	0
E_2	2	0	−2	2	0	−2	0	0	−2	0	0	2	0	0
E_3	2	0	−2	2	0	−2	0	0	2	0	0	−2	0	0
E_4	2	0	−2	−2	0	2	0	0	0	2	−2	0	0	0
E_5	2	0	−2	−2	0	2	0	0	0	−2	2	0	0	0

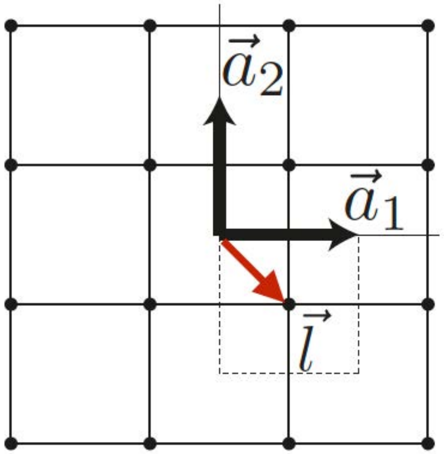
SDW order:

$$\boldsymbol{S}(\boldsymbol{r}) = \boldsymbol{m}_1 \sin(\boldsymbol{Q}_1 \cdot \boldsymbol{r}) + \boldsymbol{m}_2 \sin(\boldsymbol{Q}_2 \cdot \boldsymbol{r})$$



- Symmetry group: $\mathcal{G} = C_{4v}''' \otimes \mathrm{SO}(3)$
- Primary order $\eta_\alpha = (\boldsymbol{m}_1, \boldsymbol{m}_2)_\alpha$
- Transforms as $\Gamma = E_5 \otimes \Gamma^{S=1}$

Extended point group C_{4v}'''



- Symmetry group: $\mathcal{G} = C_{4v}''' \otimes \mathrm{SO}(3)$
- Primary order $\eta_\alpha = (\boldsymbol{m}_1, \boldsymbol{m}_2)_\alpha$
- Transforms as $\Gamma = E_5 \otimes \Gamma^{S=1}$

$$\boxed{\Gamma \otimes \Gamma = (E_5 \otimes E_5) \otimes (\Gamma^{S=1} \otimes \Gamma^{S=1})}$$

Spatial irreps: $E_5 \otimes E_5 = A_1 \oplus B_2' \oplus A_2' \oplus B_1$

Spin irreps

$$\Gamma^{S*} \otimes \Gamma^{S} = \bigoplus_{j=0}^{2S} \Gamma^{j}.$$

Construct all possible bilinears:

$$\boxed{\phi^{\mu}_{m\equiv(r,j)} = \sum_{A,B} \eta_A \Lambda^{m,\mu}_{A,B} \eta_B,}$$

Pauli matrices τ^r (4x)

$$\Gamma \otimes \Gamma = (A_1 \oplus B_2' \oplus A_2' \oplus B_1) \otimes (\Gamma^0 \oplus \Gamma^1 \oplus \Gamma^2)$$

Gell-Mann matrices λ^j (9x)

$$\Lambda^{m\equiv(r,j),\mu}_{A,B} = \tau^r_{ab} \lambda^{j,\mu}_{\alpha\beta}\,.$$

Singlet (scalar) vestigial orders

$$\Gamma \otimes \Gamma = (A_1 \oplus B_2' \oplus A_2' \oplus B_1) \otimes (\Gamma^0 \oplus \Gamma^1 \oplus \Gamma^2)$$

- S=0 singlet in spin space: scalar vestigial orders

$$\phi^{\mu}_{m\equiv(r,j)} = \sum_{A,B} \eta_A \Lambda^{m,\mu}_{A,B} \eta_B, \quad \text{with} \quad \lambda^{0,0}_{\alpha\beta} = \delta_{\alpha\beta}$$

$$\phi_{(0,0)} = \mathbf{m}_1 \cdot \mathbf{m}_1 + \mathbf{m}_2 \cdot \mathbf{m}_2$$ A_1 amplitude fluctuations (no broken symmetry

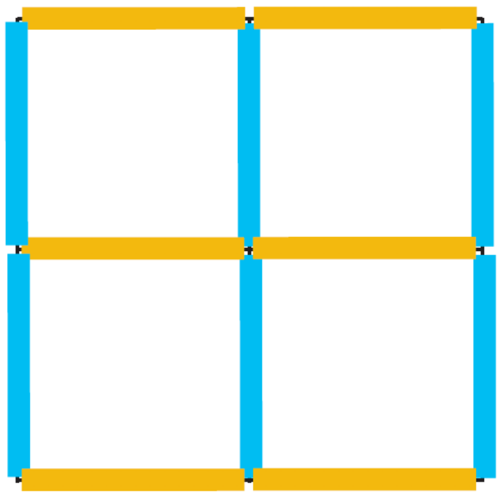
$$\phi_{(1,0)} = 2\mathbf{m}_1 \cdot \mathbf{m}_2$$ Composite order transforming as B_2' (CSDW)

$$\phi_{(3,0)} = \mathbf{m}_1 \cdot \mathbf{m}_1 - \mathbf{m}_2 \cdot \mathbf{m}_2$$ Composite order transforming as B_1 (nematic)

Nematic composite B_1 order

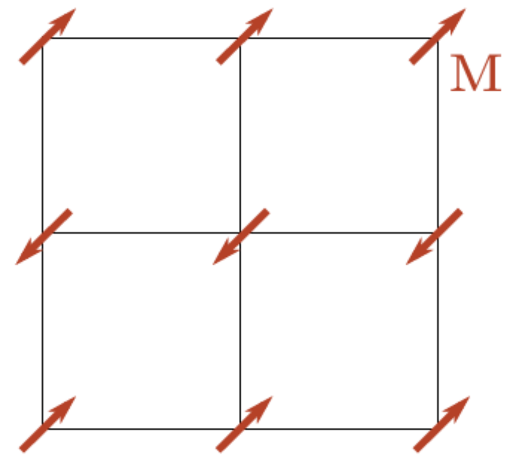
$$\phi_{(3,0)} = \mathbf{m}_1 \cdot \mathbf{m}_1 - \mathbf{m}_2 \cdot \mathbf{m}_2$$

Composite order transforming as B_1 (nematic)



Real-space: bond order

- Translations t_1, t_2 preserved
- C_4 and σ_d broken



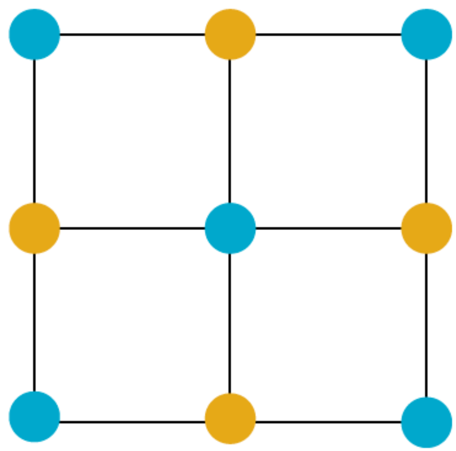
Primary order: SSDW

Conjugacy class		t_1, t_2					C_4		σ_v				σ_d	
Point group C'''_{4v}	C'''_1	C'''_2	C'''_3	C'''_4	C'''_5	C'''_6	C'''_7	C'''_8	C'''_9	C'''_{10}	C'''_{11}	C'''_{12}	C'''_{13}	C'''_{14}
$\bar{B}_1$	1	1	1	1	1	1	−1	−1	1	1	1	1	−1	−1

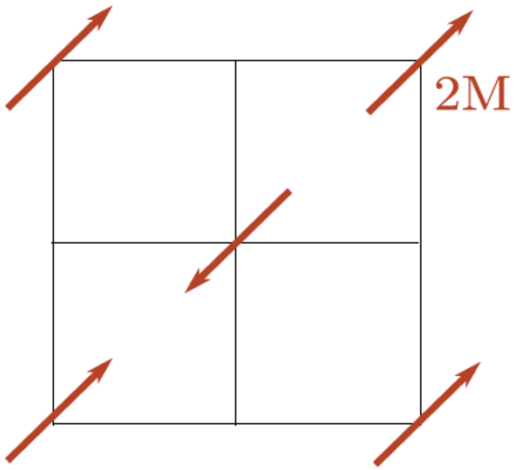
CDW (checkerboard) composite B_2' order

$$\phi_{(1,0)} = 2\mathbf{m}_1 \cdot \mathbf{m}_2$$

Composite order transforming as B_2' (SVDW)



- Translations t_1, t_2 broken
- C_4 and σ_v broken



Real-space: site order

Primary order: CSDW

Conjugacy class		t_1, t_2					C_4		σ_v				σ_d	
Point group C_{4v}'''	C_1'''	C_2'''	C_3'''	C_4'''	C_5'''	C_6'''	C_7'''	C_8'''	C_9'''	C_{10}'''	C_{11}'''	C_{12}'''	C_{13}'''	C_{14}'''
B_2'	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1
B_1	1	1	1	1	1	1	−1	−1	1	1	1	1	−1	−1

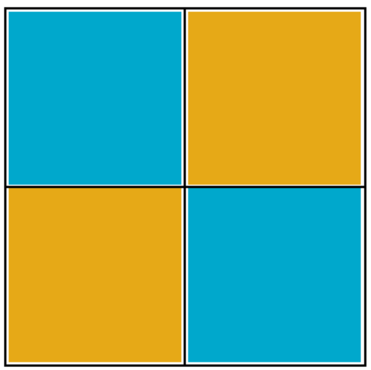
(Pseudo)-vector vestigial orders

$$\Gamma \otimes \Gamma = (A_1 \oplus B_2' \oplus A_2' \oplus B_1) \otimes (\Gamma^0 \oplus \Gamma^1 \oplus \Gamma^2)$$

- S=1 triplet in spin space: vector vestigial orders

$$\phi^{\mu}_{m\equiv(r,j)} = \sum_{A,B} \eta_A \Lambda^{m,\mu}_{A,B} \eta_B,$$ with $\lambda^{1,\mu}_{\alpha\beta} = i\epsilon_{\alpha\beta\mu}$

$$\phi_{(2,1)} = 2\mathbf{m}_1 \times \mathbf{m}_2$$



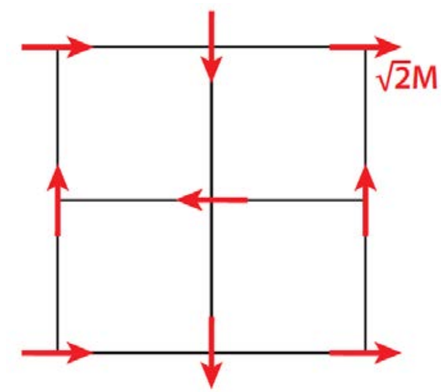
Real-space: plaquette order

Composite order transforming as A_2':

➢ preserves C_4

➢ breaks mirrors

➢ breaks translation



Primary order: spin-vortex crystal

Conjugacy class		t_1, t_2					C_4		σ_v				σ_d	
Point group C_{4v}'''	C_1'''	C_2'''	C_3'''	C_4'''	C_5'''	C_6'''	C_7'''	C_8'''	C_9'''	C_{10}'''	C_{11}'''	C_{12}'''	C_{13}'''	C_{14}'''
A_2'	1	−1	1	1	−1	1	1	−1	−1	1	1	−1	−1	1

Tensorial vestigial orders

$$\Gamma \otimes \Gamma = (A_1 \oplus B_2' \oplus A_2' \oplus B_1) \otimes (\Gamma^0 \oplus \Gamma^1 \oplus \Gamma^2)$$

- S=2 symmetric tensor in spin space

$$\phi^{\mu}_{m\equiv(r,j)} = \sum_{A,B} \eta_A \Lambda^{m,\mu}_{A,B} \eta_B, \text{ with } \lambda^{2,(\mu,\mu')}_{\alpha\beta} = \frac{1}{2}\left(\delta_{\alpha\mu}\delta_{\beta\mu'} + \delta_{\alpha\mu'}\delta_{\beta\mu}\right) - \frac{1}{3}\delta_{\alpha\beta}\delta_{\mu\mu'}.$$

$$\phi^{\mu\mu'}_{(0,2)} = m_1^{\mu} m_1^{\mu'} + m_2^{\mu} m_2^{\mu'} - \frac{1}{3}\delta_{\alpha\beta}\left(\mathbf{m}_1 \cdot \mathbf{m}_1 + \mathbf{m}_2 \cdot \mathbf{m}_2\right)$$

$$\phi^{\mu\mu'}_{(1,2)} = m_1^{\mu} m_2^{\mu'} + m_2^{\mu} m_1^{\mu'} - \frac{1}{3}\delta_{\mu\mu'}\left(\mathbf{m}_1 \cdot \mathbf{m}_2 + \mathbf{m}_2 \cdot \mathbf{m}_2\right)$$

$$\phi^{\mu\mu'}_{(3,2)} = m_1^{\mu} m_1^{\mu'} - m_2^{\mu} m_2^{\mu'} - \frac{1}{3}\delta_{\mu\mu'}\left(\mathbf{m}_1 \cdot \mathbf{m}_1 - \mathbf{m}_2 \cdot \mathbf{m}_2\right)$$

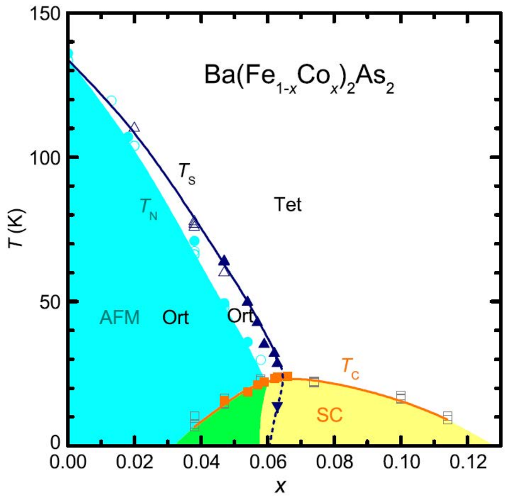
Composite tensorial orders:

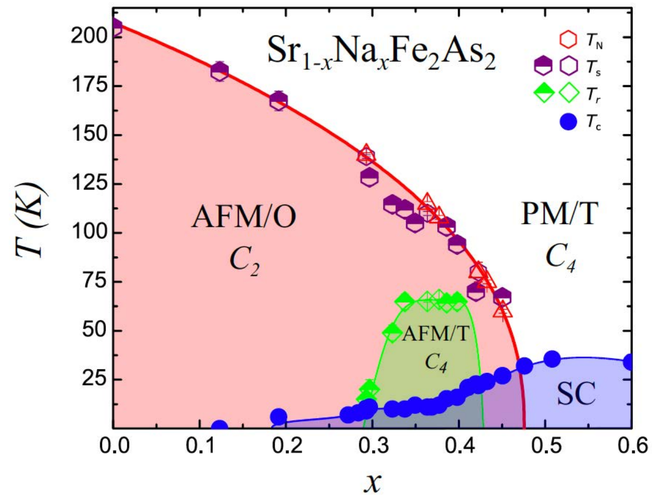
➢ $\phi_{(0,2)}$ transforms spatially as A_1: nematicity purely in spin space ($NiGa_2S_4$)

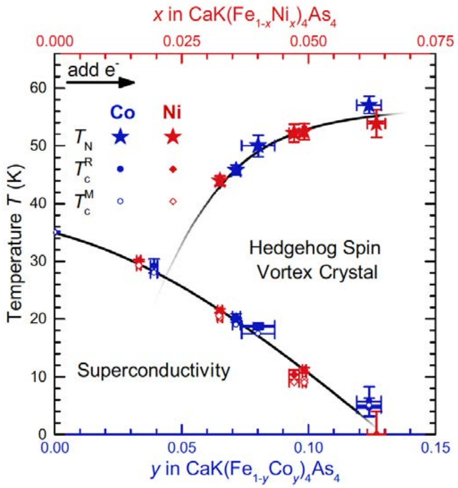
➢ Presence of spin-orbit coupling would induce nematicity in real-space as well.

[1] Chandra, Coleman (1991), [2] Nakatsuji, Science (2005).

Materials: vestigial orders in iron-based superconductors







- Size of vestigial phase depends on microscopics, size of fluctuations
  - Vestigial orders (often) found on hole-doped side
  - Not found on electron-doped side: joint first-order transiton
- Vestigial order can be induced by breaking corresponding symmetry (1144)
  - Path to control magnetic order

[1] S. Nandi *et al.*, PRL **104**, 057006 (2010); [2] Taddei et al., PRB (2016); [3] W. R. Meier et al,, npj Quantum Materials (2018).

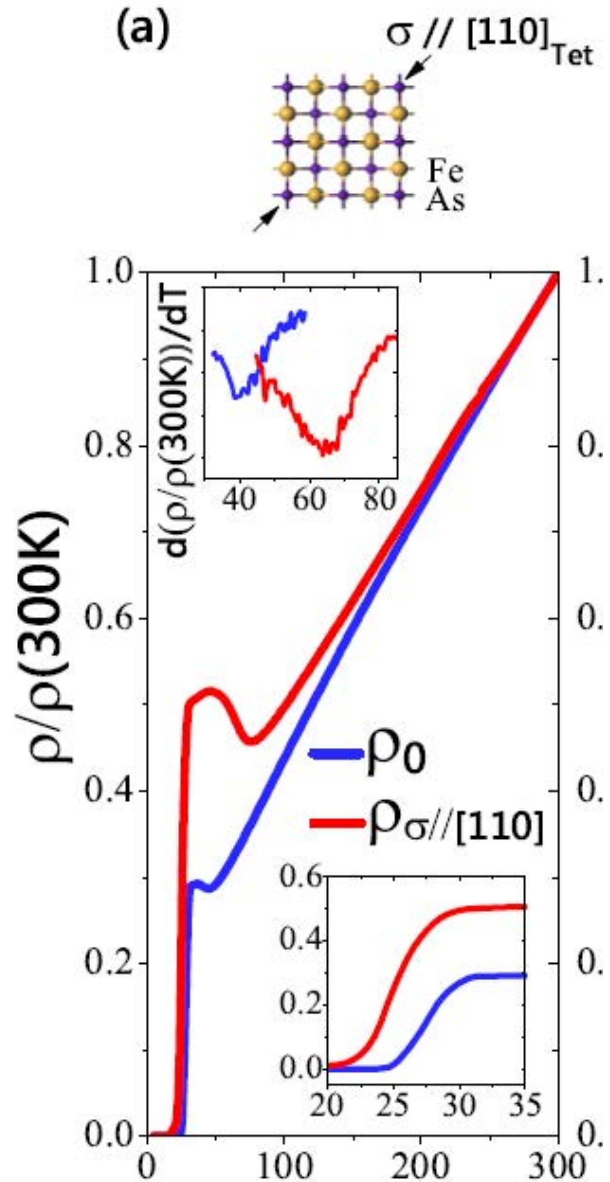
Controlling magnetic order

- **How can we control phases?**
  - Doping, pressure, magnetic field
  - Other possibilities

Coupling to vestigial order parameter

Simplest case: SSDW

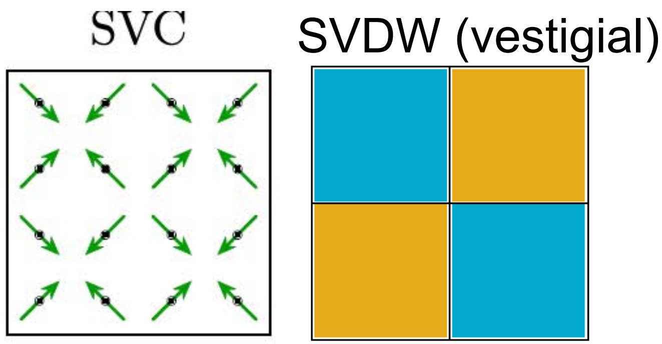
- Apply external strain σ to cause orthorhombic distortion
- Acts as "conjugate field" for emergent order parameter $\phi_{nematic}$
- $\Delta F = \sigma\phi_{nematic}$
- Transition temperature T_N to stripe order increases (27K in example)



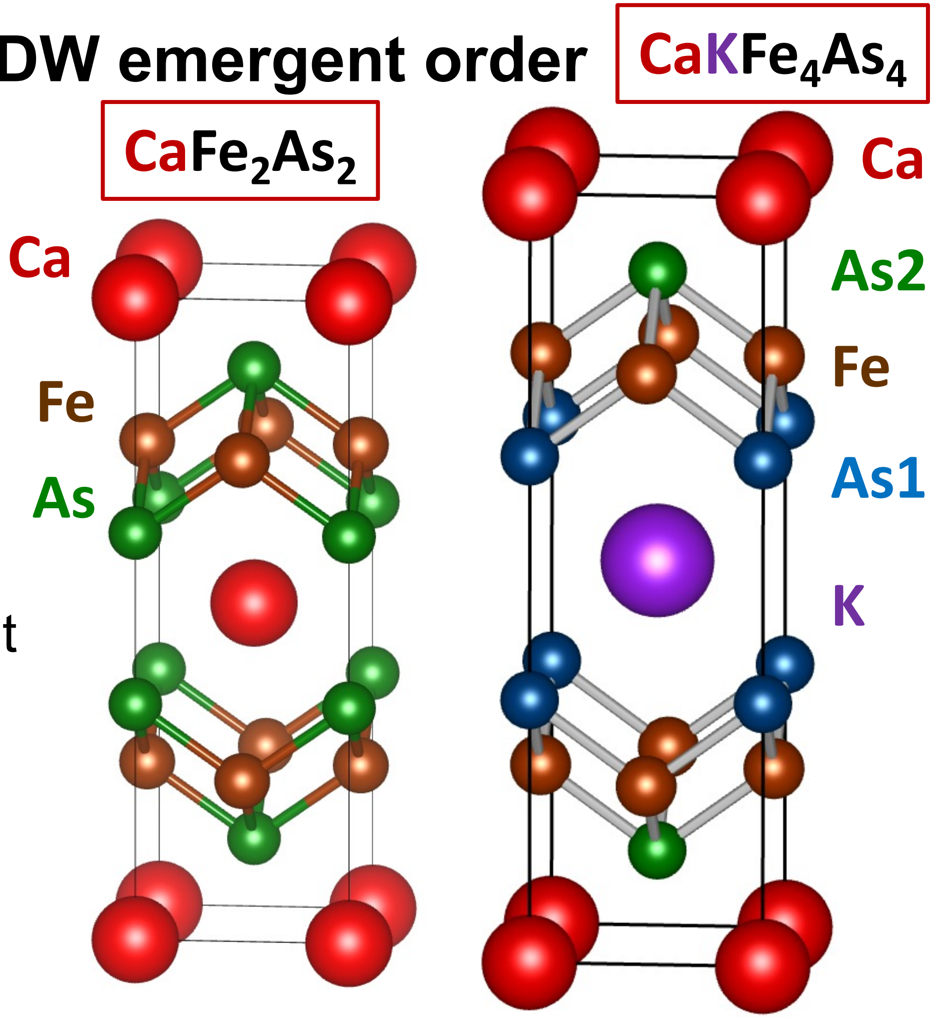
[1] Kuo et al., PRB 86, 134507 (2012)

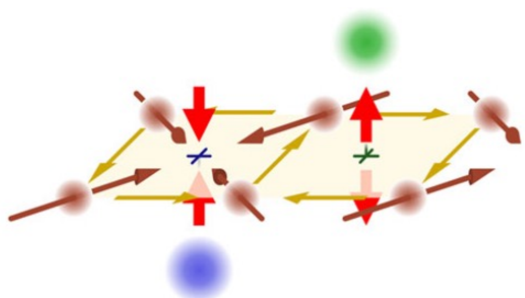
$CaKFe_4As_4$: coupling to SVDW emergent order

How to generate Spin Vortex Crystal magnetic order?



- Realize conjugate field to emergent SVDW order
- Two inequivalent As sites
- Breaks glide plane (-4) symmetry
- Lowers SVC magnetic state to be ground state



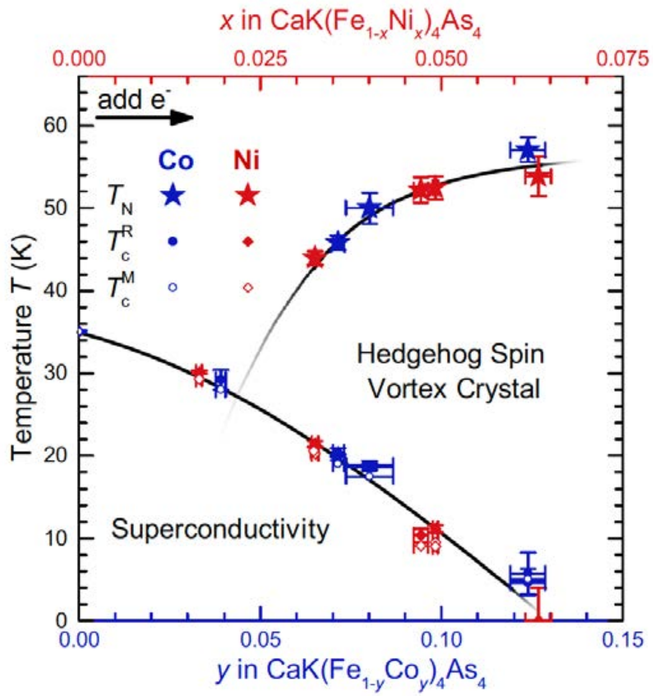


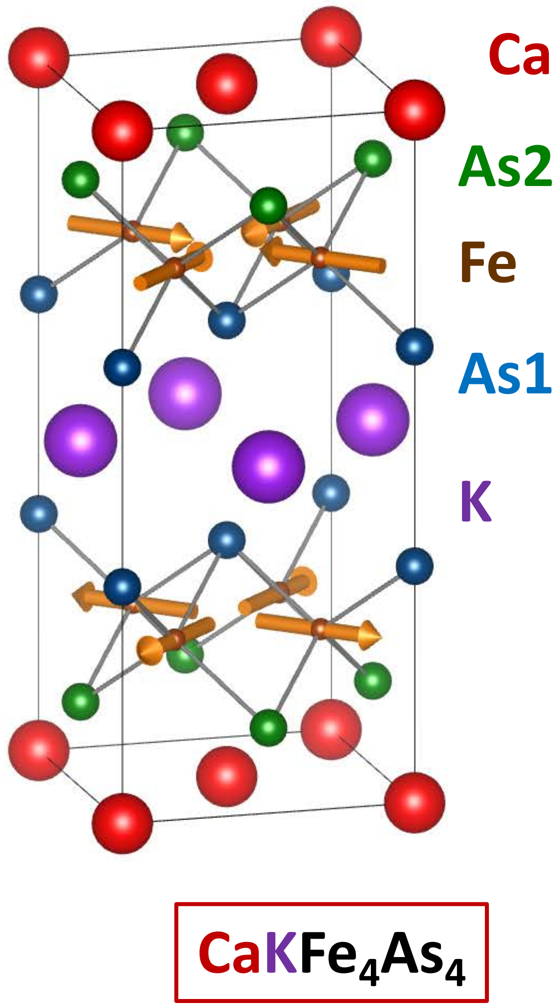
$$F_\eta = -\frac{1}{2}\eta \cdot (M_1 \times M_2)\,.$$

CaKFe$_4$As$_4$: coupling to SVDW emergent order

- Crystal structure generates conjugate field for emergent SVDW
- Lowers **SVC magnetic state** $\frac{\eta_c}{2|a|} = \sqrt{\frac{E_{GS}}{E_{SVC}(0)}} - 1$
- **First time SVC phase is experimentally realized!**

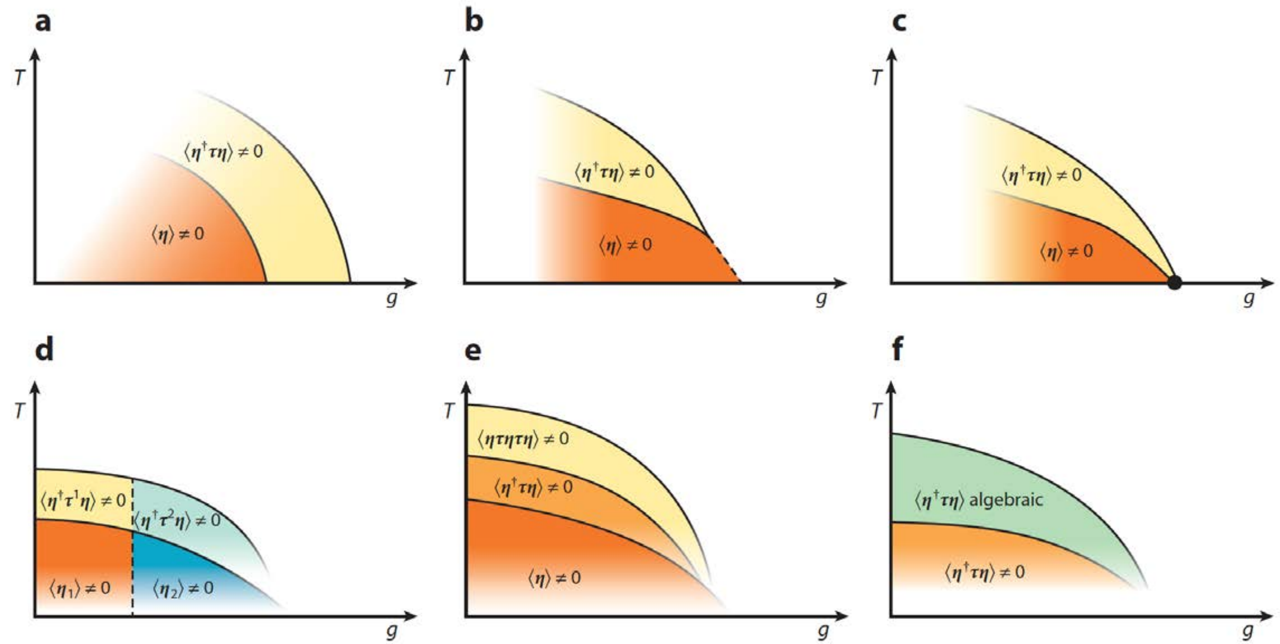
$$F_\eta = -\frac{1}{2}\boldsymbol{\eta} \cdot (\boldsymbol{M}_1 \times \boldsymbol{M}_2)\,.$$





W. R. Meier, …, PPO, ..., P. C. Canfield, npj Quantum Materials (2018).

Summary



- Vestigial orders naturally leads to complex phase diagrams with multiple phases having comparable transition temperatures
- Powerful concept to explain complexity of quantum materials beyond competing orders
- Vestigial order can be used to control phase diagram, including the associated primary “mother” phase

[1] R. M. Fernandes, PPO, J. Schmalian, Annu. Rev. Cond. Mat., in press (2019).