Algebraic identification of fluid parameters using transmission line dynamics

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***Abstract*—The dynamic input-output behaviour of cylindrical rigid-walled fluid transmission lines with frequency dependent viscous friction can be described in terms of the Laplace domain model proposed by D'Souza and Oldenburger [1]. Operational calculus is used to generate a set of equations from that model, which can easily be interpreted in the time domain. From these equations physical fluid parameters can be identified. In the present case, both the kinematic viscosity and the product of mass density and bulk modulus of the fluid are computed by evaluating multiple convolutions and integrals of pressure and flow rate signals at two different ends of the line.**

I. Introduction

The knowledge of fluid parameter values is a matter of interest in different technical applications involving fluid power systems. The information provided by these parameters is used not only in model-based controller and observer design but also in condition monitoring and fault detection applications in order to judge the system health status. For example, Chinniah et al. in [2] used an extended Kalman filter approach for estimating the effective bulk modulus in this context.

Another application for fluid parameter identification was presented by Manhartsgruber in [3] where the transmission line dynamics was utilized for flow rate calculation via inverse Laplace transform. In this approach, fluid parameters are obtained as a solution of a frequency domain optimization problem based on the transmission line model derived by D'Souza and Oldenburger in [1].

In the present contribution, on the same transmission line model, algebraic identification techniques are used. A method to calculate the kinematic viscosity as well as the product of mass density and bulk modulus of the fluid from boundary measurements is presented.

The framework for algebraic identification in linear finite dimensional systems was first proposed by Fliess et al. (see [4], [5]). It is based on the use of Mikusiński's operational calculus (see [6]) for generating algebraic equations suitable for parameter identification. This framework has been extended to linear distributed parameter systems governed by a partial differential equation (p.d.e.) with one spatial variable by Rudolph and Woittennek in [7]. The main idea is to reduce the p.d.e. to an ordinary differential equation (o.d.e.) w.r.t. the spatial coordinate. This is achieved by replacing the derivatives w.r.t. time by the operator s assuming homogeneous initial conditions. In the same manner, time signals of system variables are replaced by operational functions. In the case of boundary measurements, the solution of the obtained boundary value problem involves operators which do not directly reveal their time domain correspondent. Hence, they have to be eliminated in order to generate time domain equations for parameter identification. To this end, the knowledge of differential equations w.r.t. s satisfied by those operators is exploited. Once suitable time domain equations have been generated, parameter identification is performed solving linear systems of equations.

This paper is organized as follows: In section II, the transmission line model is briefly discussed, including the solution of the operational domain boundary value problem. Section III is divided into two parts each addressing the elimination of a special operator. The generation of the system of equations used for the identification and the solution of these equations are described in section IV. An illustration of the considered identification method is provided in section V where results from numerical simulations are presented. As an outline for further development, section VI shows an approach accommodating difficulties in flow rate measurements. Finally, section VII summarizes the main aspects of this contribution.

II. Transmission line model

The transmission line model described in this section was first proposed in [1]. In order to give a better understanding of the structure of the operators and the resulting identification equations presented in the following sections, the derivation of the operational domain model is summarized from the identification point of view.

The considered fluid transmission line sketched in Fig. 1 consists of a rigid-walled small diameter pipe with a circular cross section of constant radius R. Assuming laminar flow, small temperature fluctuations, and exploiting rotational sym-

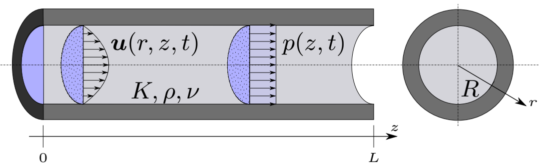

Fig. 1. Fluid transmission line.

metry, the governing p.d.e.s read

$$\frac{\partial}{\partial z}p(z,t)+\left(\frac{\partial}{\partial t}-\nu\frac{\partial^2}{\partial r^2}-\frac{\nu}{r}\frac{\partial}{\partial r}\right)\rho u_z(r,z,t)=0 \tag{1a}$$

$$\frac{1}{K}\frac{\partial}{\partial t}p(z,t)+\left(\frac{1}{r}+\frac{\partial}{\partial r}\right)u_r(r,z,t)+\frac{\partial}{\partial z}u_z(r,z,t)=0 \tag{1b}$$

where $r\in[0,R]$ and $z\in[0,L]$ denote coordinates in radial and axial direction. The system variables p, u_r, and u_z are the steady-state deviations of the pressure and the components of the fluid velocity in radial and axial directions. The density ρ, kinematic viscosity ν, and bulk modulus K of the fluid are assumed to be constant.

Assuming steady-state conditions[1] at time $t=0$, the time derivative is replaced by the operator s. Additionally, the system variables are substituted[2] by operational functions labeled by $\hat{(\cdot)}$. Hence, equation (1a) can be rewritten as a Bessel-type differential equation w.r.t. r:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{s}{\nu}\right)\hat{u}_z(r,z)=\frac{1}{\rho\nu}\frac{\mathrm{d}}{\mathrm{d}z}\hat{p}(z). \tag{2}$$

The no-slip condition at the pipe wall boundary, $r=R$, and the demand for a finite solution at the center of the cross section, $r=0$, lead to the solution

$$\hat{u}_z(r,z)=\left(\frac{I_0\left(r\sqrt{\frac{s}{\nu}}\right)}{I_0\left(R\sqrt{\frac{s}{\nu}}\right)}-1\right)\frac{1}{\rho s}\frac{\mathrm{d}}{\mathrm{d}z}\hat{p}(z). \tag{3}$$

Here, the modified Bessel function of first kind and n-th order is denoted by I_n. In the following, the abbreviation $\hat{I}_n=I_n\left(R\sqrt{\frac{s}{\nu}}\right)$ is used.

Due to the fact that the measurement of the volumetric flow rate

$$q(z,t)=\int_0^{2\pi}\int_0^R u_z(r,z,t)r dr d\phi$$

is technically simpler than measuring the fluid velocity profile, a more favorable model formulation can be achieved by integrating equation (3) over the cross sectional area of the pipe. The application of the same integration to the operational representation of equation (1b) leads to

$$\frac{\rho s}{\pi R^2}\frac{\hat{I}_0}{\hat{I}_2}\hat{q}(z)+\frac{\mathrm{d}}{\mathrm{d}z}\hat{p}(z)=0 \tag{4a}$$

$$\pi R^2\frac{s}{K}\hat{p}(z)+\frac{\mathrm{d}}{\mathrm{d}z}\hat{q}(z)=0. \tag{4b}$$

The system (4) consisting of first order o.d.e.s w.r.t. z can be rewritten in vector notation:

$$\frac{\mathrm{d}}{\mathrm{d}z}\hat{\boldsymbol{y}}(z)=\begin{pmatrix}0 & -\hat{\Gamma}\\ -\hat{\Gamma} & 0\end{pmatrix}\hat{\boldsymbol{y}}(z)=\hat{A}\boldsymbol{y}(z), \tag{5}$$

where $\hat{\boldsymbol{y}}(z)=\left(\hat{p}(z),\hat{Z}\hat{q}(z)\right)$. The transmission characteristic of the line is described by the impedance operator $\hat{Z}$ and the propagation operator $\hat{\Gamma}$:

$$\hat{Z}=\frac{\sqrt{\rho K}}{\pi R^2}\sqrt{\frac{\hat{I}_0}{\hat{I}_2}},\qquad \hat{\Gamma}=s\sqrt{\frac{\rho}{K}}\sqrt{\frac{\hat{I}_0}{\hat{I}_2}}. \tag{6}$$

The fundamental matrix of (5) reads

$$\hat{\Phi}(z)=\begin{pmatrix}\hat{C}(z) & -\hat{S}(z)\\ -\hat{S}(z) & \hat{C}(z)\end{pmatrix}, \tag{7}$$

where $\hat{C}(z)=\cosh(\hat{\Gamma}z)$ and $\hat{S}(z)=\sinh(\hat{\Gamma}z)$. Consequently, any solution of (5) satisfies

$$\hat{p}(z_1)=\hat{C}(z_1-z_0)\hat{p}(z_0)-\hat{S}(z_1-z_0)\hat{Z}\hat{q}(z_0) \tag{8a}$$

$$\hat{Z}\hat{q}(z_1)=\hat{C}(z_1-z_0)\hat{Z}\hat{q}(z_0)-\hat{S}(z_1-z_0)\hat{p}(z_0) \tag{8b}$$

for any $z_0,z_1\in[0,L]$. It can easily be shown that the transfer matrix derived in [1] is equivalent to (8) which is a more convenient model representation in the present context.

III. Elimination of operators

In the following, a transmission line with a full set of boundary measurements[3]

$$\begin{aligned}\hat{p}_0&=\hat{p}(z)\big|_{z=0}, & \hat{q}_0&=\hat{q}(z)\big|_{z=0},\\ \hat{p}_1&=\hat{p}(z)\big|_{z=L}, & \hat{q}_1&=\hat{q}(z)\big|_{z=L}\end{aligned} \tag{9}$$

available is considered for fluid parameter identification. First, the hyperbolic operational expressions $\hat{C}(L)$ and $\hat{S}(L)$ are eliminated by exploiting the fact that they are the fundamental solutions of the o.d.e. system (5). This includes an incidental elimination of the propagation operator $\hat{\Gamma}$ which occurs only in the argument of the hyperbolic functions. In a second step, the impedance operator is eliminated, too.

[1] This assumption implies homogeneous initial conditions at $t=0$ since the system variables are defined as deviations from steady-state values.

[2] The substitutions made can be interpreted either in the sense of Laplace transform or in the sense of Mikusiński's operational calculus [6].

[3] For the sake of brevity, the operators representing boundary measurement signals are referred to simply as boundary measurements.

A. Hyperbolic operators and propagation operator $\hat{\Gamma}$

For the time being, it is assumed that at neither boundary there is a non-reflecting line termination[4]. This implies that the condition $\hat{p}^2(z) - \hat{Z}^2\hat{q}^2(z) \neq 0$ holds for all $z \in [0, L]$. Note that with regard to the structure of the impedance operator $\hat{Z}$, this is not a strong restriction from a technical point of view. Nevertheless, the theoretic case of non-reflecting termination is included in the considerations in section VI.

With (8), it is possible to express the hyperbolic operators in terms of the measurements (9):

$$\hat{C}(L) = \frac{\hat{p}_0\hat{p}_1 - \hat{Z}^2\hat{q}_0\hat{q}_1}{\hat{p}_0^2 - \hat{Z}^2\hat{q}_0^2}, \quad \hat{S}(L) = \frac{\hat{Z}\left(\hat{q}_0\hat{p}_1 - \hat{p}_0\hat{q}_1\right)}{\hat{p}_0^2 - \hat{Z}^2\hat{q}_0^2}. \tag{10}$$

An equation which is suitable for the elimination of $\hat{C}(L)$ and $\hat{S}(L)$ is given by

$$\hat{C}^2(z) - \hat{S}^2(z) - 1 = 0 \tag{11}$$

which can easily be found by examining the determinant of the fundamental matrix $\hat{\Phi}(z)$ in (7),

$$\det\hat{\Phi}(z) = \hat{C}^2(z) - \hat{S}^2(z). \tag{12}$$

At the same time, using the matrix exponential properties of $\hat{\Phi}(z)$, yields

$$\det\hat{\Phi}(z) = \det \mathrm{e}^{\hat{A}z} = \mathrm{e}^{\mathrm{tr}\hat{A}z} = 1, \tag{13}$$

where $\mathrm{tr}\,\hat{A} = 0$ is the trace of the matrix from (5). Evaluating (11) at $z = L$ and using (10), some minor calculations lead to

$$\left(\hat{p}_1^2 - \hat{p}_0^2\right) - \hat{Z}^2\left(\hat{q}_1^2 - \hat{q}_0^2\right) = 0 \tag{14}$$

which is free of any hyperbolic operational expression. Furthermore, the propagation operator $\hat{\Gamma}$ does not occur any longer. Obviously, the only undesirable expression left is the impedance operator $\hat{Z}$. Thus, the next step is to eliminate the latter operator.

B. Impedance operator $\hat{Z}$

Using its definition (6), the impedance operator $\hat{Z}$ can be factorized into a constant part and a term depending on s. Since only the square of the impedance operator occurs in (14), the square of its residual part may be denoted by

$$\hat{\zeta} = \frac{\hat{Z}^2}{Z_0^2} = \frac{\hat{I}_0}{\hat{I}_2}. \tag{15}$$

Note that the constant part, $Z_0 = \sqrt{\rho K}/\left(\pi R^2\right)$, corresponds to the characteristic impedance of a fluid transmission line in the frictionless "water hammer" case. As might be expected, it contains only physical and geometrical parameters and, therefore, does not need to be eliminated. On the contrary, there is no simple time domain correspondent known for $\hat{\zeta}$, which determines the character of the frequency-dependent friction. Thus, an elimination is desirable.

[4]Non-reflecting termination means that due to a matching termination impedance, waves travel only in one direction, i.e. $\hat{p}(z) = \pm\hat{Z}\hat{q}(z)$ for any $z \in (0, L)$.

Accordingly, the objective is to find an equation which is satisfied by the operator $\hat{\zeta}$. Using the recurrence relations for $\hat{I}_2$ (e.g. [8]), the definition (15) is rewritten as

$$\left(\hat{\zeta} - 1\right)\sqrt{s}\hat{I}_0 - 2\sqrt{\frac{\nu}{R^2}}\hat{\zeta}\hat{I}_1 = 0. \tag{16}$$

By deriving equation (16) w.r.t. s and substituting

$$\hat{I}_0' = \frac{1}{2}\sqrt{\frac{R^2}{s\nu}}\hat{I}_1, \qquad \hat{I}_1' = \frac{1}{2}\sqrt{\frac{R^2}{s\nu}}\hat{I}_0 - \frac{1}{2s}\hat{I}_1,$$

an equation

$$\sqrt{\frac{\nu}{R^2}}\left[2s\hat{\zeta}' - \hat{\zeta} - 1\right]\sqrt{s}\hat{I}_0 + \left[s\left(\hat{\zeta} - 1\right) - 2\frac{\nu}{R^2}\left(2s\hat{\zeta}' - \hat{\zeta}\right)\right]\hat{I}_1 = 0 \tag{17}$$

is generated, where $(\cdot)'$ abbreviates the derivative $\mathrm{d}/\mathrm{d}s(\cdot)$. Obviously, (16) and (17) constitute a linear system of equations for $\sqrt{s}\hat{I}_0$ and $\hat{I}_1$. The obvious existence of nontrivial solutions implies that the equations must be linearly dependent and, therefore, the determinant is zero, i.e.,

$$4s\frac{\nu}{R^2}\hat{\zeta}' + s\hat{\zeta}^2 - \left(2s + 4\frac{\nu}{R^2}\right)\hat{\zeta} + s = 0. \tag{18}$$

The Riccati-type differential equation (18) can be used to eliminate $\hat{\zeta}$.

To this end, the operator $\hat{\zeta}$ is expressed in terms of measurements again using equations (14) and (15):

$$\hat{\zeta} = \frac{1}{Z_0^2}\frac{\hat{p}_1^2 - \hat{p}_0^2}{\hat{q}_1^2 - \hat{q}_0^2}. \tag{19}$$

Finally, by replacing $\hat{\zeta}$ with (19) and multiplying by $s^{-1}Z_0^4(\hat{q}_1^2 - \hat{q}_0^2)^2$, equation (18) can be rewritten as

$$\hat{y}_1 Z_0^4 + \hat{y}_2 Z_0^2\frac{\nu}{R^2} + \hat{y}_3 Z_0^2 = \hat{y}_0 \tag{20}$$

with the new operators

$$\begin{aligned} &\hat{y}_0 = -\hat{P}^2, && \hat{y}_1 = \hat{Q}^2, \\ &\hat{y}_2 = 4\left(\hat{P}\hat{Q}' - \hat{P}'\hat{Q} - s^{-1}\hat{P}\hat{Q}\right), && \hat{y}_3 = -2\hat{P}\hat{Q}, \\ &\hat{P} = \hat{p}_1^2 - \hat{p}_0^2, && \hat{Q} = \hat{q}_1^2 - \hat{q}_0^2. \end{aligned}$$

IV. Identification

With (20), one main objective has been achieved: An equation has been found which involves only constant parameters and operators representing measurement signals, and, above all, it is easy to interpret in the time domain. To this end, it is helpful to point out in which way the newly defined operators $\hat{y}_i$ depend on the original measurements. Obviously, there are three basic operations:

1) the multiplication of two operators $\hat{a}$ and $\hat{b}$ which corresponds to the convolution of their time domain signals $t \mapsto a(t)$ and $t \mapsto b(t)$,

$$\hat{a}\hat{b} \quad \bullet\!\!-\!\!\circ \quad \int_0^t a(\tau)b(t-\tau)d\tau,$$

2) the derivation of operators w.r.t. s which corresponds to a multiplication of the time domain signals with $-t$,

$$\hat{a}' \quad \bullet\!\!-\!\!\circ \quad -ta(t),$$

3) and the multiplication with negative powers of s which corresponds to the integration of the time domain signals

$$s^{-\beta}\hat{a} \quad \bullet\!\!-\!\!\circ \quad \frac{1}{(\beta-1)!}\int_0^t (\tau - t)^{\beta-1} a(\tau) d\tau.$$

Consequently, the time domain representation of (20) can be used as a first equation for parameter identification. Since there are two parameters $c_1 = Z_0^2$ and $c_2 = \nu/R^2$, it is necessary to generate at least one further equation for identification. Such generation of further equations can easily be achieved by applying a differential operator

$$\mathrm{D}_s^{(\alpha,\beta)} = s^{-\beta}\left(\frac{\mathrm{d}}{\mathrm{d}s}\right)^{\alpha}, \quad \alpha, \beta \in \mathbb{N}\setminus\{0\}$$

to (20). The resulting time domain equation reads

$$c_1^2 y_1^{(\alpha,\beta)}(t) + c_1 c_2 y_2^{(\alpha,\beta)}(t) + c_1 y_3^{(\alpha,\beta)}(t) = y_0^{(\alpha,\beta)}(t) \quad (21)$$

where

$$y_i^{(\alpha,\beta)}(t) = \frac{(-1)^\alpha}{(\beta-1)!}\int_0^t (t-\tau)^{\beta-1}\tau^\alpha y_i(\tau) d\tau.$$

Obviously, equation (21) is nonlinear w.r.t. the parameters c_1 and c_2. One possibility for parameter identification would be to generate a nonlinear system of equations of such type. Here, another approach proposed in [7] is used in order to avoid solving nonlinear systems of equations.

Therefore, the three components of

$$\bar{\boldsymbol{c}} = (\bar{c}_1, \bar{c}_2, \bar{c}_3)^T = \left(c_1^2, c_1 c_2, c_1\right)^T$$

are considered to be mutually independent. Using n equations of type (21), an overparametrized linear system of equations

$$M(t)\bar{\boldsymbol{c}} = \boldsymbol{b}(t) \quad (22)$$

is constructed in such a way that $|\alpha_i - \alpha_j| + |\beta_i - \beta_j| \neq 0$ holds for all $i, j \in \{1, \ldots, n\}$ where $i \neq j$, that is, the rows of both

$$M(t) = \begin{pmatrix} y_1^{(\alpha_1,\beta_1)}(t) & y_2^{(\alpha_1,\beta_1)}(t) & y_3^{(\alpha_1,\beta_1)}(t) \\ y_1^{(\alpha_2,\beta_2)}(t) & y_2^{(\alpha_2,\beta_2)}(t) & y_3^{(\alpha_2,\beta_2)}(t) \\ \vdots & \vdots & \vdots \\ y_1^{(\alpha_n,\beta_n)}(t) & y_2^{(\alpha_n,\beta_n)}(t) & y_3^{(\alpha_n,\beta_n)}(t) \end{pmatrix}$$

and

$$\boldsymbol{b}(t) = \left(y_0^{(\alpha_1,\beta_1)}(t), \quad y_0^{(\alpha_2,\beta_2)}(t), \quad \ldots, \quad y_0^{(\alpha_n,\beta_n)}(t)\right)^T$$

are pairwise different.

If $M(t)$ has full column rank[5], a solution $\tilde{\boldsymbol{c}}(t)$ minimizing the squared norm of the residual $\boldsymbol{r}(t) = M(t)\tilde{\boldsymbol{c}}(t) - \boldsymbol{b}(t)$ is given by

$$\tilde{\boldsymbol{c}}(t) = \left(M^T(t)M(t)\right)^{-1} M^T(t)\boldsymbol{b}(t). \quad (23)$$

[5]Obviously, it is necessary to generate at least three equations since $\mathrm{rank}\,(M(t)) \leq \min(n, 3)$.

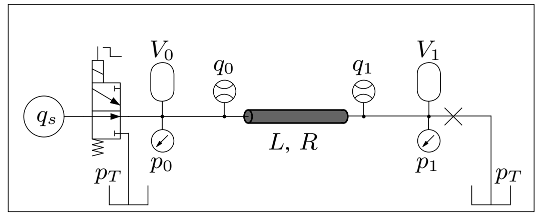

Fig. 2. Simulated circuit.

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Unit
length L	20	m
radius R	$7.5 \cdot 10^{-3}$	m
valve eigenvalues	$70\pi(-1 \pm j)$	s^{-1}
kinematic viscosity ν	$34 \cdot 10^{-6}$	m^2/s
mass density ρ	860	kg/m^3
bulk modulus K	$1.4 \cdot 10^9$	N/m^2
flow rate q_s	20	l/min
orifice nominal flow rate	20	l/min
orifice nominal pressure drop	15	bar
tank pressure p_T	15	bar
dead volumes V_0, V_1	1	l

This solution $\tilde{\boldsymbol{c}}(t)$, which is computed point-wise in time, is an estimate for the constant parameter vector $\bar{\boldsymbol{c}}$. Since the radius R of the cross section is considered to be known, for every instant in time, estimates for the product $\Pi = \rho K$ and the viscosity ν can be obtained as

$$\tilde{\Pi}(t) = \pi^2 R^4 \tilde{c}_3(t) \quad (24a)$$

$$\tilde{\nu}(t) = R^2 \frac{\tilde{c}_2(t)}{\tilde{c}_3(t)}. \quad (24b)$$

In summary, the parameter identification is performed by solving a linear system of equations the coefficients of which are obtained by integration and multiple convolution of boundary measurements and time polynomials. The equations have been generated by utilizing operational calculus to eliminate the hyperbolic operators $\hat{C}(L)$ and $\hat{S}(L)$ as well as the impedance operator $\hat{Z}$.

V. SIMULATION RESULTS

The identification method under consideration has been tested in a simulation both with uncorrupted signals and signals with additive zero-mean white noise. It should be emphasized that the main focus of this contribution is the derivation of the equations for algebraic identification rather than the numerical implementation of the identification method. The foremost purpose of the simulations is to show the usefulness of the algebraic identification approach for fluid transmission lines.

The simulation has been performed utilizing the *hydroLib* Toolbox [9] for Matlab/Simulink which provides a simulation model for hydraulic transmission lines based on the Zielke-Suzuki simulation method (see [10], [11]). A sketch of the

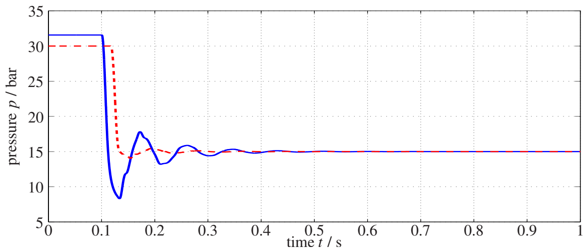

Fig. 3. Boundary pressures $p_0(t)$ (solid) and $p_1(t)$ (dashed).

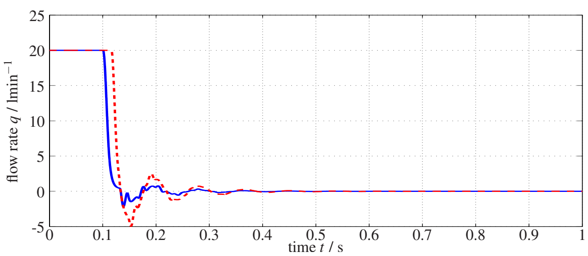

Fig. 4. Boundary flow rates $q_0(t)$ (solid) and $q_1(t)$ (dashed).

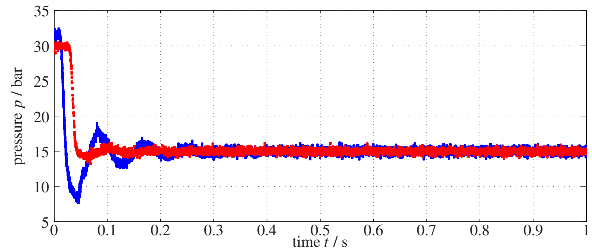

Fig. 6. Boundary pressures $p_0(t)$ (solid) and $p_1(t)$ (dashed).

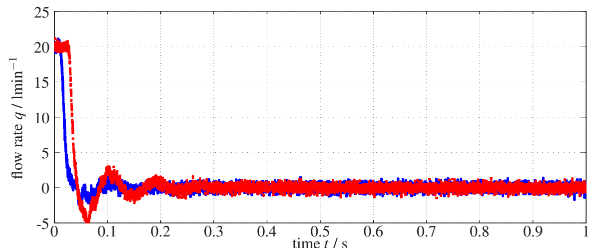

Fig. 7. Boundary flow rates $q_0(t)$ (solid) and $q_1(t)$ (dashed).

simulated system is depicted in Fig. 2. The system is driven by an idealized constant flow source of flow rate q_s. The upstream end of the pipe is connected to a 3/2 directional valve which is modeled as a simple second order lag with a switching time of about 20 ms. The numerical values of the simulation parameters are provided in Table I. An orifice for setting the steady-state pressure is located downstream. Additionally, there are two dead volumes V_1 and V_2, one at each end of the line. As mentioned already in the previous section, the steady state deviations of the boundary pressures and flow rates all serve as measurement data.

The switching valve is closed by a step signal at $t = 0.1$ s. An illustration of the corresponding pressure and flow rate signals is given in Fig. 3 and Fig. 4. Since all geometrical parameters as well as the mass density of the fluid are considered to be known, both the kinematic viscosity and the bulk modulus of the fluid can be estimated using (24).

The result of the identification procedure is displayed in Fig. 5. It can be seen that, after a short amount of time, the identification algorithm provides fairly good estimates for the parameters. The stage of failure in the very beginning is due the fact that the system remains in steady-state, i.e., there are only zero measurements. As a consequence, the matrix $M(t)$ in (22) is rank deficient. Hence, there is no solution of the system and, therefore, identification is not possible in the very beginning. Taking a closer look at the duration of the failure stage reveals that even after the valve closure there is no reasonable output of the identification algorithm. This is due to the fact that the entries of the matrix involve quadruple convolutions of the measurement signals. Therefore, the duration of the stage of failure is exactly four times the length of the period of steady-state.

As a matter of fact, in an experimental environment it is reasonably difficult to get measurement signals with very low corrupting noise. Thus, a second set of simulation results with zero-mean white noise added to each measurement signal is shown in Fig. 6 to Fig. 8. Although the presence of noise obviously has a negative influence on the quality of the estimates, the identification method still provides admissible results.

VI. A METHOD TO OMIT FLOW RATE MEASUREMENTS

Besides the presence of measurement noise, there is yet another drawback when it comes to experiment. Whereas the availability for sensors for highly accurate and fast pressure measurements is no problem, similar requirements for flow rate measurements are difficult to be complied with. Therefore,

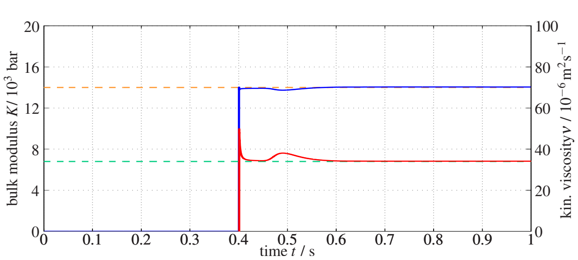

Fig. 5. Estimates $\tilde{\Pi}(t)/\rho$, $\tilde{\nu}(t)$ (solid) and parameters K,ν (dashed).

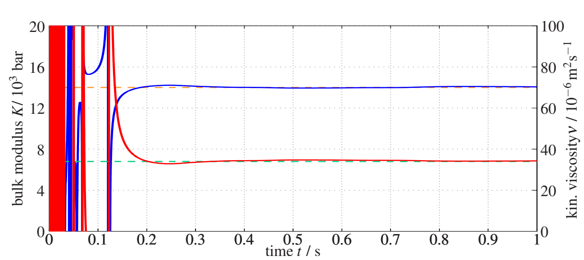

Fig. 8. Estimates $\tilde{\Pi}(t)/\rho$, $\tilde{\nu}(t)$ (solid) and parameters K,ν (dashed).

an outline for further development of the considered identification method is given.

The main idea is to derive an identification algorithm that is based only on pressure measurements, provided that a linear[6] line termination condition

$$\hat{Y}\hat{p}_1 - \hat{q}_1 = 0 \tag{25}$$

is known. The combination of (25) with boundary evaluation of (8) provides three equations that allow for the elimination of both flow rate measurements $\hat{q}_0$ and $\hat{q}_1$. Considering again a transmission line of length L this yields

$$\hat{p}_1\hat{C}(L) + \hat{W}\hat{p}_1\hat{S}(L) - \hat{p}_0 = 0, \tag{26}$$

where $\hat{W} = \hat{Z}\hat{Y}$. Since only one equation is left after eliminating the flow rates, a second equation has to be generated in order to eliminate the hyperbolic operators $\hat{C}(L)$ and $\hat{S}(L)$. By taking the derivative of (26) w.r.t. s, a second equation is obtained:

$$\left(\hat{p}_1' + L\hat{\Gamma}'\hat{W}\hat{p}_1\right)\hat{C}(L) + \left(L\hat{\Gamma}'\hat{p}_1 + (W\hat{p}_1)'\right)\hat{S}(L) - \hat{p}_0' = 0. \tag{27}$$

Solving (26) and (27) for the hyperbolic operators leads to

$$\hat{C}(L) = \frac{\hat{p}_0\hat{p}_1\left(\hat{W}' + L\hat{\Gamma}'\right) + \hat{W}\left(\hat{p}_0\hat{p}_1' - \hat{p}_0'\hat{p}_1\right)}{\hat{p}_1^2\left(L\hat{\Gamma}'\left(\hat{W}^2 - 1\right) - \hat{W}'\right)}, \tag{28a}$$

$$\hat{S}(L) = \frac{\hat{p}_0\hat{p}_1\hat{W}L\hat{\Gamma}' + \hat{p}_0\hat{p}_1' - \hat{p}_0'\hat{p}_1}{\hat{p}_1^2\left(L\hat{\Gamma}'\left(\hat{W}^2 - 1\right) - \hat{W}'\right)}. \tag{28b}$$

As in the case of four measurements, the hyperbolic operators can be eliminated using the expressions (28) in (11) for $z = L$. In order to derive an equation which is qualified for identification, i.e., it can easily be interpreted in the time domain, both $\hat{W}$ and $\hat{\Gamma}$ have to be eliminated from the resulting equation. Due to its lengthy character, this equation is not being stated here.

Note that a necessary condition[7] for the application of the outlined identification method is that $\hat{W}^2 - 1 \neq 0$ which again means that there must not be a non-reflecting line termination. As mentioned in section III-A, the latter case shall be discussed separately at this point. If the output impedance corresponds to the characteristic line impedance, i.e. $\hat{Y} = \pm\hat{Z}^{-1}$, equation (26) reduces to

$$\hat{p}_1 \mathrm{e}^{\pm L\hat{\Gamma}} - \hat{p}_0 = 0. \tag{29}$$

The operator $\exp(\pm L\hat{\Gamma})$ could be interpreted as a dispersive transport operator due to the frequency-dependent friction characteristic. It satisfies

$$\left(\frac{\mathrm{d}}{\mathrm{d}s} \mp L\hat{\Gamma}'\right)\mathrm{e}^{\pm L\hat{\Gamma}} = 0. \tag{30}$$

Replacing the transport operator by $\hat{p}_0/\hat{p}_1$ yields

$$\hat{p}_0'\hat{p}_1 - \hat{p}_0\hat{p}_1' \mp L\hat{\Gamma}'\hat{p}_0\hat{p}_1 = 0. \tag{31}$$

Obviously, an equation for identification could be derived by finally eliminating $\hat{\Gamma}'$ which is closely related to the elimination of $\hat{\zeta}$.

[6] The requirement for linearity could possibly be met by restricting the excitation to small deviations from steady state and using a linear approximation as in (25) for the downstream system model. The impact of such an approximation must be analyzed separately.

[7] If $\hat{W} = \pm 1$, (26) and (27) are linearly dependent w.r.t. $\hat{C}(L)$ and $\hat{S}(L)$ and, therefore, they cannot be solved for these operators.

VII. Conclusion and Future Work

A method for the identification of fluid parameters using algebraic identification techniques has been considered. An operational model for transmission lines with frequency-dependent friction has been examined in order to derive equations for parameter identification. Utilizing operational calculus, an algorithm for calculating the kinematic viscosity as well as the product of mass density and bulk modulus from measurements of boundary pressures and flow rates has been developed. The usefulness of the considered method has been illustrated in numerical examples. In addition, an outline for further development has been given which involves omitting the flow rate measurements. Future work should concern the implementation of the outlined method in an experimental environment.

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