Problem 1

In[3]:=
```
vv = {v1, v2, v3};
op = vv.{PauliMatrix[1], PauliMatrix[2], PauliMatrix[3]}
esys = Eigensystem[op] // FullSimplify[#, Assumptions → {v1^2 + v2^2 + v3^2 == 1}] &
```

Out[4]= $\{\{v3, v1 - i\, v2\}, \{v1 + i\, v2, -v3\}\}$

Out[5]= $\left\{\{-1, 1\}, \left\{\left\{\frac{-1 + v3}{v1 + i\, v2}, 1\right\}, \left\{\frac{1 + v3}{v1 + i\, v2}, 1\right\}\right\}\right\}$

In[6]:=
```
vv = {Sin[θ] * Cos[ϕ], Sin[θ] * Sin[ϕ], Cos[θ]};
op = vv.{PauliMatrix[1], PauliMatrix[2], PauliMatrix[3]}
esys = Eigensystem[op] // FullSimplify[#, Assumptions → {θ ≠ 0}] &
```

Out[7]= $\{\{\text{Cos}[\theta], \text{Cos}[\phi]\ \text{Sin}[\theta] - i\ \text{Sin}[\theta]\ \text{Sin}[\phi]\}, \{\text{Cos}[\phi]\ \text{Sin}[\theta] + i\ \text{Sin}[\theta]\ \text{Sin}[\phi], -\text{Cos}[\theta]\}\}$

Out[8]= $\left\{\{-1, 1\}, \left\{\left\{-e^{-i\,\phi}\ \text{Tan}\left[\frac{\theta}{2}\right], 1\right\}, \left\{e^{-i\,\phi}\ \text{Cot}\left[\frac{\theta}{2}\right], 1\right\}\right\}\right\}$

In[9]:=
```
op.{Cos[θ / 2], Exp[I * ϕ] * Sin[θ / 2]}
% // FullSimplify
```

Out[9]= $\left\{\text{Cos}\left[\frac{\theta}{2}\right]\ \text{Cos}[\theta] + e^{i\,\phi}\ \text{Sin}\left[\frac{\theta}{2}\right]\ (\text{Cos}[\phi]\ \text{Sin}[\theta] - i\ \text{Sin}[\theta]\ \text{Sin}[\phi]),\right.$
$\left. -e^{i\,\phi}\ \text{Cos}[\theta]\ \text{Sin}\left[\frac{\theta}{2}\right] + \text{Cos}\left[\frac{\theta}{2}\right]\ (\text{Cos}[\phi]\ \text{Sin}[\theta] + i\ \text{Sin}[\theta]\ \text{Sin}[\phi])\right\}$

Out[10]=
$\left\{\text{Cos}\left[\frac{\theta}{2}\right], e^{i\,\phi}\ \text{Sin}\left[\frac{\theta}{2}\right]\right\}$

In[11]:=
```
op.{Sin[θ / 2], -Exp[I * ϕ] * Cos[θ / 2]}
% // FullSimplify
```

Out[11]=
$\left\{\text{Cos}[\theta]\ \text{Sin}\left[\frac{\theta}{2}\right] - e^{i\,\phi}\ \text{Cos}\left[\frac{\theta}{2}\right]\ (\text{Cos}[\phi]\ \text{Sin}[\theta] - i\ \text{Sin}[\theta]\ \text{Sin}[\phi]),\right.$
$\left. e^{i\,\phi}\ \text{Cos}\left[\frac{\theta}{2}\right]\ \text{Cos}[\theta] + \text{Sin}\left[\frac{\theta}{2}\right]\ (\text{Cos}[\phi]\ \text{Sin}[\theta] + i\ \text{Sin}[\theta]\ \text{Sin}[\phi])\right\}$

Out[12]=
$\left\{-\text{Sin}\left[\frac{\theta}{2}\right], e^{i\,\phi}\ \text{Cos}\left[\frac{\theta}{2}\right]\right\}$

Problem 2

In[13]:=
```
MM = 10^4;
MM1 = 3847;
ZExp = 2 * MM1 / MM - 1 // N
MMPlus = 6523;
XExp = 2 * MMPlus / MM - 1 // N
```

Out[15]=
```
-0.2306
```

Out[17]=
```
0.3046
```

In[18]:=
```
ArcCos[ZExp]
% / 2 / Pi * 360
ArcCos[XExp / Sqrt[1 - ZExp^2]]
% / 2 / Pi * 360
```

Out[18]=
```
1.80349
```

Out[19]=
```
103.332
```

Out[20]=
```
1.25241
```

Out[21]=
```
71.7577
```

Problem 3

In[22]:=
```
alpha = 3 * Pi / 4
Cos[alpha]^2
Sin[alpha]^2 // N
```

Out[22]=
$$\frac{3\pi}{4}$$

Out[23]=
$$\frac{1}{2}$$

Out[24]=
```
0.5
```

In[25]:=
```
mm = {{1 / 2, -1 / 2}, {-1 / 2, -1 / 2}}
```

Out[25]=
$$\left\{\left\{\frac{1}{2}, -\frac{1}{2}\right\}, \left\{-\frac{1}{2}, -\frac{1}{2}\right\}\right\}$$

In[26]:=
```
Tr[Sqrt[ConjugateTranspose[mm].mm]]
```
Out[26]=
$$\sqrt{2}$$

In[27]:=
```
ee1 = cc * {{0, 0}, {0, 1}}
ee2 = cc * {{1 / 2, -1 / 2}, {-1 / 2, 1 / 2}}
ee3 = DiagonalMatrix[{1, 1}] - ee1 - ee2
% // MatrixForm
evals3 = Eigenvalues[ee3]
Solve[evals3[[1]] == 0, cc]
ccVal = %[[1, 1, 2]]
Solve[evals3[[2]] == 0, cc]
```

Out[27]=
$$\{\{0, 0\}, \{0, cc\}\}$$

Out[28]=
$$\left\{\left\{\frac{cc}{2}, -\frac{cc}{2}\right\}, \left\{-\frac{cc}{2}, \frac{cc}{2}\right\}\right\}$$

Out[29]=
$$\left\{\left\{1 - \frac{cc}{2}, \frac{cc}{2}\right\}, \left\{\frac{cc}{2}, 1 - \frac{3\,cc}{2}\right\}\right\}$$

Out[30]//MatrixForm=
$$\begin{pmatrix} 1 - \frac{cc}{2} & \frac{cc}{2} \\ \frac{cc}{2} & 1 - \frac{3\,cc}{2} \end{pmatrix}$$

Out[31]=
$$\left\{\frac{1}{2}\left(2 - 2\,cc - \sqrt{2}\,cc\right), \frac{1}{2}\left(2 - 2\,cc + \sqrt{2}\,cc\right)\right\}$$

Out[32]=
$$\left\{\left\{cc \to \frac{2}{2 + \sqrt{2}}\right\}\right\}$$

Out[33]=
$$\frac{2}{2 + \sqrt{2}}$$

Out[34]=
$$\left\{\left\{cc \to -\frac{2}{-2 + \sqrt{2}}\right\}\right\}$$

In[35]:=
```
Sqrt[2] / (1 + Sqrt[2]) * 1 / Sqrt[2] // N
1 - %
```
Out[35]=
$$0.414214$$

Out[36]=
$$0.585786$$

In[37]:=
```
ρ1 = {{1, 0}, {0, 0}}
Tr[ρ1.ee1] //. cc → ccVal;
% // N
Tr[ρ1.ee2] //. cc → ccVal
% // N
Tr[ρ1.ee3] //. cc → ccVal // FullSimplify;
% // N
```

Out[37]=
$$\{\{1, 0\}, \{0, 0\}\}$$

Out[39]=
$$0.$$

Out[40]=
$$\frac{1}{2+\sqrt{2}}$$

Out[41]=
$$0.292893$$

Out[43]=
$$0.707107$$

In[44]:=
```
ρ2 = {{1 / 2, 1 / 2}, {1 / 2, 1 / 2}}
Tr[ρ2.ee1] //. cc → ccVal
% // N
Tr[ρ2.ee2] //. cc → ccVal
Tr[ρ2.ee3] //. cc → ccVal // FullSimplify
% // N
```

Out[44]=
$$\left\{\left\{\frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{1}{2}\right\}\right\}$$

Out[45]=
$$\frac{1}{2+\sqrt{2}}$$

Out[46]=
$$0.292893$$

Out[47]=
$$0$$

Out[48]=
$$\frac{1}{\sqrt{2}}$$

Out[49]=
$$0.707107$$

In[50]:=
```
1 / 2 * Tr[Sqrt[ConjugateTranspose[ρ1 - ρ2].(ρ1 - ρ2)]]
```

Out[50]=

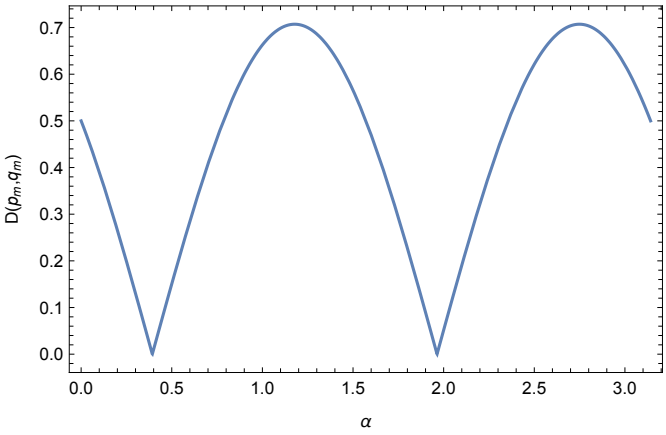
$$\frac{1}{\sqrt{2}}$$

In[51]:=
```
f[α_] = 1 / 2 * (Abs[Cos[α]^2 - Cos[α - Pi / 4]^2] + Abs[Sin[α]^2 - Sin[α - Pi / 4]^2])
Plot[1 / 2 * (Abs[Cos[α]^2 - Cos[α - Pi / 4]^2] + Abs[Sin[α]^2 - Sin[α - Pi / 4]^2]),
 {α, 0, Pi}, Frame → True, FrameLabel → {{"D(pₘ,qₘ)", ""}, {"α", ""}}]
```

Out[51]=

$$\frac{1}{2}\left(\text{Abs}\left[-\text{Cos}\left[\frac{\pi}{4}-\alpha\right]^2+\text{Cos}[\alpha]^2\right]+\text{Abs}\left[-\text{Sin}\left[\frac{\pi}{4}-\alpha\right]^2+\text{Sin}[\alpha]^2\right]\right)$$

Out[52]=



In[53]:=
```
FindMaximum[f[α], α]
%[[2, 1, 2]] / Pi
```

Out[53]=

```
{0.707107, {α → 1.1781}}
```

Out[54]=

```
0.375
```

Problem 4

In[55]:=
```
werner[λ_] := {{λ / 4, 0, 0, 0}, {0, 1 / 2 * (1 - λ / 2), (λ - 1) / 2, 0},
   {0, (λ - 1) / 2, 1 / 2 * (1 - λ / 2), 0}, {0, 0, 0, λ / 4}}
werner[λ] // MatrixForm
```

Out[56]//MatrixForm=

$$\begin{pmatrix} \frac{\lambda}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\left(1-\frac{\lambda}{2}\right) & \frac{1}{2}(-1+\lambda) & 0 \\ 0 & \frac{1}{2}(-1+\lambda) & \frac{1}{2}\left(1-\frac{\lambda}{2}\right) & 0 \\ 0 & 0 & 0 & \frac{\lambda}{4} \end{pmatrix}$$

In[57]:=
```
evals = Eigenvalues[werner[λ]]
Solve[%[[4]] == 0, λ]
```

Out[57]=
$$\left\{\frac{\lambda}{4}, \frac{\lambda}{4}, \frac{\lambda}{4}, \frac{1}{4}\ (4-3\ \lambda)\right\}$$

Out[58]=
$$\left\{\left\{\lambda \to \frac{4}{3}\right\}\right\}$$

In[59]:=
```
SvN[λ_] = -evals.Log[evals]
SRenyi2[λ_] = -Log[evals.evals]
participation[λ_] = evals.evals
```

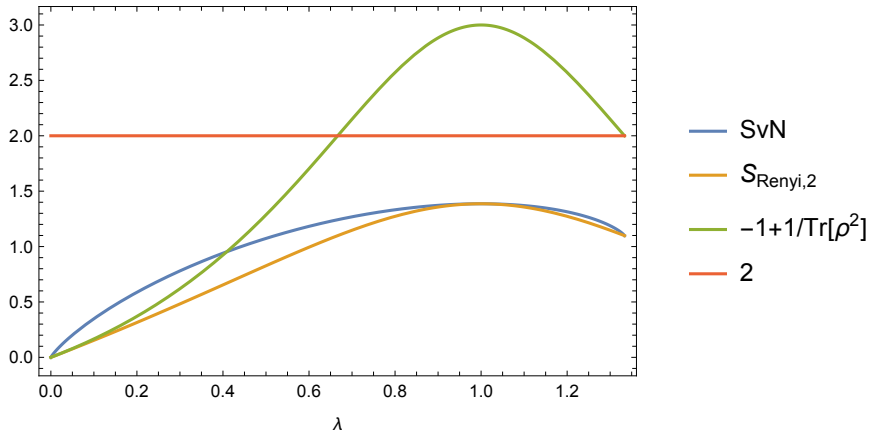
Out[59]=
$$-\frac{1}{4}\ (4-3\ \lambda)\ \text{Log}\left[\frac{1}{4}\ (4-3\ \lambda)\right] - \frac{3}{4}\ \lambda\ \text{Log}\left[\frac{\lambda}{4}\right]$$

Out[60]=
$$-\text{Log}\left[\frac{1}{16}\ (4-3\ \lambda)^2 + \frac{3\ \lambda^2}{16}\right]$$

Out[61]=
$$\frac{1}{16}\ (4-3\ \lambda)^2 + \frac{3\ \lambda^2}{16}$$

In[62]:=
```
Plot[{SvN[λ], SRenyi2[λ], 1 / participation[λ] - 1, 2},
 {λ, 0, 4 / 3}, Frame → True, FrameLabel → {{"", ""}, {"λ", ""}},
 PlotLegends → {"SvN", "S_Renyi,2", "-1+1/Tr[ρ^2]", "2"}]
```

Out[62]=



In[63]:=
```
Solve[1 / participation[λ] - 1 == 2, λ]
```

Out[63]=
$$\left\{\left\{\lambda \to \frac{2}{3}\right\}, \left\{\lambda \to \frac{4}{3}\right\}\right\}$$