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Randomized benchmarking

Model-free benchmarking technique that yields the average fidelity F_{av} over the full set of Clifford gates $Clif_n$. The Clifford group is the normalizer of the Pauli group $\mathcal{P}$, i.e., it maps Pauli operators $P_n \in \mathcal{P}$ to Pauli operators: $g P_n g^{-1} = P_m \quad \forall g \in Clif_n$.

The Clifford group is generated by $\{H, S, CNOT\}$ and circuits composed only out of Clifford gates can be efficiently simulated classically (Gottesmann-Knill theorem).

Clifford group plays important role in QEC using stabilizer codes and forms a unitary 2-design, which implies

$$\frac{1}{|Clif_n|} \sum_{j=1}^{|Clif_n|} \left(C_j \Lambda(C_j^{\dagger} \rho C_j) C_j^{\dagger} \right) =$$

uniform Haar measure of $\mathcal{U}(d)$

w/o proof; we show a similar statement explicitly later for Pauli twirls

$$= \int_{\mathcal{U}(d)} d\mathcal{U} \left(\mathcal{U} \Lambda(\mathcal{U}^{\dagger} \rho \, \mathcal{U}) \mathcal{U}^{\dagger} \right) = \Lambda_{dep}(\rho)$$

Λ_{dep} is depolarizing channel with same average fidelity $\mathcal{F}_{ave}$ as Λ. Now, $\Lambda_{dep} = (1-\tilde{p})\rho + \tilde{p}\frac{I}{d}$ and thus $\mathcal{F}_{ave} = 1 - \tilde{p} + \frac{\tilde{p}}{d}$ (shown below).

RB is scalable in the number of qubits n ($O(n^2)$ gates and $O(n^4)$ classical preprocessing costs). However, RB only provides partial information about the noise (= error channel), specifically the average fidelity $\mathcal{F}_{ave}$. This is related to the χ_{00} element of the χ process matrix as

$$\mathcal{F}_{ave} = \frac{\chi_{00}\, d + 1}{d+1} .$$

RB is unaffected by SPAM errors (unlike quantum process tomography), where SPAM refers to state preparation & measurement errors. RB in fact gives estimates for SPAM errors.

Fidelity of depolarizing channel:

$$\mathcal{E}(\rho) = (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z) =$$

$$= (1-\tfrac{3}{4}\tilde{p})\rho + \frac{\tilde{p}}{4}(X\rho X + Y\rho Y + Z\rho Z) =$$

$$= (1-\tilde{p})\rho + \tilde{p}\frac{I}{d}$$

($d=2$ here, generally 2^n).

Here, $\frac{3}{4}\tilde{p} = p \Leftrightarrow \tilde{p} = \frac{4}{3}p$.

The average fidelity of the depolarizing channel is defined as

$$\bar{F}_{av}(\mathcal{E}) = \int d\psi \; F\big(|\psi\rangle, \mathcal{E}(|\psi\rangle\langle\psi|)\big)$$

Haar measure

with state-dependent channel

$$F\big(|\psi\rangle, \mathcal{E}(|\psi\rangle\langle\psi|)\big) = \langle\psi|\,\mathcal{E}(|\psi\rangle\langle\psi|)\,|\psi\rangle.$$

Inserting the depolarizing channel yields (use def. w/o square root here)

$$\langle\psi|\Big[(1-\tilde{p})\underbrace{\rho}_{=|\psi\rangle\langle\psi|} + \tilde{p}\frac{I}{d}\Big]|\psi\rangle = 1-\tilde{p} + \underbrace{\frac{\tilde{p}}{d}}_{=\tilde{p}/2} = F\big(|\psi\rangle, \mathcal{E}(|\psi\rangle\langle\psi|)\big)$$

RB protocol:

Generate random sequences of Clifford gates $C_i \in \mathrm{Clif}_n$ of length $m \leq M-1$. Fix initial state $|\psi\rangle$ and maximal sequence length M.

Step 1: fix $m \leq M-1$ and generate K_m sequences consisting of $(m+1)$ quantum operations.

The first m are chosen uniformly random from Clif_n.

The $(m+1)$th operation is chosen such that (possible as Clif_n is a group)

$$C_{i_{m+1}} \circ \left[C_{i_m} \circ \cdots C_{i_1} \right] = I$$

Each Clifford gate is associated with an error channel $\Lambda_{i_j, j}$ such that the sequence K_m corresponds to the quantum operation

$$S_{\vec{i}_m} = \bigcirc_{j=1}^{m+1} \left(\Lambda_{i_j, j} \circ C_{i_j} \right), \quad \vec{i}_m = (i_1, \dots, i_m)$$

Note that i_{m+1} is uniquely determined by $\vec{i}_m$.

Step 2: For each of K_m sequences, measure the survival probability

$$\mathrm{Tr}\left[E_\psi\, S_{\vec{i}_m}(\rho_\psi)\right]$$

$$\left.\begin{aligned} \rho_\psi &= \text{noisy version of } |\psi\rangle\langle\psi| \ (\text{state prep. error}) \\ E_\psi &= \text{noisy measurement of } |\psi\rangle\langle\psi| \ (\text{measurement error}) \end{aligned}\right\} \text{SPAM error}$$

Step 3: Average over K_m random realizations to find the averaged sequence fidelity

$$F_{seq}(m,\psi) = \mathrm{Tr}\left[E_\psi\, S_{K_m}(\rho_\psi)\right]$$

where

$$S_{K_m} = \frac{1}{K_m}\sum_{\vec{i}_m} S_{\vec{i}_m}$$

Step 4: Repeat steps 1 through 3 for different values of m and fit results to the model

$$F_g(m, \psi) = A_0 (1-p)^m + B_0$$

$\left.\begin{array}{l} A_0 : \text{state preparation error} \\ B_0 : \text{measurement error} \end{array}\right\}$ SPAM error

The average error rate $r = 1 - F_{ave} =$

$$= 1 - \left[1 - \tilde{p} + \frac{\tilde{p}}{d}\right] =$$

$$= \tilde{p} - \frac{\tilde{p}}{d}.$$ ($d = 2^n$ for n qubits)

Note that it becomes obvious that we measure the twirled channel over the Clifford group, i.e., a depolarizing channel with the same average fidelity as the original channel.

For this define: (1) $D_{i_1} = C_{i_1}$

(2) $D_{i_2} = C_{i_2} \circ C_{i_1} \Rightarrow C_{i_2} = D_{i_2} \circ D_{i_1}^{\dagger}$

(3) general if $C_{i_1}, \dots, C_{i_j}$ have been chosen and $D_{i_1}, \dots, D_{i_j}$ have been defined accordingly, define $D_{i_{j+1}}$ such that $C_{i_{j+1}} = D_{i_{j+1}} \circ D_{i_j}$, i.e.,

$$D_{i_{j+1}} = C_{i_j} \circ \dots \circ C_{i_1} .$$

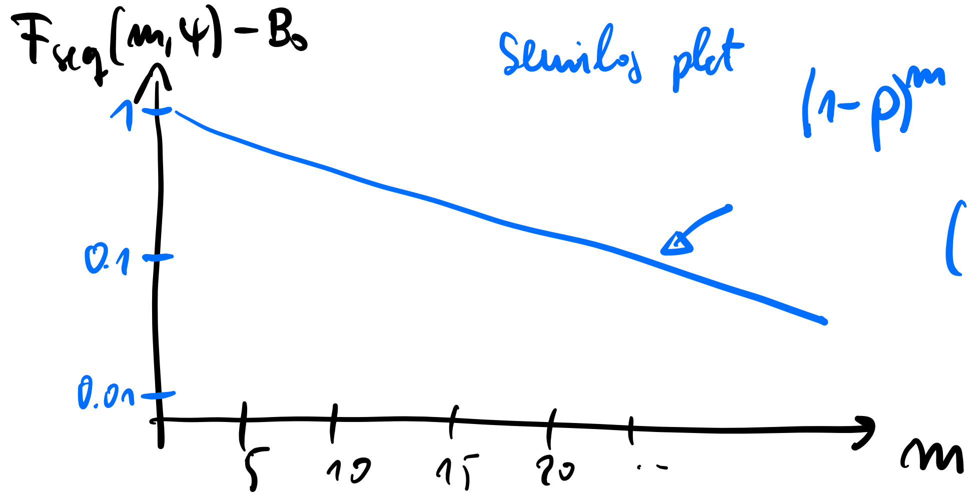
Then,

$$S_{\vec{i}_m} = \Lambda_{i_{m+1}} \circ C_{i_{m+1}} \circ \Lambda_{i_m} \circ C_{i_m} \circ \dots \Lambda_{i_1} \circ C_{i_1} =$$

$$= \Lambda_{i_{m+1}} \circ D_{i_{m+1}} \circ D^{\dagger}_{i_m} \circ \Lambda_{i_m} \circ D_{i_m} \circ D^{\dagger}_{i_{m-1}} \circ \Lambda_{i_{m-1}} \circ D_{i_{m-1}}$$

$$\circ \dots \circ \underbrace{D^{\dagger}_{i_1} \circ \Lambda_{i_1} \circ D_{i_1}}_{\text{Clifford twirled channel}} .$$

$\Rightarrow$ depolarizing noise channel with same average fidelity p as original channel Λ_{i_j}.



$$(1-p)^m = \exp[m \log(1-p)]$$

$$(1-p) \approx 1-\varepsilon.$$

Forest benchmarking suite

Ideal of interleaved RB: determine av. error of particular Clifford gate G

(1) run original RB: K_m sequences $C_{i_{m+1}} \circ C_{i_m} \circ \dots \circ C_{i_1}$

(2) for every sequence $\vec{i}_m$, run the circuit

$$\tilde{C}_{i_{m+1}} \circ G \circ C_{i_m} \circ \dots \circ G \circ C_{i_2} \circ G \circ C_{i_1}$$

needs to be updated compared to standard RB (1) to invert the sequence containing G.

$\Rightarrow$ compute r for (1) & (2) $\Rightarrow$ difference is r_G.

Hamiltonian simulation:

Goal: compute time-evolution of a quantum system described by Hamiltonian (= energy functional) H.

Many applications:

- Real-time dynamics of quantum systems
  - investigate nonequilibrium behavior
    * chemical reactions
    * scattering experiments
    * phase transformations, synthesis, modeling experimental measurements (optics, quenches, (non) linear transport)
    * fundamental interest in understanding nonequilibrium matter (criticality, ETH - thermalization, MIPT)
  - adiabatic state preparation: $H(t) = H_0\left(1-\frac{t}{T}\right) + H_1 \frac{t}{T}$.
    * slowly preparing ground states of desired Hamiltonians
    * general optimization problems, e.g., find GS of MFIM $H = \sum_{j=1}^{m} \left(h_{x,j} X_j + h_{z,j} Z_j + J_{ij} Z_i Z_j \right)$

- Imaginary-time evolution

  * prepare GS of Hamiltonians : $|\psi_0\rangle$.

  * prepare thermal states of Hamiltonians [e.g. Gibbs state $\frac{e^{-\beta H}}{Z}$)

We will focus on NISQ-implementable approaches :

- Lie-Suzuki-Trotter product formula (PF) approach

- Randomized compilation

- Multi-product formulas

- Variational quantum algorithms (hybrid quantum classical algorithms)