Energy-Based Control of Multicopters

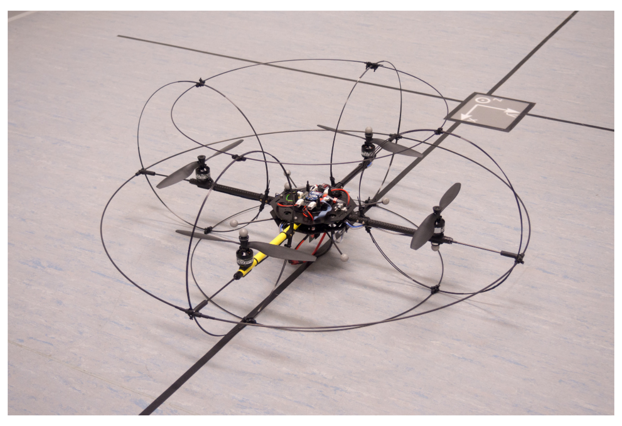
Task and motivation

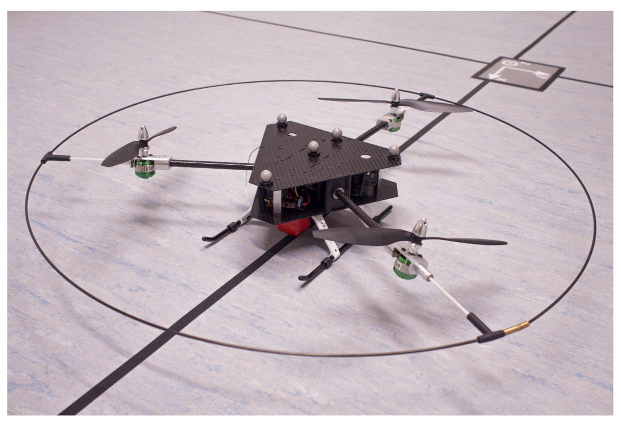
Multicopters
- UAVs with multiple propellers
- increasingly popular in hobby and industry

State of the art
- position controlled by a pilot
- *linear* stabilization of attitude

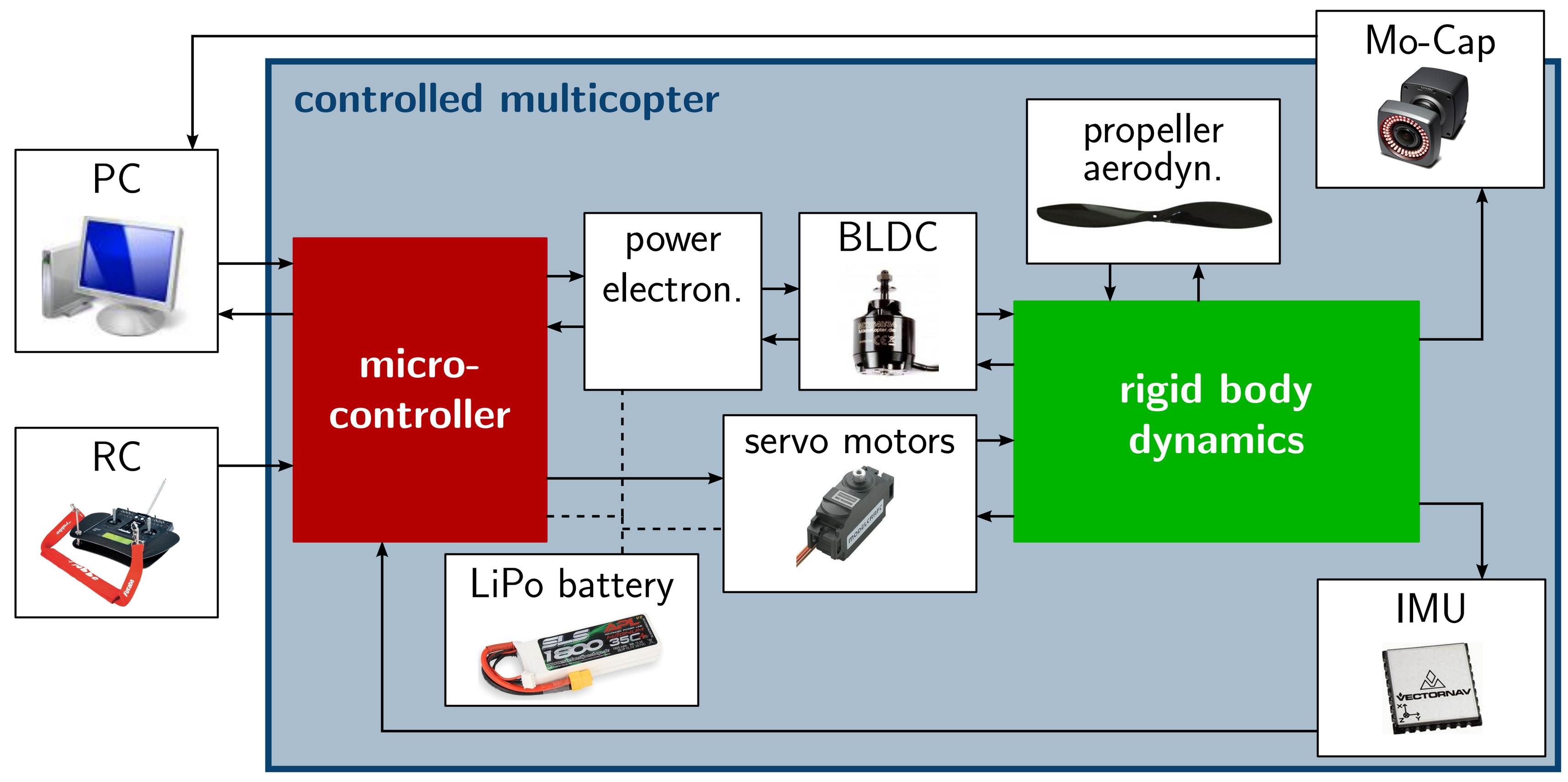
$\hookrightarrow$ not capable of large tilt angles

Goal
- nonlinear tracking control of position and attitude capable of aerobatic maneuvers





Experimental setup



Microcontroller
- ATMEL AVR32UC3C: 32bit, 66MHz, FPU, compiler for C and C++
- sampling time for controller implementation 5 ms

Some practical challenges
- Inertial meas. unit (IMU): gyroscope and accelerometer
- VICON motion capture sys. (Mo-Cap): accurate but delayed $\approx 70\,\mathrm{ms}$

$\hookrightarrow$ dynamic, model-based fusion for real-time state estimate

- Propellers are driven by brushless DC motors (BLDC)

$\hookrightarrow$ Fast underlying control of propeller speed

[D. Kastelan, M. Konz, J. Rudolph: *Fully actuated tricopter with pilot-supporting control*, IFAC-PapersOnLine 28(9), pp. 79–84, 2015]

Theoretical challenges

Rigid body dynamics are the **dominant** part of the system
- unstable system $\Rightarrow$ feedback control is inevitable
- nonlinear configuration manifold $\Rightarrow$ minimal parameterizations are only local

Lagrange's formalism

Redundant configuration coordinates $x(t)$ and configuration manifold $\mathbb{X}$

$$\mathbb{X} = \{x \in \mathbb{R}^\nu \,|\, \phi(x) = 0\}, \qquad n = \dim \mathbb{X} = \nu - \operatorname{rank} \frac{\partial \phi}{\partial x}$$

Minimal velocity coordinates $\xi(t) \in \mathbb{R}^n$ and kinematics

$$\dot{x}^\alpha = A_i^\alpha(x)\xi^i, \ \alpha = 1, \dots, \nu \qquad \frac{\partial \phi^\kappa}{\partial x^\alpha} A_i^\alpha \equiv 0, \ \operatorname{rank} A = n$$

Directional derivative ∂_i and commutator coefficients γ_{ij}^k

$$\partial_i = A_i^\alpha \frac{\partial}{\partial x^\alpha}, \qquad \partial_i \partial_j - \partial_j \partial_i = \gamma_{ij}^k \partial_k, \quad \gamma_{ij}^k = (\partial_i A_j^\alpha - \partial_j A_i^\alpha)(A^+)_\alpha^i$$

Lagrange's equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \xi^i} + \gamma_{ij}^k \xi^j \frac{\partial L}{\partial \xi^k} - \partial_i L = f_i, \ i = 1, \dots, n$$

[M. Konz, J. Rudolph: *Equations of Motion with Redundant Coordinates for Mechanical Systems on Manifolds*, Proc. 8th Vienna International Conference on Mathematical Modelling, pp. 697-698, 2015]

Rigid body model

Coordinates

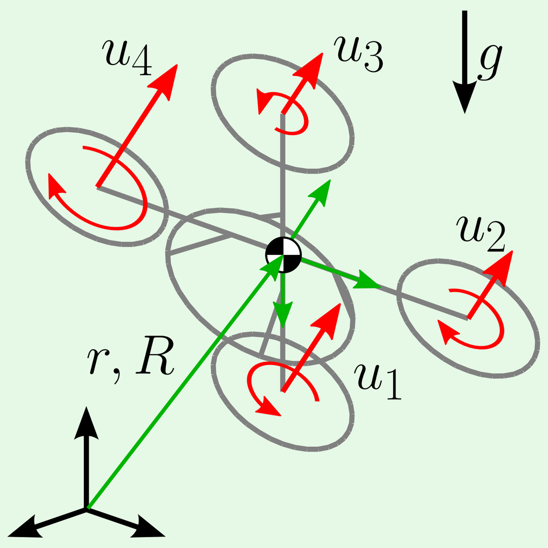
$$x \cong (r, R) \in \mathbb{R}^3 \times \mathrm{SO}(3), \ \phi(x) \cong R^\top R - I_3 = 0$$
$$\xi \cong (v, \omega) \in \mathbb{R}^6$$

Kinematics

$$\dot{r} = Rv, \ \dot{R} = R\widehat{\omega} \quad \cong \quad \dot{x} = A(x)\xi$$

Kinetics from Lagrange's equation

$$M(x)\dot{\xi} + c(x,\xi) + \nabla V(x) = Bu$$



Desired controlled system

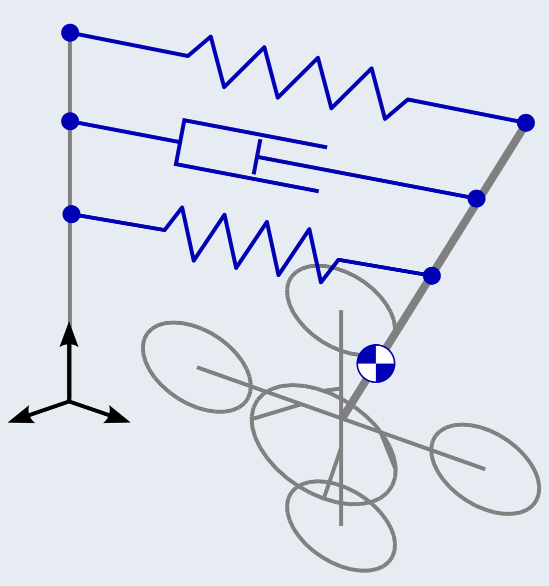
- desired inertia $\bar{M}(x) > 0$
- desired dissipation $\bar{D}(x) \geq 0$
- desired potential $\bar{V}(x) \geq 0$, $\bar{V} = 0 \Leftrightarrow x = x_{\mathrm{R}}$

$\hookrightarrow$ closed loop kinetics

$$\bar{M}(x)\dot{\xi} + \bar{c}(x,\xi) + \bar{D}(x)\xi + \nabla\bar{V}(x) = 0$$

Total energy $\bar{H}$ is a Lyapunov function

$$\bar{H} = \tfrac{1}{2}\xi^\top \bar{M}\xi + V \geq 0, \quad \dot{\bar{H}} = -\xi^\top \bar{D}\xi \leq 0$$



Control law

Insert desired kinetics into the model kinetics to get required generalized force

$$f = c + D\xi + \nabla V - M\bar{M}^{-1}(\bar{c} + \bar{D}\xi + \nabla\bar{V})$$

$\hookrightarrow$ control law and matching condition

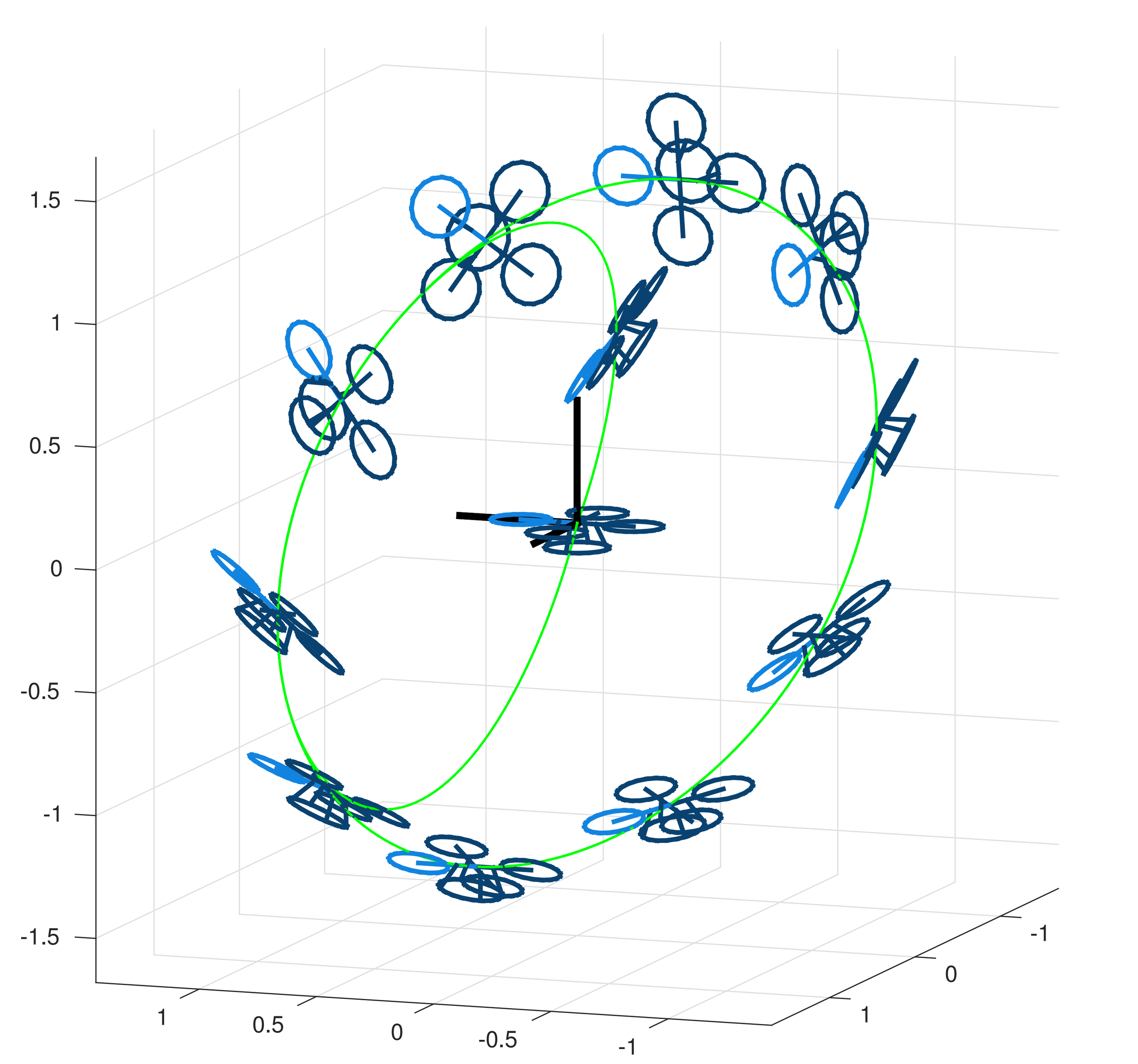
$$u = B^+ f, \qquad B^\perp f \stackrel{!}{=} 0$$

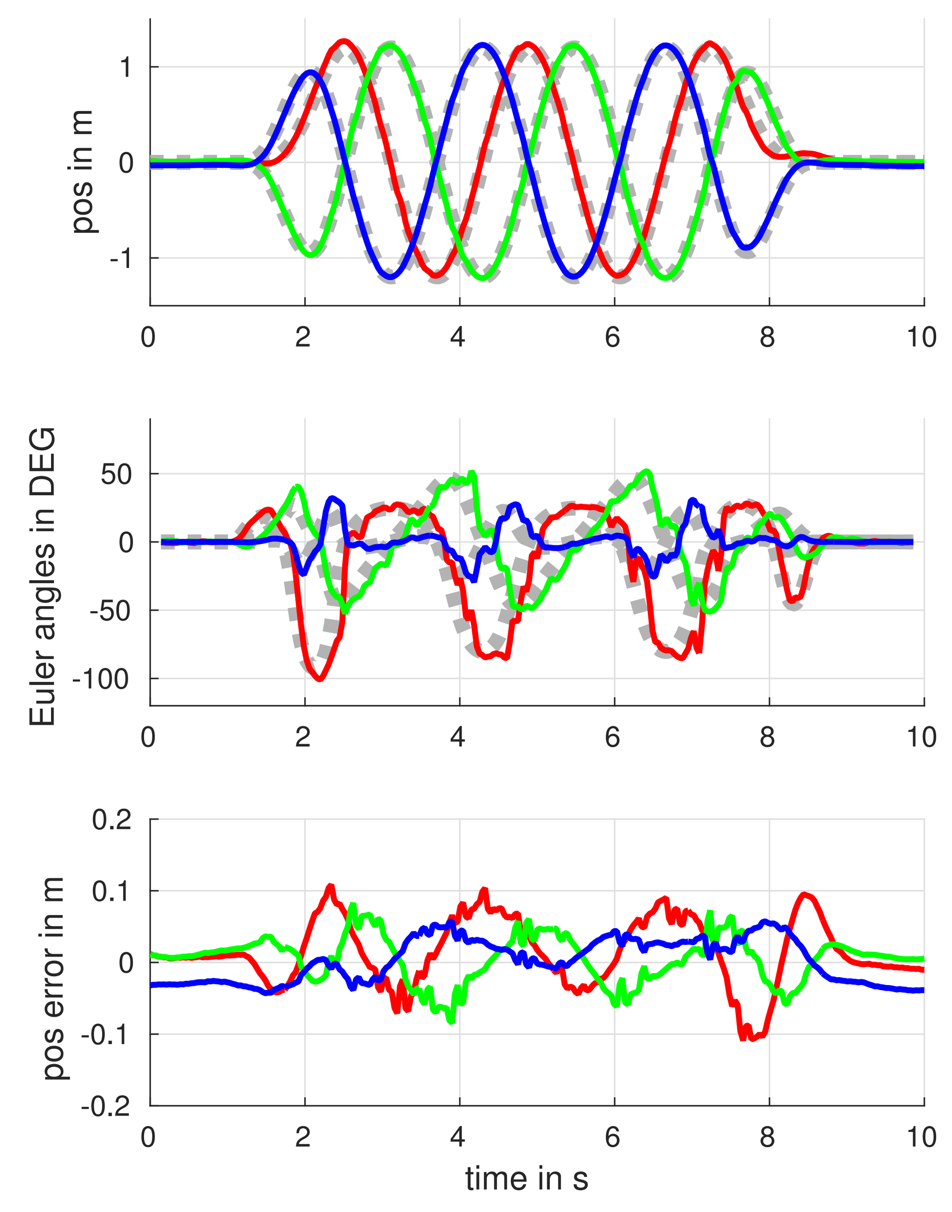
Extension to *tracking control* is more sophisticated but follows the same basic idea

[M. Konz, J. Rudolph: *Beispiele für einen direkten Zugang zu einer globalen, energiebasierten Modellbildung und Regelung von Starrkörpersystemen*, at - Automatisierungstechnik 64(2), pp. 96–109, 2016]

Experimental result

Tracking a circular reference trajectory (velocity on circle $\approx 5\,\mathrm{m/s}$, max. acceleration $\approx 18\,\mathrm{m/s^2}$)





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