Universal Post-quench Dynamics at a Quantum Critical Point

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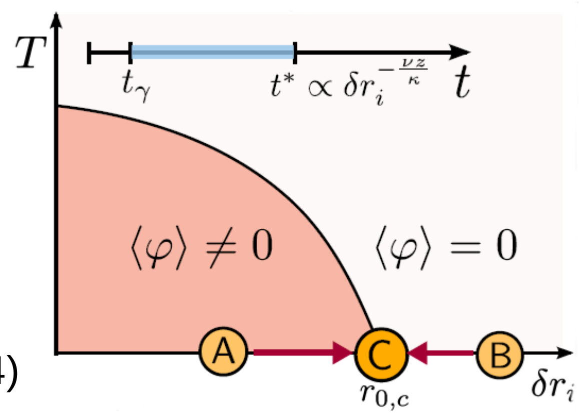
Rutgers University, 10 March 2016



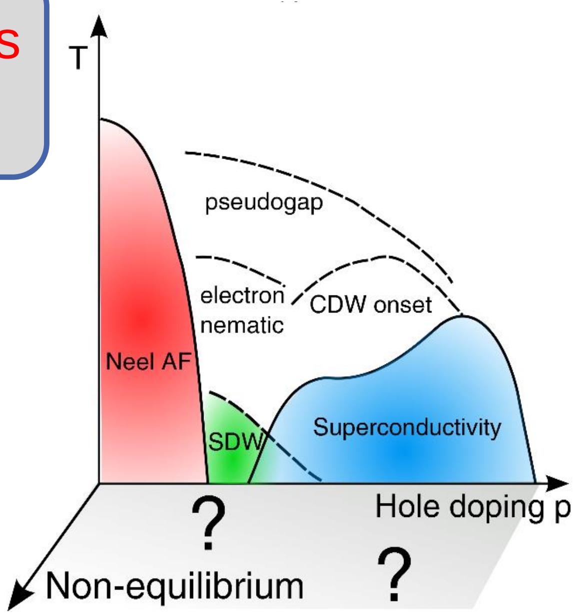
References:
P. Gagel, P. P. Orth, J. Schmalian
Phys. Rev. Lett. **113**, 220401 (2014)
Phys. Rev. B **92**, 115121 (2015).

Why probing in non-equilibrium?

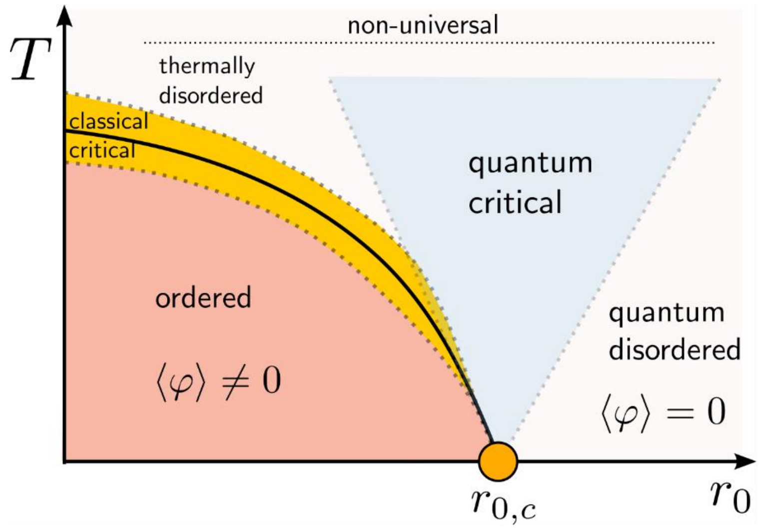
- Give access to excitations and their relaxation
  - Inelastic scattering (neutron, resonant x-ray) probes dynamic response in equilibrium
  - Pump-probe spectroscopic techniques probe non-equilibrium response
- Measure fluctuations arising from nearby competing phases

Primary interest and puzzle of correlated materials often lies in properties of excited states.

- Examples:
  - Linear in temperature resistivity in cuprates and heavy-fermions
  - Fractionalized excitations in spin liquids (e.g. α-$RuCl_3$)



Universality at classical and quantum criticality



Materials:

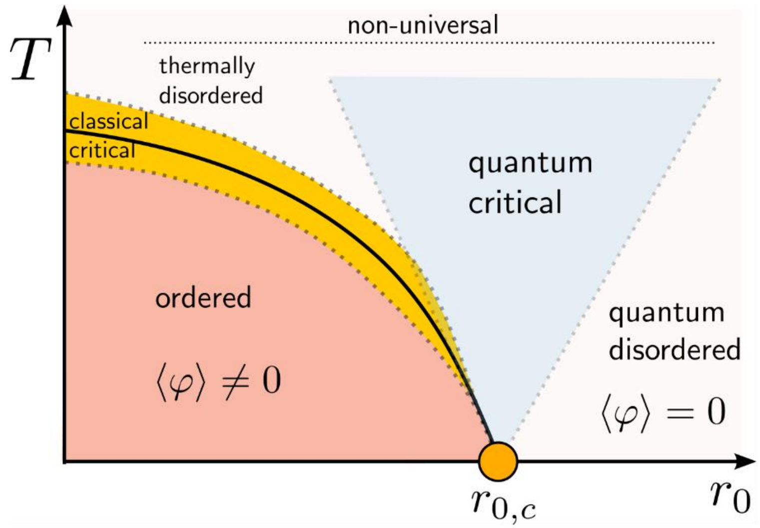
- Rare-earth magnetic insulators
- Heavy-fermion compounds
- Unconventional superconductors
- 2D electron gases

Universal behavior:

- Divergent correlation length and time
  $\xi \propto \delta r^{-\nu}$ and $\xi_\tau \propto \delta r^{-\nu z}$
- Power-laws, critical exponents
- Data collapse due to scaling
- Precise experiment-theory comparison

[1] S. Sachdev "Quantum Phase Transitions, (1999); [2] M. Vojta, Rep. Prog. Phys. **66**, 2069 (2003); [3] P. C.Hohenberg, B. I Halperin, RMP **49**, 435 (1977); [4] N. Markovic *et al.*, PRB **60**, 4320 (1999).

Universality at classical and quantum criticality



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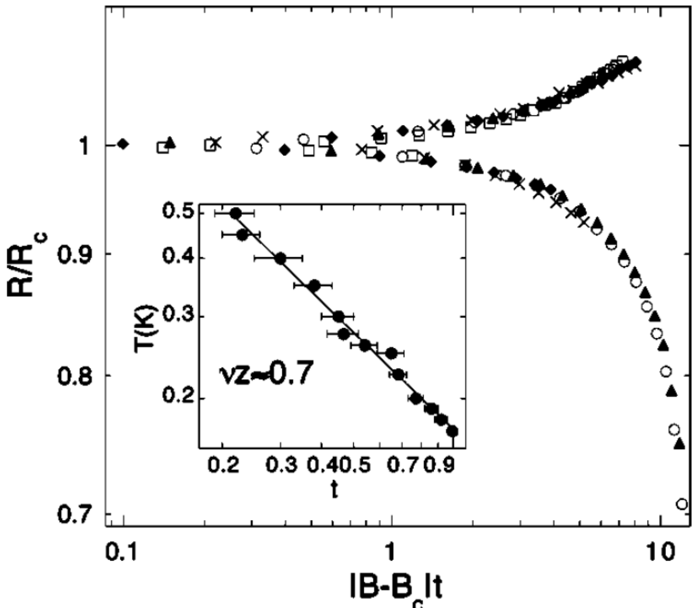

From [4]

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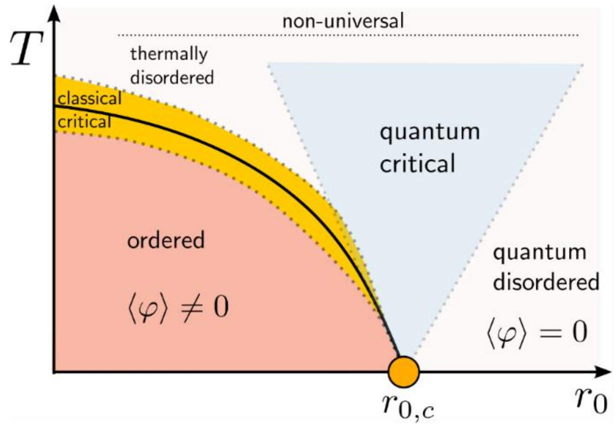
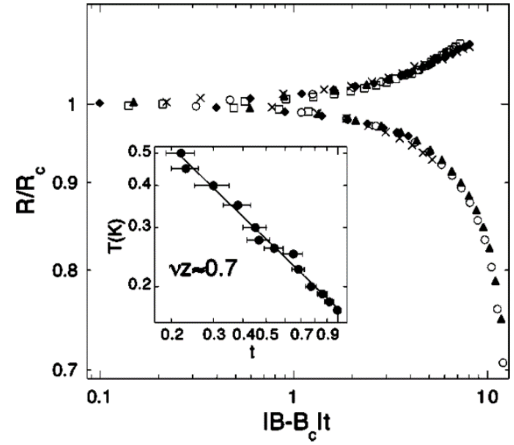




From [4]

Universality comes with potential for quantitative predictions for strongly interacting systems far from equilibrium.

Example: N-component φ[4] field theory

N=1: Ising model in transverse field ($CoNbO_6$)
N=2: Sc-insulator QPT (XY model)
N=3: quantum dimer systems ($TlCuCl_3$)

$$S = \frac{1}{2}\int d^d x \int_0^{1/T} d\tau \Big((\partial_\tau \varphi)^2 + c^2 (\nabla\varphi)^2 + r_0 \varphi^2 + \frac{u}{2}(\varphi^2)^2 \Big)$$

Here: z=1

[1] S. Sachdev "Quantum Phase Transitions, (1999); [2] M. Vojta, Rep. Prog. Phys. **66**, 2069 (2003); [3] P. C.Hohenberg, B. I Halperin, RMP **49**, 435 (1977); [4] N. Markovic *et al.*, PRB **60**, 4320 (1999).

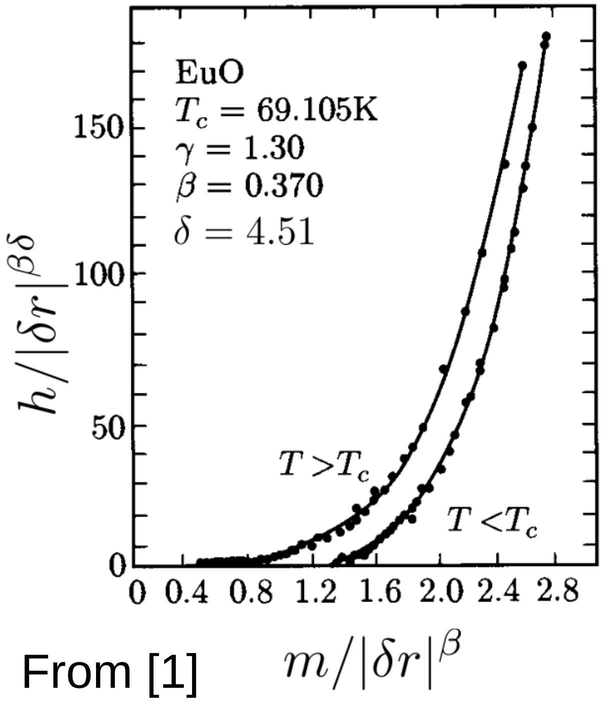
Universality in equilibrium

- Scaling of magnetization in equilibrium [1]

$$m(\delta r, h) = b^{-\beta/\nu} m(b^{1/\nu}\delta r, b^{\delta\beta/\nu} h)$$

$\Rightarrow$

$$m(\delta r, 0) \propto \delta r^{\beta}$$
$$m(0, h) \propto h^{1/\delta}$$



From [1]

Data collapses onto universal curve:

$$m(\delta r, h) = \delta r^{\beta} \Phi_m \left(h/\delta r^{\beta\delta}\right)$$

- Critical exponents define universality class:
  - Depends only on dimensionality and symmetry
  - Calculate exponents using the renormalization group in small $\epsilon = d_{up} - d$ or $1/N$

Theoretical prediction from ε-expansion [2]:

$$\gamma = 1.40, \beta = 0.38, \delta = 4.68$$

[1] C. C. Huang, J. T. Ho, PRB **12**, 5255 (1975); [2] J. C. le Guillou, J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980).

Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

Is universality in non-equilibrium characterized by new critical exponents?

Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

YES!

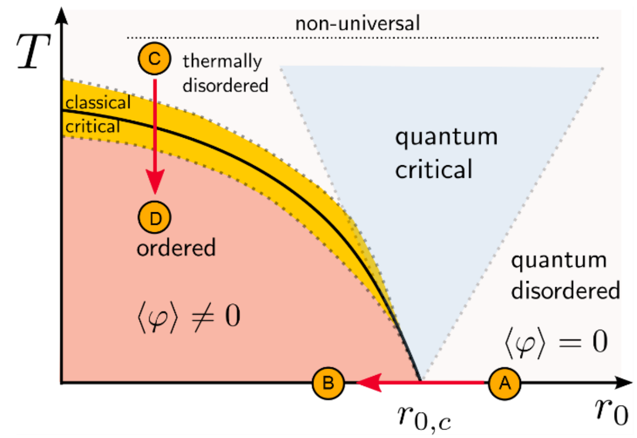
- Near-equilibrium dynamics and long-time approach to equilibrium described by power laws with equilibrium exponents [1]
- Kibble-Zurek mechanism describing defect formation in parameter sweeps through critical points [2-6]

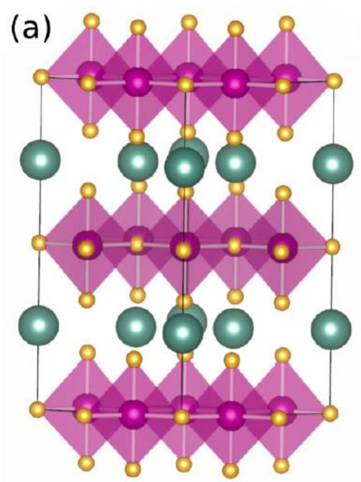
[1] P. C.Hohenberg, B. I Halperin, RMP **49**, 435 (1977); [2] T. Kibble, J. Phys. A **9**, 1387 (1976). [3] W. H. Zurek, Nature **317**, 505 (1985); [4] A. Polkovnikov *et al.* RMP **83**, 863 (2011); [5] S.-Z. Lin *et al*., Nat. Phys. **10**, 970 (2014); [6] S. M. Griffin *et al*., PRX **2**, 041022 (2012).

Universality and scaling in non-equilibrium

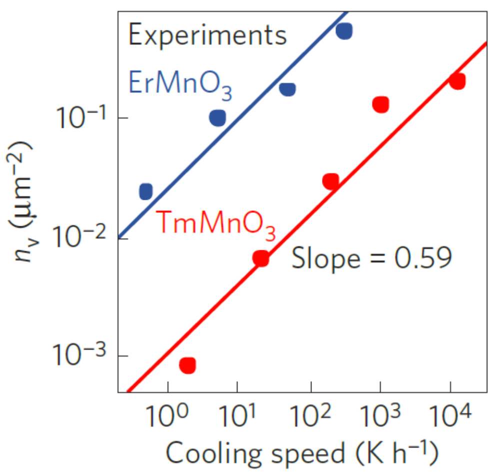
Does universality occur also in non-equilibrium situations?

<u>Example (Cheong group):</u>
Thermal Kibble-Zurek quench in hexagonal manganites $RMnO_3$ with R = Sc, Y, Dy, Lu





From [5]



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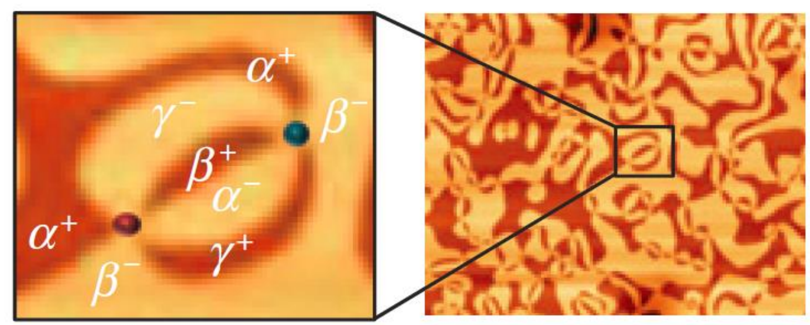

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Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

YES!

Is universality in non-equilibrium characterized by new critical exponents?

YES, sometimes! Topic of today's talk

→ Can use **non-equilibrium dynamics** as a **new tool** to study quantum critical materials.

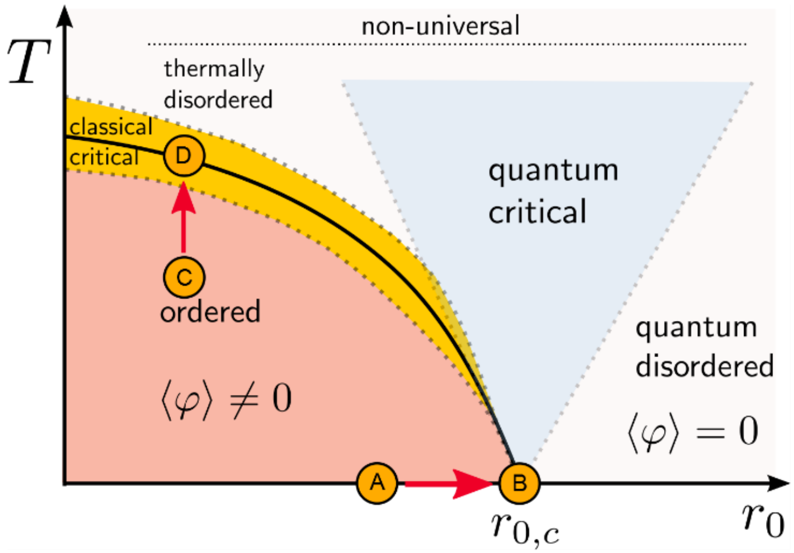
For classical systems pionnered by [1] H. Janssen *et al.*, Z. Phys. B **73**, 539 (1989).
See also [2] J. Bonart *et al.*, J. Stat. Mech. (2012) P01014.

General protocol of a quench

- Bring system to non-equilibrium by rapid change of parameter
  - Strain [1], magnetic field [2]: correlated materials
  - Temperature [3, 4]: ferroelectric materials $RMnO_3$
  - Laser intensity [5, 6, 7]: cold-atom setups

- Theoretically well-defined protocol
  - Prepare system in ground state of initial Hamiltonian
  - Perform unitary time evolution with a different Hamiltonian

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$$



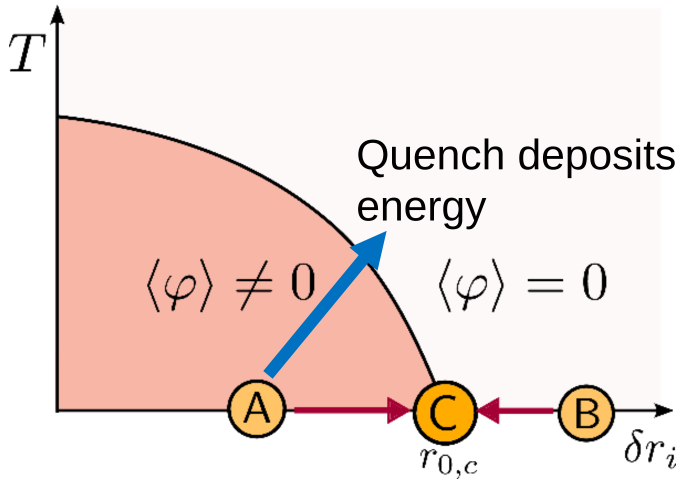
Non-equilibrium dynamics: $\mathcal{O}(t) = \langle\Psi(t)|\hat{\mathcal{O}}|\Psi(t)\rangle$

[1] C. W. Hicks et al., Science 344, 284 (2014).; [2] C. Ruegg (private communication); [3] S.-Z. Lin *et al*., Nat. Phys. **10**, 970 (2014); [4] S. M. Griffin *et al*., PRX **2**, 041022 (2012); [5] E. Nicklas *et al*., PRL **115**, 245301 (2015); [6] P. M. Preiss *et al.,* Science **347**, 1229 (2015); [7] M. Schreiben *et al.*, Science **349**, 842 (2015)

Model and quench protocol

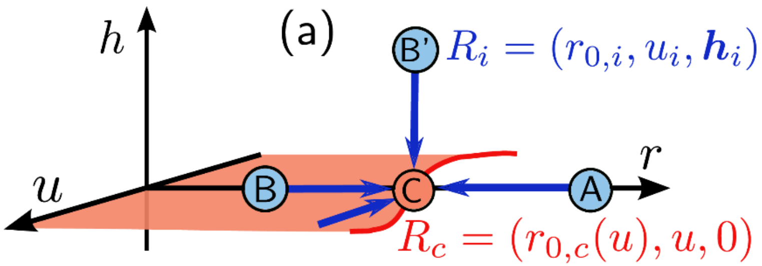
Hamiltonian: N-component φ[4]-field theory

$$H_s(t) = \frac{1}{2}\int d^dx\left(\boldsymbol{\pi}^2 + (\nabla\boldsymbol{\varphi})^2 + r_0(t)\boldsymbol{\varphi}^2 + \frac{u(t)}{2N}(\boldsymbol{\varphi}\cdot\boldsymbol{\varphi})^2 - \boldsymbol{h}(t)\boldsymbol{\varphi}\right)$$



Equilibration?
At finite temperature?

Sudden **quench protocol** (fast KZ sweep)



$$r_0(t) = r_{0,i} + \theta(t)(r_{0,c} - r_{0,i})$$

Model and quench protocol

Hamiltonian: N-component φ[4]-field theory coupled to a bath

$$H_s(t) = \frac{1}{2}\int d^dx\Big(\boldsymbol{\pi}^2 + (\nabla\boldsymbol{\varphi})^2 + r_0(t)\boldsymbol{\varphi}^2 + \frac{u(t)}{2N}(\boldsymbol{\varphi}\cdot\boldsymbol{\varphi})^2 - \boldsymbol{h}(t)\boldsymbol{\varphi}\Big)$$

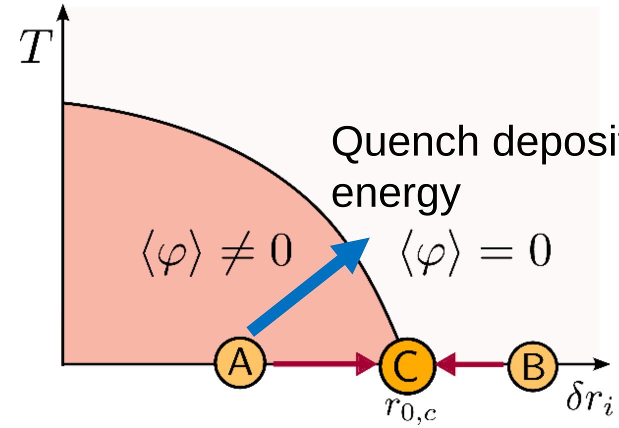
$$H_{sb} = \frac{1}{2}\sum_j \int d^dx \boldsymbol{X}_j \cdot \boldsymbol{\varphi} \qquad H_b = \frac{1}{2}\sum_j \int d^dx\Big(\boldsymbol{P}_j^2 + \Omega_j^2 \boldsymbol{X}_j^2\Big)$$

Bath spectral function: $\mathrm{Im}\eta(\omega) = \gamma\omega|\omega|^{-1+2/z}e^{-|\omega|/\omega_c}$

Ohmic z = 2
Sub-Ohmic, z < 2
Super-Ohmic, z > 2

Induces dissipation, dynamic exponent z > 1

Bath ensures equilibration at T = 0 at long times.



Quench in closed system: [1] A. Chandran *et al.*, Phys. Rev. B **86**, 064304 (2012),
[2] A. Chiocchetta *et al.* Phys. Rev. B **91**, 220302 (2015), A. Maraga *et al.* arXiv:1506.04528 (2015).

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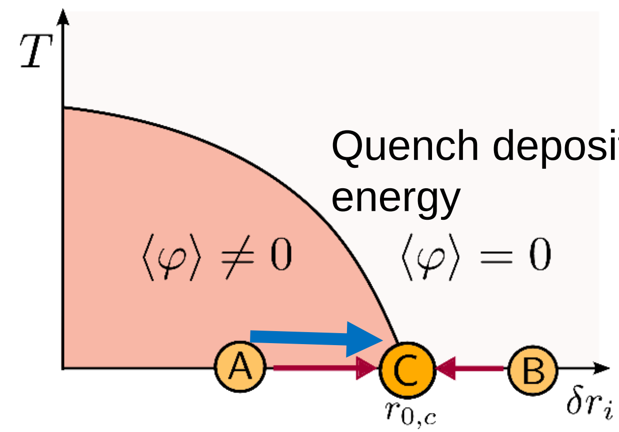

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Main results and scaling analysis

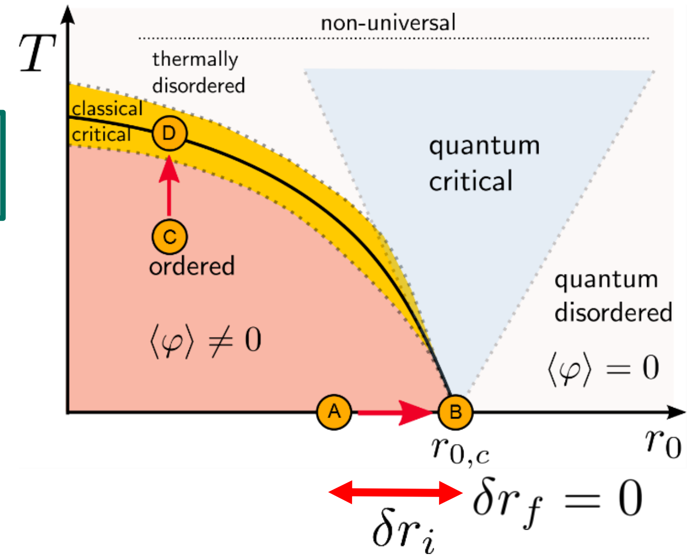
Scaling in non-equilibrium: order parameter

- Universal dynamics of the order parameter

$$m(\delta r_i, \delta r_f, t) = b^{-\beta/\nu} m(b^{\kappa/\nu}\delta r_i, b^{1/\nu}\delta r_f, b^{-z}t)$$

Quench right to the critical point

$$m(\delta r_i, 0, t) = t^{-\beta/(\nu z)}\Phi(t^{\kappa/(\nu z)}\delta r_i)$$



Classical post-quench dynamics: [1] H. Janssen *et al*. Z. Phys. B **73**, 539 (1989); [2] J. Bonart *et al.*, J. Stat. Mech. (2012) P01014.

Quantum post-quench dynamics: [3] P. Gagel, PPO, J. Schmalian, PRL **113**, 220401 (2014); [4] P. Gagel, PPO, J. Schmalian, PRB **92**, 115121 (2015).
For closed systems see, e.g., [5] A. Chiocchetta *et al.* Phys. Rev. B **91**, 220302 (2015)

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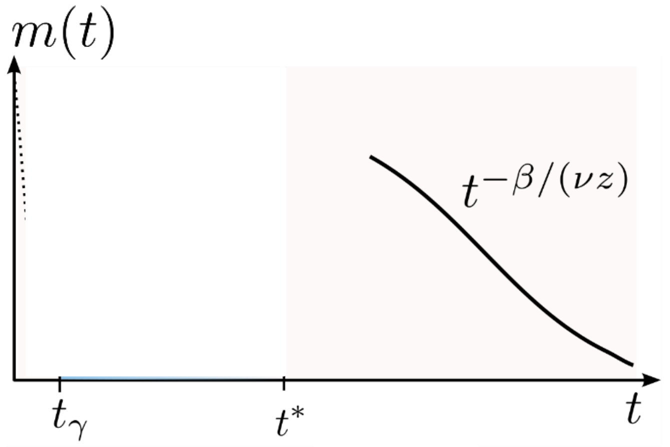

Long times: $\Phi(y \gg 1) = \text{const.}$

$$\Rightarrow \boxed{m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)}}$$

Long-time approach to equilibrium is described by equilibrium scaling exponents.

Scaling in non-equilibrium: order parameter

Quench right to the critical point

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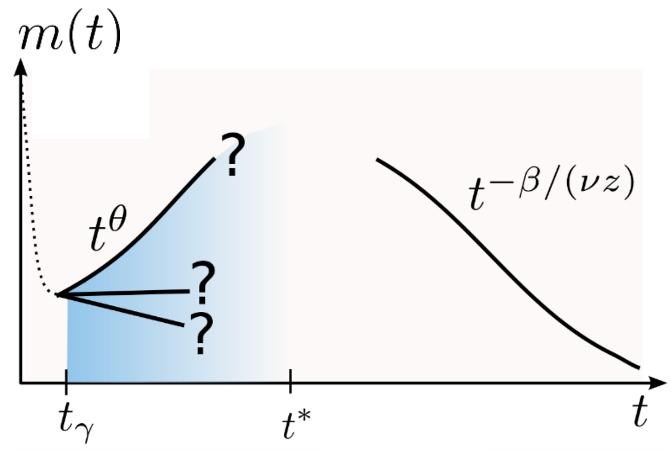

Long times: $m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)}$

Short times: $\Phi(y \ll 1) = y^{\beta}$

$$\Rightarrow \quad m(\delta r_i, 0, t) \propto t^{\beta(\kappa-1)/(\nu z)} = t^{\theta}$$

Potential for a new dynamical exponent.

[1] P. Gagel, PPO, J. Schmalian, PRL **113**, 220401 (2014); [2] P. Gagel, PPO, J. Schmalian, PRB **92**, 115121 (2015).

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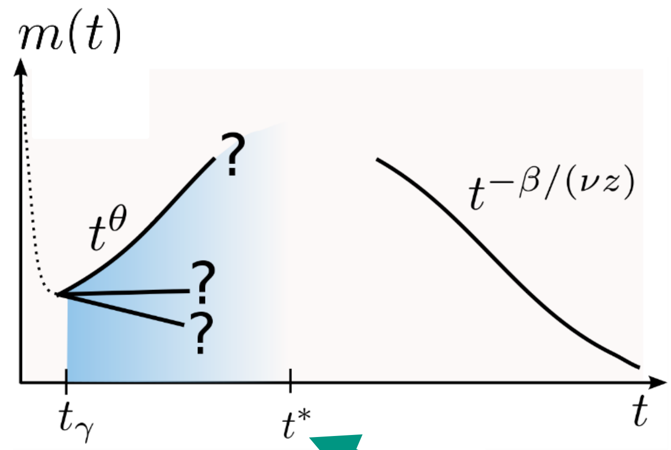


Crossover timescale:

$$t^* = \delta r_i^{-\nu z/\kappa}$$

Damping sets beginning of universal regime

$$t_\gamma \propto \gamma^{-z/(2(z-1))}$$

[1] P. Gagel, PPO, J. Schmalian, PRL **113**, 220401 (2014); [2] P. Gagel, PPO, J. Schmalian, PRB **92**, 115121 (2015).

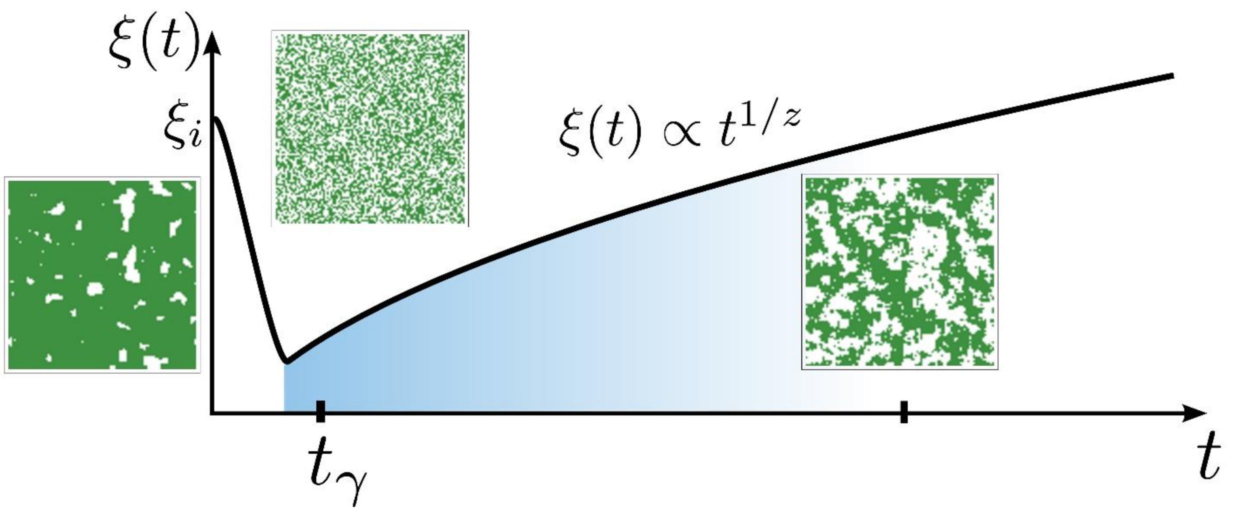
Scaling in non-equilibrium: correlation length

- Correlation length becomes time-dependent. At critical point we find

$$\xi(t) \propto t^{1/z}$$

- Rapid quench first leads to a non-universal collapse of the correlation length [1, 2]
- Then, dynamic build-up of correlations

Light-cone growth of correlation length



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Scaling in non-equilibrium: correlation functions

- Dynamic scaling of correlation and response functions

In equilibrium:

$$\{\varphi_{\mathrm{eq}}(q,t),\varphi_{\mathrm{eq}}(q,t')\} = G^K_{\mathrm{eq}}(q,t-t')$$

Non-equilibrium: now depends in general on both time variables t and t'

$$\{\varphi(q,t),\varphi(q,t')\} = \left(\frac{t}{t'}\right)^{\theta} \frac{F\left(q^z t/\gamma^{z/2}, t/t'\right)}{q^{2-z}\gamma^{z/2}}$$

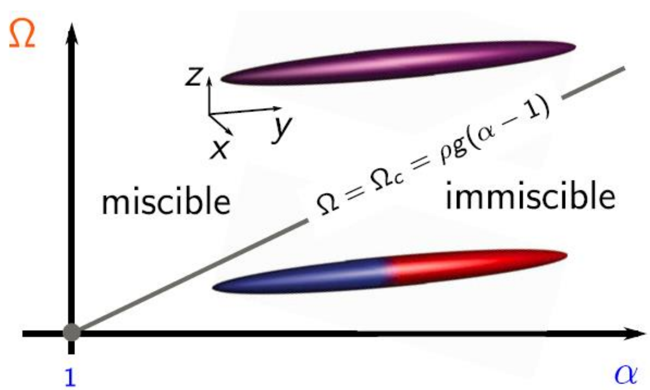
Singular dependence captured by new exponent θ

Smooth scaling function F

Cold-atom experimental realization of quench to quantum critical point

Dynamic scaling after quench in cold-atom gas

- Two-component 1D degenerate Bose gas = spin gas $S^\alpha = \rho^{-1} b^\dagger_\tau \sigma^\alpha_{\tau\tau'} b_\tau$
- Miscible-Immiscible quantum phase transition



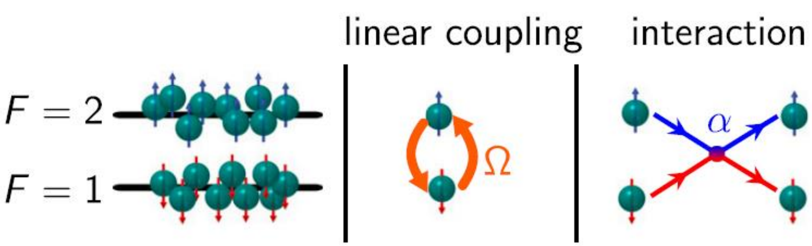
Hamiltonian:

Rabi coupling ∝ laser intensity

$$H = \int_x \frac{\rho}{2}\left(|\partial_x \boldsymbol{S}|^2 + \Omega S^x - \Omega_c (S^z)^2\right)$$

Single-ion anisotropy ∝ $a_{\uparrow\downarrow}$

Heisenberg exchange from kinetic energy



Density difference = Spin z-component

$$S^z = (\rho_\uparrow - \rho_\downarrow)/\rho$$

Miscible-Immiscible quantum phase transition

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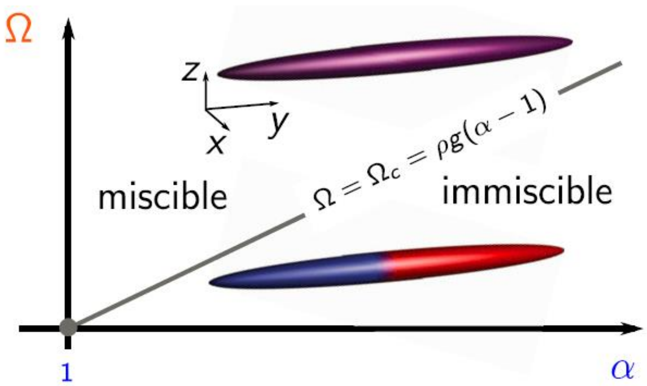

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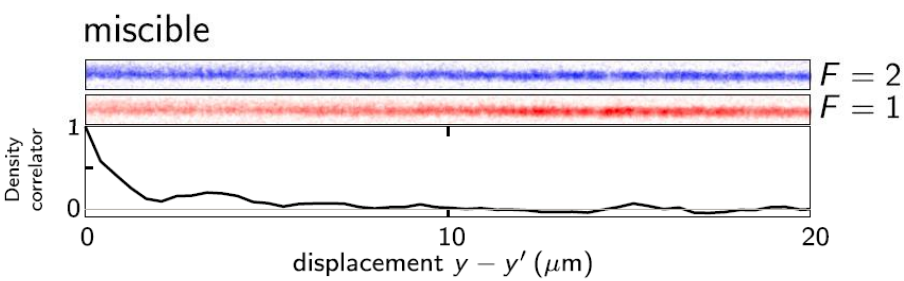
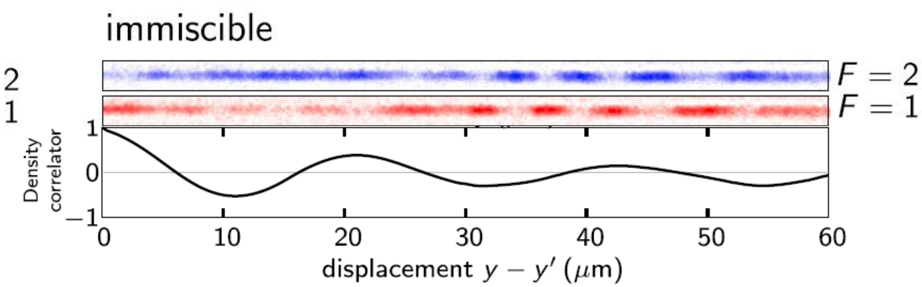
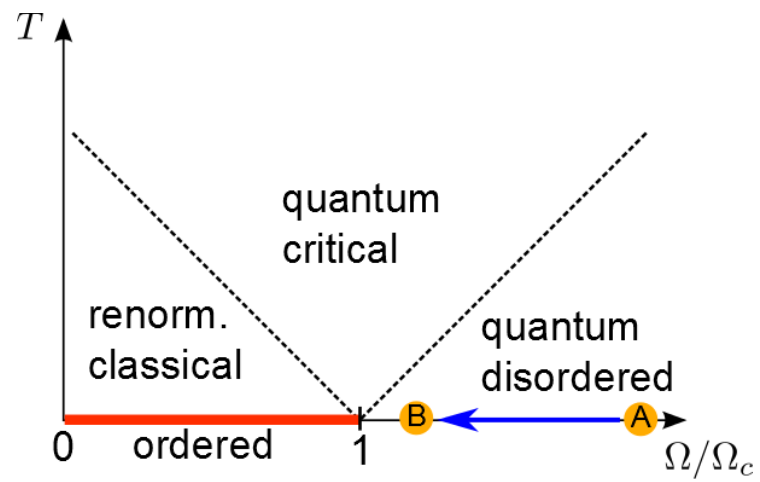


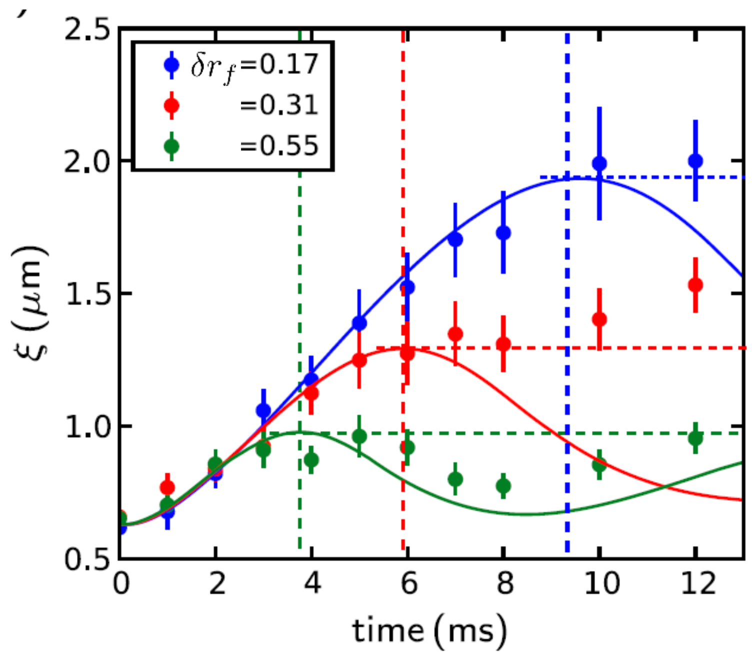


$S^z = (\rho_{\uparrow} - \rho_{\downarrow})/\rho$ ⟶ Miscible = Paramagnetic
Immiscible = Ferromagnetic ⟶ Ising quantum phase transition

Quench of Rabi coupling

- Sudden quench from paramagnetic state towards critical point
- Measure spin-spin correlation function to extract correlation length ξ





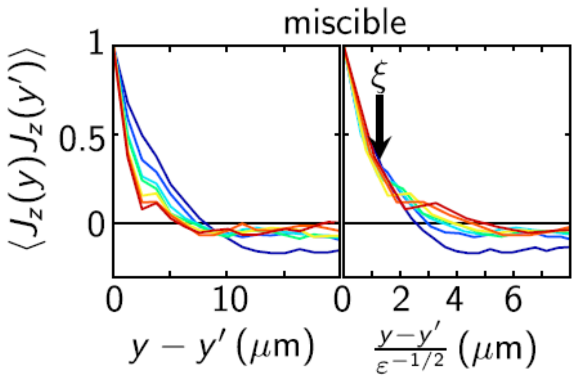
Correlation length increases as

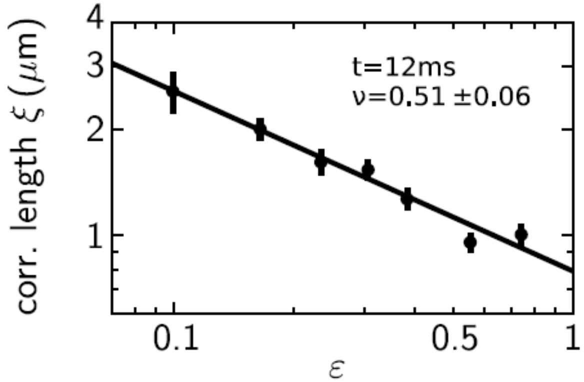
$$\xi(t, 0) \propto t^{1/z}$$

Here: z = 1 (mean-field result)

Reason: technical limitation to come close enough to critical point to reach critical regime

Scaling of correlation function at fixed time





- Data collapse of correlation function at long times when rescaling lengths with ξ_{eq}
- Equilibrium correlation length scales as

$$\xi(\delta r_f) \propto \delta r_f^{-\nu}$$

Here: ν = ½ (mean-field result)

- Interaction effects only visible for quenches closer to critical point
- Short-time scaling could be observed in non-equal time correlation functions and when quenching out of ordered phase

Dynamic scaling after rapid quench to quantum criticality

Quench in non-interacting model coupled to bath

- Post-quench retarded Green's functions for u=0 in presence of bath

Heisenberg equations of motion

$$\left(\partial_t^2 + r_{0,f} + q^2\right) \boldsymbol{\varphi}(q,t) = \int_0^{\infty} ds \eta(t-s) \boldsymbol{\varphi}(q,s) + \boldsymbol{\Xi}(q,t) + \mathbf{h}(q,t)$$

Source operator depends on bath initial states

$$\boldsymbol{\Xi}(q,t) = -\sum_j c_j \left(\mathbf{X}_j^0(q) \cos(\Omega_j t) + \frac{1}{\Omega_j} \mathbf{P}_j^0(q) \sin(\Omega_j t) \right)$$

Quench in non-interacting model coupled to bath

- Post-quench retarded Green's functions for u=0 in presence of bath

Heisenberg equations of motion

$$\left(\partial_t^2 + r_{0,f} + q^2\right)\boldsymbol{\varphi}(q,t) = \int_0^\infty ds\eta(t-s)\boldsymbol{\varphi}(q,s) + \boldsymbol{\Xi}(q,t) + \mathbf{h}(q,t)$$

Source operator depends on bath initial states

$$\boldsymbol{\Xi}(q,t) = -\sum_j c_j\left(\mathbf{X}_j^0(q)\cos(\Omega_j t) + \frac{1}{\Omega_j}\mathbf{P}_j^0(q)\sin(\Omega_j t)\right)$$

Solve via Laplace transformation $\boldsymbol{\varphi}(q,\omega) = \int_0^\infty dt e^{i(\omega+i0^+)t}\boldsymbol{\varphi}(q,t)$

$\Rightarrow$ $$\boldsymbol{\varphi}(q,\omega) = \boldsymbol{F}(q,\omega)\, g_f^R(q,\omega)$$

Force operator $\boldsymbol{F}(q,\omega) = \boldsymbol{\pi}_0(q) - i\omega\boldsymbol{\varphi}_0(q) + \boldsymbol{\Xi}(q,\omega) + \boldsymbol{h}(q,\omega)$

Bare retarded post-quench Green's function

$$g_f^R(q,\omega) = \frac{1}{\omega^2 - r_{0,f} - q^2 + \eta(\omega)}$$

Bare post-quench Keldysh Green's function

- Find G via commutators of $\boldsymbol{\varphi}\left(q,\omega\right)=\boldsymbol{F}\left(q,\omega\right)g_f^R\left(q,\omega\right)$

  Retarded: $G^R\left(t,t'\right)=-i\theta\left(t-t'\right)\left\langle\left[\varphi_H\left(t\right),\varphi_H\left(t'\right)\right]\right\rangle$

  Depends on (t - t') only (no longer the case for u > 0)

$$g_f^R\left(q,\omega\right)=\frac{1}{\omega^2-r_{0,f}-q^2+\eta\left(\omega\right)}$$

  Correlation: $G^K\left(t,t'\right)=-i\left\langle\left\{\varphi_H\left(t\right),\varphi_H\left(t'\right)\right\}\right\rangle$

  Depends on both t and t'. Use double Laplace transform

$$g_f^K\left(q,\omega,\omega'\right)=M\left(q,\omega,\omega'\right)g_f^R\left(q,\omega\right)g_f^R\left(q,\omega'\right)$$

Memory function M depends on initial conditions:

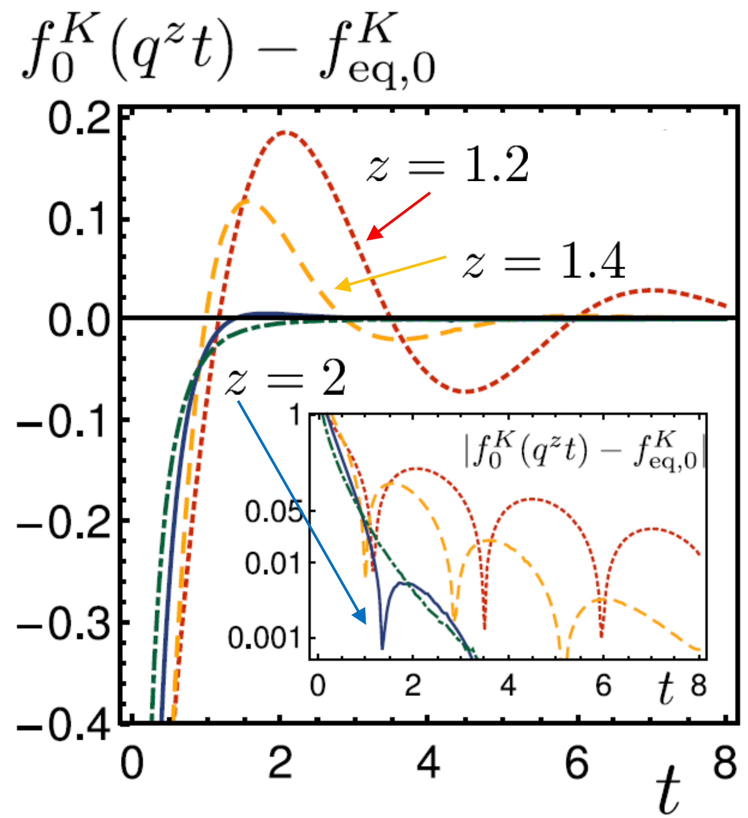
$$M\left(q,\omega,\omega'\right)=i\frac{g_i^K\left(q,\omega\right)+g_i^K\left(q,\omega'\right)}{\omega+\omega'+i0^+}g_i^R\left(q,\omega\right)^{-1}g_i^R\left(q,\omega'\right)^{-1}$$

Bare post-quench Keldysh Green's function

Exponential decay to equilibrium at equal times and u=0.

$$g_f^K(q,t,t) = \frac{f_0^K(q^z t/\gamma^{z/2}, 1)}{q^{2-\eta-z}\gamma^{z/2}}$$

For scaling limit of large initial mass δr_i .



- Overdamped for z > 2: sub-Ohmic
- Underdamped for z ≤ 2: (super)-Ohmic

Equal time correlations in presence of interactions

- Free Keldysh function approaches equilibrium exponentially
- Interacting Keldysh function exhibits power-law decay
  - Amplitude depends on universal exponent θ

$$G_r^K(q,t,t) = G_{\text{eq}}^K(q) - \frac{f(\gamma,z)}{q^{4-z}} \frac{\theta}{t^{2/z}}$$

→ Critical fluctuations significantly slow down equilibration.

with $f(\gamma,z) = \frac{2\Gamma(2/z)}{c_K \sin(\pi/z)} \frac{1}{\gamma}$ and coefficient: $c_K = \frac{4\sin(\pi z/2)}{z(2-z)\sin^{z/2}(\pi/z)}$

Distribution Wigner function at long times

- At long times it holds

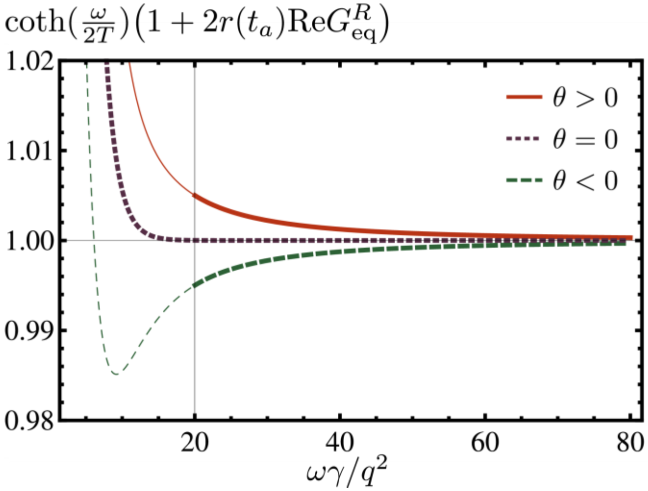
$$G_r^K(q,t_a,\omega) = 2i\coth\left(\frac{\omega}{2T}\right)\left[1+2r(t_a)\mathrm{ReG}_{\mathrm{eq}}^{\mathrm{R}}(\mathrm{q},\omega)\right]\mathrm{Im}G_r^R(q,t_a,\omega)$$

- Introduce time-dep. distribution function $n(t_a,\omega) = n_B(\omega) + \delta n(t_a,\omega)$

Deviation from equilibrium

$$\delta n(t,\omega) = -\coth\left(\frac{\omega}{2T}\right)\frac{\gamma\theta\Gamma(2/z)}{\sin(\pi/z)t^{2/z}}\mathrm{Re}\,G_{\mathrm{eq}}^R(q,\omega)$$

- Non-thermal since algebraically decaying at large frequencies $\delta n \propto |\omega|^{-2/z}$
- Slow approach to equilibrium described by power-law $\delta n \propto t_a^{-2/z}$
- Can change sign: density matrix non-diagonal in energy basis (coherence)



Solving the (non-)equilibrium large-N equations

Interactions in large-N approximation

- Pre-quench equilibrium large-N equations

$$\begin{aligned} h_i &= r_i \phi_i \\ r_i &= \bar{r}_{0,i} + \frac{u_i}{2}\phi_i^2 + u_i \int_{q,\omega_n} G^M_{r_i}(q,\omega_n) \end{aligned}$$

Pre-quench order parameter value

Matsubara Green's function

$$G^M_{r_i}(q,\omega_n) = \frac{-1}{\omega_n^2 + r_i + q^2 - \delta\eta^M(\omega_n)}$$

Initial distance to QCP (in presence of bath and interactions)

Bath induced self-energy

$$\delta\eta^M(\omega_n) = -\frac{\gamma}{\sin\frac{\pi\alpha}{2}}|\omega_n|^\alpha$$

Dynamic critical exponent $z = 2/\alpha$

Interactions in large-N approximation

- Pre-quench equilibrium large-N equations

$$\begin{aligned} h_i &= r_i \phi_i \\ r_i &= \bar{r}_{0,i} + \frac{u_i}{2}\phi_i^2 + u_i \int_{q,\omega_n} G_{r_i}^M (q,\omega_n) \end{aligned}$$

Ordered phase: $\phi_i \neq 0 \Rightarrow r_i(h_i = 0) = 0$ Massless spectrum (in 1/N)

Phase transition when $\phi_i = 0$ and $r_i = 0$

$$\bar{r}_{0,c} = -u \int_{q,\omega}^{\Lambda} \frac{1}{\omega^2 + q^2 + \delta\eta^M(\omega)}$$

Depends on cutoffs Λ and ω_c

Universality as function of $\delta r_i = \bar{r}_{0,i} - r_{0,c}$

For example: $\phi_i \propto (-\delta r_i)^\beta, \xi = r_i^{-1/2} \propto \delta r_i^{-\nu}$ with $\beta = 1/2, \nu = d + z - 2$

Post-quench large-N equations

- Non-equilibrium large-N equations:
  - Time-dependent mass $r_i \to r(t)$
  - Time-dependent order parameter $\phi_i \to \phi(t)$
  - Self-energy given by Keldysh Green's function $\int_{q,\omega_n} G^M_{r_i} \to \int_{q,t} G^K_r(q,t,t)$

$$h_f = -\int_0^\infty dt' (G^R_r)^{-1}(t,t')\,\phi(t') - \phi_i \int_{-\infty}^0 dt' \delta\eta(t-t')$$

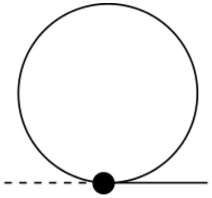
$$r(t) = \bar{r}_{0,f} + \frac{u_f}{2}\phi^2(t) + \frac{u_f}{2}\int_q iG^K_r(q,t,t).$$

Retarded Green's function contains time-dependent mass as well

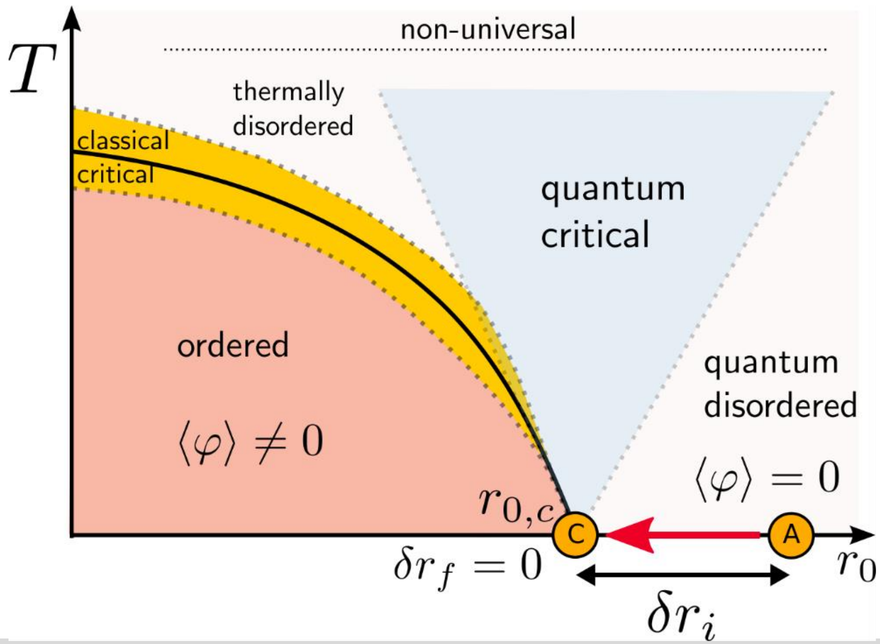
$$(G^R_r)^{-1}(t,t') = -\left(\partial_t^2 + r(t) - \nabla^2\right)\delta(t-t') + \delta\eta(t-t')$$

Quench from disordered phase to critical point

- Initial magnetization vanishes $\phi_i = 0 \Rightarrow \phi(t) = 0$
- Quench right to critical point $\bar{r}_{0,f} = 0$

$r(t)$ 

$$r(t) = \frac{u}{2} \int \frac{d^d k}{(2\pi)^d} \left(iG_r^K(k,t,t) - iG_{\text{eq}}^K(k) \right)$$



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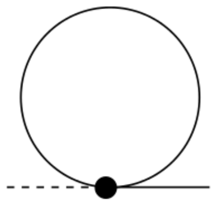
$r(t)$



$$r(t) = \frac{u}{2} \int \frac{d^d k}{(2\pi)^d} \left(iG_r^K(k,t,t) - iG_{\mathrm{eq}}^K(k) \right)$$

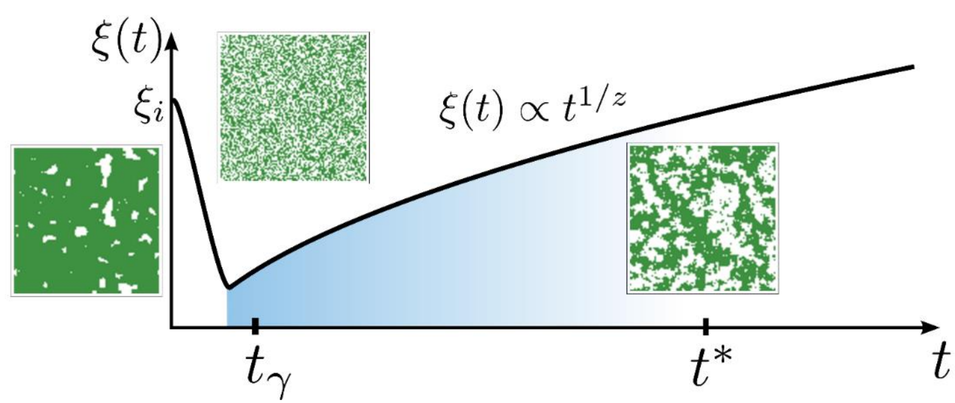
- Ansatz for mass term (that provides self-consistent solution)

$$r(t) = \frac{\gamma a}{t^{2/z}}$$

Light-cone amplitude

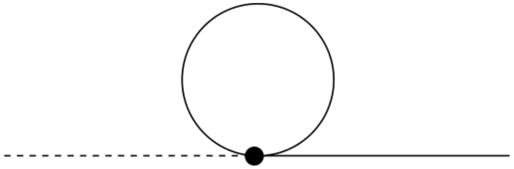
Light-cone dynamical growth of correlation length

$$\xi(t) = r(t)^{-1/2} \propto t^{1/z}$$



Consequence of dynamic mass r(t)

- Logarithmic divergencies at leading order $(t' \ll t)$



$$\delta G^R(k,t,t') = \int_{t_\gamma}^{t} ds g^R(k,t-s)\frac{\gamma a}{s^{2/z}} g^R(k,s-t')$$

- Bare Green's function completely local at short times $(\sqrt{\gamma}/q)^z > t, t' > t_\gamma$

$$g^R(q,\omega) \approx \frac{1}{\delta\eta(\omega)} \Rightarrow g^R(q,t) \approx -\frac{\sin(\pi/z)}{\gamma\Gamma(2/z)} t^{2/z-1}$$

Justification for deep-quench limit: locality corresponds to small correlation length directly after the quench.

Consequence of dynamic mass r(t)

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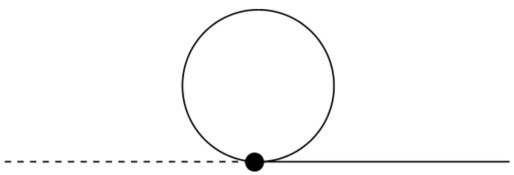


$$\delta G^R(k,t,t') = \int_{t_\gamma}^{t} ds g^R(k,t-s)\frac{\gamma a}{s^{2/z}} g^R(k,s-t')$$
$$= -\frac{a\sin(\pi/z)}{\gamma\Gamma(2/z)} g^R(t-t') \log(t/t')$$

New non-equilibrium critical exponent

$$\theta = -\frac{a\sin(\pi/z)}{\gamma\Gamma(2/z)}$$

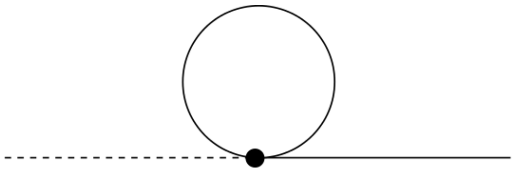
Determined by light-cone amplitude a.

Scaling form of retarded Green's function

$$G^R(k,t,t') = \left(\frac{t}{t'}\right)^{\theta} \frac{f^R(k^z t/\gamma^{z/2}, t'/t)}{k^{2-\eta-z}\gamma^{z/2}}$$

Consequence of dynamic mass r(t)

- Logarithmic divergencies at leading order $(t' \ll t)$



$$\delta G^R(k,t,t') = \int_{t_\gamma}^{t} ds g^R(k,t-s)\frac{\gamma a}{s^{2/z}} g^R(k,s-t')$$
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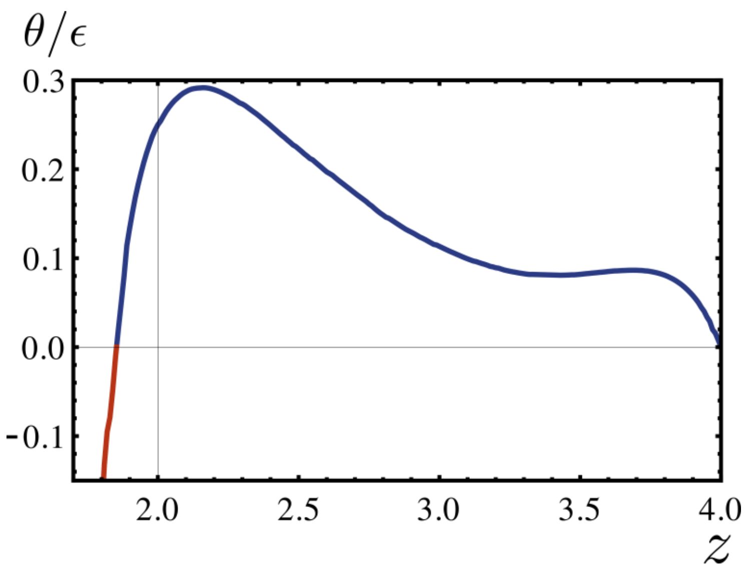
New non-equilibrium critical exponent

$$\theta = -\frac{a\sin(\pi/z)}{\gamma\Gamma(2/z)}$$

For Ohmic bath: $\theta = \frac{\epsilon}{4}$



Determining light-cone amplitude a

- Crucial: interacting Keldysh function decays as power-law at long times

$$G_r^K(q,t,t) = G_{\mathrm{eq}}^K(q) + \frac{2r(t)}{c_K q^{4-z}\gamma^{z/2}}$$

Critical fluctuations slow down equilibration.

Inserting into self-consistency equation: $G_r^K = g^K + G_1^K$

$$r(t) = \frac{uK_d}{2}\int_0^\Lambda dq\, q^{3-z-\epsilon}\left[iG_r^K(q,t,t) - iG_{\mathrm{eq}}^K(q)\right]$$

Yields:

$$\frac{a\gamma}{t^{2/z}} = \frac{uK_d}{2z\gamma^{z/2}}\frac{\gamma C_0 t^{\epsilon/z}}{\gamma^{\epsilon/z}t^{2/z}} + \frac{ua\gamma K_d}{c_K\epsilon\gamma^{z/2}t^{2/z}}\left(\Lambda^{-\epsilon} - \frac{t^{\epsilon/z}}{\gamma^{\epsilon/2}}\right)$$

Exponent:

$$a = \frac{c_K C_0}{2z}\epsilon$$

Solve numerically for general z

$$C_0 = i\int_0^\infty dx\, x^{\frac{2}{z}-1}\left(f^K(x,1) - F_{\mathrm{eq}}^K\right)$$

Interaction fixed-point:

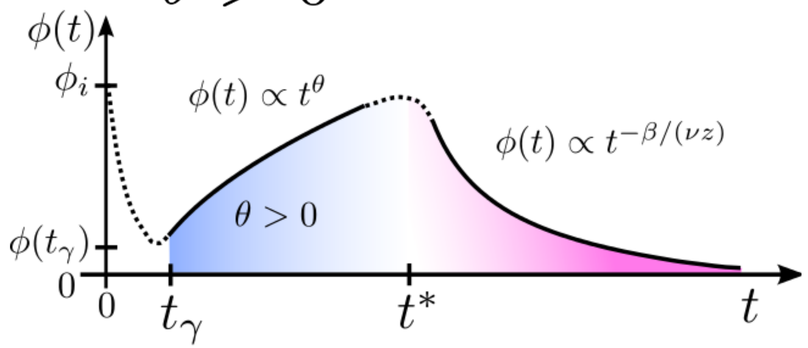
$$u = u^* \equiv \frac{c_K\gamma^{z/2}\Lambda^\epsilon}{K_d}\epsilon$$

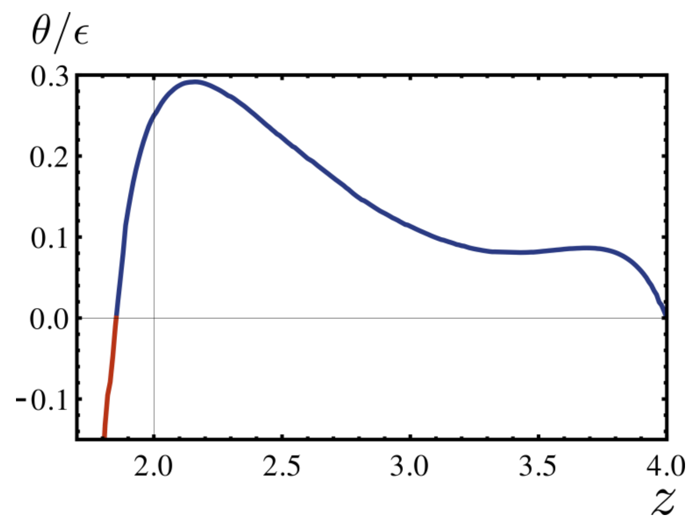
Order parameter dynamics

Two different universal time regimes

- Short time $m(\delta r_i, 0, t) \propto t^{\theta}$ for $t < t^* \propto \delta r_i^{-\nu z/\kappa}$
- Long time $m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)}$ for $t > t^*$

$\theta > 0$ Ohmic and sub-Ohmic





New short time critical exponent depends on z

- Recovery of magnetization due to fast growing correlation length
- Depends on dynamic critical exponent z
- Test hyperscaling, since θ vanishes in mean-field

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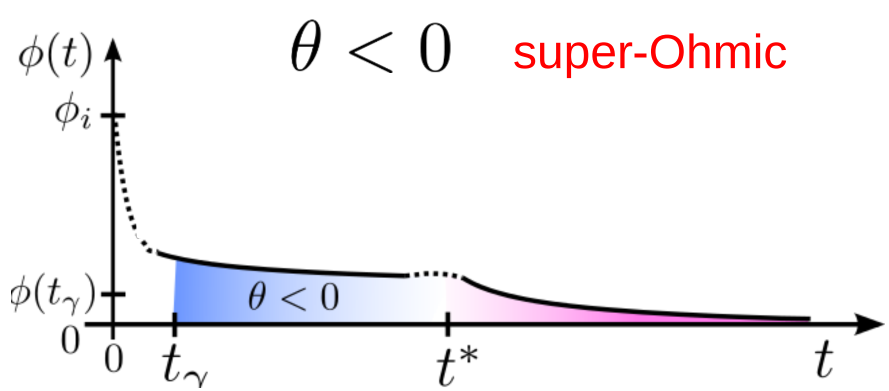

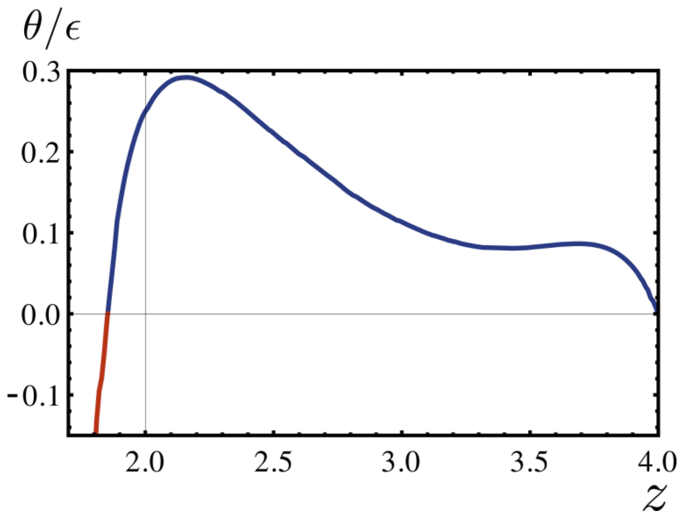

New short time critical exponent depends on z

- Recovery of magnetization due to fast growing correlation length
- Depends on dynamic critical exponent z
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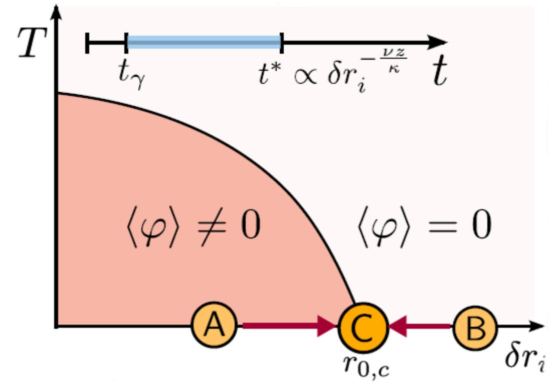
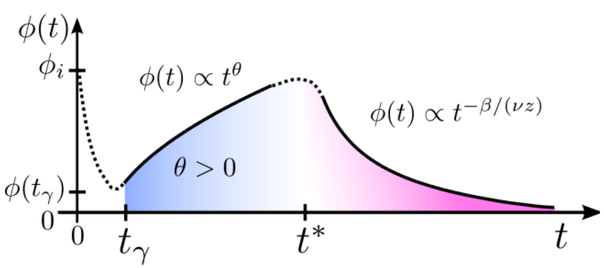
Summary and Outlook

- Quench to quantum critical points results in universal post-quench dynamics
- Characterized by a new critical exponent
- Correlation length collapses after quench and recovers in a light-cone fashion





- Quench in closed system
- Coupled bosonic order parameters, e.g. competition between superconductivity and magnetism.
- Fermionic field theory (metallic magnets, graphene)
- Propagation of entanglement

References:
P. Gagel, P. P. Orth, J. Schmalian,
Phys. Rev. Lett. **113**, 220401 (2014)
Phys. Rev. B **92**, 115121 (2015).

Thank you for your attention.