Nonlinear Observer Design for State Estimation during Anti-lock Braking

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Abstract—A novel model based scheme for the state estimation during anti-lock braking is presented based on a LuGre tyre model and a quarter car model. The wheel angular speed and longitudinal vehicle acceleration information are used to design a nonlinear observer based on the unscented Kalman filter theory. This nonlinear observer provides the necessary information for the anti-lock control despite the presence of measurement noise, the unknown parameter of the dynamic tyre model, the road friction coefficient, and the road inclination.

Keywords: Model-based design, LuGre tyre model, quarter car model, anti-lock control, nonlinear observer, unscented Kalman filter.

I. INTRODUCTION

A. Anti-lock brake systems

The Anti-lock braking system is an important component of the vehicle safety system. The main objective of the anti-lock brake system is to avoid wheel lock-up in order to preserve the vehicle steerability and stability on one hand and to maximize tyre braking force in order to minimize the vehicle stopping distance on the other hand. For these purposes, the anti-lock brake system must work robustly in different situations such as road inclination, different road friction coefficients (e.g. asphalt or ice), and different tyre types (e.g. winter or summer tyre).

There are several kinds of anti-lock control designs. Wheel slip control based on different control methods (e.g. [1], [2]) is widely used in the anti-lock brake systems. Compared with anti-lock strategies based on logic switching [3], those with wheel slip control can make the wheel converge to a fixed value of slip ratio in the stable region as well as in the unstable region of the tyre. Wheel slip controllers work even if there is no clear maximum in the tyre characteristics. In addition, the wheel slip control can also be used by other functions of the vehicle safety system, e.g. a vehicle dynamic controller [4]. Therefore, a wheel slip control.

Observer design is indispensable for wheel slip control for different reasons. Firstly, for the optimal performance of wheel slip control, the vehicle velocity which cannot be directly measured is required in addition to the measured wheel angular velocity to have a precise value of wheel slip ratio. Therefore, the vehicle velocity must be precisely estimated by an appropriate observer despite the parameter uncertainties of the vehicle and the environment influence. Secondly, in the study of [5], it was found that the optimal slip ratio is related to the road friction coefficient. Thus, further improvement of the performance of wheel slip control will be achieved, if the optimal slip ratio, with respect to varying road friction coefficients, is utilized as reference signal for the wheel slip control. Obviously, this is only possible if the road friction coefficient is also properly estimated by the observer.

B. State of the Art

The LuGre tyre model is a first-order dynamic friction model, introduced to capture the friction phenomenon more accurately [6]. In comparison with the empirical Pacejka model [7], the LuGre model can reproduce observed behavior on real tyres, like the moving maximum in the tyre characteristics [8] which depends on the relative velocity due to the Stribeck effect. The parameters of the LuGre model are related to material properties, such as tyre rubber stiffness. Therefore, their effects are easier to analyze. Because of its advantages, the LuGre model is widely used to design nonlinear observers for the state estimation.

A nonlinear observer was designed in [9] with adaptive estimation of the road friction coefficient using only the measured wheel angular velocity. Due to the structure of the vehicle-tyre system, the estimated vehicle velocity converges very slowly. In addition, this observer design was found not to be robust enough against measurement noise [10].

To improve the robustness of the observer, sliding mode observers were applied in [11] and [12]. In these approaches, the road friction coefficient was estimated after the sliding surface has been reached. The slow convergence of the estimated vehicle velocity remains an unsolved problem. Furthermore, the estimation is corrupted due to the chattering effect from the first order sliding mode method.

In order to overcome the slow convergence problem in the estimation, both the wheel angular velocity and the vehicle longitudinal acceleration were used in [13]. The vehicle velocity can now be estimated separately from the \dot{v}_x -measurement with the assumption that road inclination does not exist. But with the existence of road inclination the convergence of vehicle velocity estimation is no longer guaranteed.

There is still no observer approach that takes into consideration all the real conditions such as road inclination, measurement noise, changing road friction coefficient, and unknown tyre parameters. In addition, an observability analysis seems lacking in the literature.

C. Contribution of the Paper

A nonlinear observer will be designed in this paper, which should estimate both the vehicle velocity and the changing road friction coefficient for the anti-lock control. To ensure the accuracy and the robustness of the observer, the LuGre tyre internal state and the tire rubber stiffness are also defined as state variables and included in the estimation. For the nonlinear observation, the measurements of both the wheel angular velocity and the vehicle longitudinal acceleration will be used. Furthermore, an observability analysis will be carried out for the plant in order to prove the observability of defined state variables with respect to the measured signals.

II. MATHEMATICAL MODELING OF PLANT

This section presents a dynamic system model for the antilock braking system consisting of a quarter car model and the LuGre tyre model.

A. LuGre Tyre Model

In this paper, the lumped LuGre model suggested by [2] will be used

$$\dot{z} = v_r - \sigma_0 \theta \frac{|v_r|}{\varphi(v_r)} z - \kappa r \omega z,$$
 (1)

$$\varphi(\nu_r) = \mu_c + \frac{\mu_s - \mu_c}{1 + (\frac{\nu_r}{\nu_s})^2}, \qquad (2)$$

where z is the friction internal state, $v_r = r\omega - v_x$ is the relative velocity with the tyre rolling radius r, the angular velocity of the wheel ω , and the vehicle velocity v_x , $\varphi(v_r)$ is the normalized static friction coefficient with the normalized Coulomb friction μ_c , the normalized static friction μ_s , and the Stribeck relative velocity v_s . The parameter θ is used to model the effect of different road friction coefficients ($\theta = 1$ for dry asphalt and $\theta = 4$ for ice). The model accuracy is improved with the extra parameter κ which depends on the shape-profile of the tyre normal force distribution.

Following [2], the tyre rubber damping σ_1 and the viscous relative damping σ_2 are neglected in this paper. The friction coefficient μ is given by

$$\mu = -\sigma_0 z, \tag{3}$$

where σ_0 is the tyre rubber stiffness. After defining wheel slip as

$$\lambda = -\frac{\mathbf{r}\mathbf{0} - \mathbf{v}_x}{\mathbf{v}_x} = -\frac{\mathbf{v}_r}{\mathbf{v}_x},\tag{4}$$

the friction coefficient μ depending on wheel slip λ , vehicle velocity v_x , and road friction coefficient θ can be calculated for the stationary friction internal state $\dot{z} = 0$, as shown in Fig.1.



Fig. 1. Static view of the lumped LuGre model at $\dot{z} = 0$ under different values of θ and v_x .

B. Quarter Car Model

The quarter car model is widely applied in the anti-lock control design because of its proper description of the longitudinal vehicle braking dynamics and its simple structure:

$$\dot{\omega} = \frac{1}{I} \left(\operatorname{rmg} \cos \alpha \mu - M_b \right) = f_{\omega}(\mu, M_b), \tag{5}$$

$$\dot{v}_x = -g\cos\alpha\mu - g\sin\alpha - C_w v_x^2 = f_v(\mu, v_x), \qquad (6)$$

where J is the moment of wheel inertia, m is the mass of the quarter car, g is the gravitational constant, α is the road inclination, M_b is the braking torque, and C_w is the normalized aerodynamic coefficient.

C. Actuator Model

In the ABS actuators, the produced brake torque is physically limited by

$$M_b = \begin{cases} M_{b,\max}^{act}, & M_{b,d} > M_{b,\max}^{act} \\ M_{b,d}, & 0 \le M_{b,d} \le M_{b,\max} \\ 0, & M_{b,d} < 0 \end{cases}$$
(7)

where $M_{b,\text{max}}^{\text{act}}$ is the maximal physically possible brake torque and 0 is the minimal physically possible brake torque. The value $M_{b,d}$ is the reference brake torque from the anti-lock control.

Besides the magnitude limits, the ABS actuators are always subject to limits in their rate. In this paper, the rate limits are assumed to be constant and they are different for increasing and decreasing brake torque. The rate limits can be written as

$$\dot{M}_{b} = \begin{cases} \dot{M}_{b,\text{inc}}, & \dot{M}_{b,d} > \dot{M}_{b,\text{inc}} \\ \dot{M}_{b,d}, & -\dot{M}_{b,\text{dec}} \le \dot{M}_{b,d} \le \dot{M}_{b,\text{inc}} \\ -\dot{M}_{b,\text{dec}}, & \dot{M}_{b,d} < -\dot{M}_{b,\text{dec}} \end{cases}$$
(8)

with the rate limit $\dot{M}_{b,\text{inc}}$ for increasing brake torque, and the rate limit $\dot{M}_{b,\text{dec}}$ for decreasing brake torque.

The actuator dead time is assumed to be small and therefore neglected.



Fig. 2. Block diagram of the anti-lock control system with observer design as the focus of the paper.

III. WHEEL SLIP CONTROL

In this paper, the main focus lies on the nonlinear observer design for vehicle state estimation. Thus, a basic wheel control functionality similar to that in [14] will be used in this paper, which consists of a reference calculator and a PI-controller with anti-windup (AW) strategy.

A. Reference Calculator

As shown in Fig.2, the reference slip ratio λ_d will be taken from a mapping function defined by [15] depending on the estimated road friction coefficient $\hat{\theta}$

$$\lambda_d = f(\hat{\theta}). \tag{9}$$

As the input of the PI control is the reference angular velocity ω_d , it has to be calculated by solving (4) for ω_d

$$\omega_d = \frac{\hat{v}_x(1 - \lambda_d)}{r}.$$
 (10)

B. PI Control

Defining $e_{\omega} = \omega_d - \omega$, the PI control is given by

$$M_{b,\rm PI} = K_p e_{\omega} + K_i I, \tag{11}$$

where K_p is the proportional gain, K_i is the integral gain, and I is the integral part which satisfies

$$\dot{I} = e_{\omega} = \omega_d - \omega = f_I(\omega_d, \omega). \tag{12}$$

To find the values of K_p and K_i , (12) is used together with (5) as system equations. After being linearized about the operating point, the system matrix of the closed loop PI control is obtained as

$$A = \begin{bmatrix} \frac{\partial f_{\omega}}{\partial \omega} & \frac{\partial f_{\omega}}{\partial I} \\ \frac{\partial f_{I}}{\partial \omega} & \frac{\partial f_{I}}{\partial I} \end{bmatrix} = \begin{bmatrix} \frac{K_{p}}{J} + \frac{\operatorname{mgcosa}}{J} \frac{\partial \mu}{\partial \omega} & -\frac{K_{i}}{J} \\ -1 & 0 \end{bmatrix}, \quad (13)$$

where $\frac{d\mu}{d\omega}$ is due to the tyre friction model depending on the vehicle velocity v_x which varies during the anti-lock braking. Therefore, the parameters K_p and K_i have to be chosen properly to guarantee the stability of the system defined by (13) despite the change of v_x .

C. Anti-windup Strategy

Integral windup could occur in the PI control through the magnitude and rate limits of the actuator, which will lead to poor behavior of the controller. With an anti-windup strategy, the magnitude and rate limits of the actuator will also be taken into account in the control law.

For the anti-windup purpose, the control input e_{ω} is now limited to

$$e_{\omega,AW} = \begin{cases} e_{\omega,\max}, & e_{\omega} > e_{\omega,\max} \\ e_{\omega}, & e_{\omega,\min} \le e_{\omega} \le e_{\omega,\max} \\ e_{\omega,\min}, & e_{\omega} < e_{\omega,\min} \end{cases}$$
(14)

where the maximal and minimal control inputs $e_{\omega,\text{max}}$ and $e_{\omega,\text{min}}$ are given by [14] as functions depending on the magnitude and rate limits of the ABS actuator

$$e_{\omega,\max} = g_{e_{\omega,\max}}(M_{b,\max}^{\text{act}}, \dot{M}_{b,\text{inc}}), \qquad (15a)$$

$$e_{\omega,\min} = g_{e_{\omega,\min}}(\dot{M}_{b,\text{dec}}). \tag{15b}$$

IV. OBSERVER DESIGN

For the optimal performance of wheel slip control, vehicle velocity v_x has to be estimated using the nonlinear observer, where the measurements of both the wheel angular velocity ω and the vehicle longitudinal acceleration \dot{v}_x will be used. To ensure the accuracy and the robustness of the observer, the tyre internal state *z*, the road friction coefficient θ , and the tyre rubber stiffness σ_0 will also be defined as state variables and included in the estimation.

For the nonlinear observer design, the quarter car model with LuGre tyre defined by (1)-(3) and (5)-(6) will be used. The road friction coefficient θ and the tyre rubber stiffness σ_0 are assumed to be constant:

$$\dot{\theta} = 0, \tag{16}$$

$$\dot{\sigma}_0 = 0. \tag{17}$$

A. Observability Analysis

In this section an observability analysis is carried out to show the observability of the state variables with respect to the defined input and output:

$$u = M_b, \quad \mathbf{x} = \begin{bmatrix} \mathbf{\omega} \\ \mathbf{v}_x \\ \mathbf{\dot{v}}_x \\ \mathbf{z} \\ \mathbf{\theta} \\ \mathbf{\sigma}_0 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{\omega} \\ \mathbf{\dot{v}}_x \end{bmatrix}. \tag{18}$$

Using the measured signals, the states x_1 and x_3 can be expressed as

$$x_1 = y_1, \tag{19}$$

$$x_3 = y_2.$$
 (20)

As the road inclination α is usually small, the simplification $\sin \alpha = \alpha, \cos \alpha = 1$ can be made. Thus, (5) and (6) can be rewritten as

$$\dot{\omega} = q\mu - \frac{1}{J}M_b, \qquad (21)$$

$$\dot{v}_x = -g\mu - g\alpha - C_w v_x^2 \tag{22}$$

with q = mgr/J. Using the derivatives of (21) and (22) with respect to time with the assumption $\dot{\alpha} = 0$, we have

$$\ddot{\omega} = q\dot{\mu} - \frac{1}{J}\dot{M}_b, \qquad (23)$$

$$\ddot{v}_x = -g\dot{\mu} - 2C_w v_x \dot{v}_x. \tag{24}$$

By solving (23) and (24) for v_x , we obtain

$$x_{2} = v_{x} = -\frac{\ddot{v}_{x} + \frac{g}{q} \ddot{\omega} + \frac{g}{J_{q}} \dot{M}_{b}}{2C_{w} \dot{v}_{x}} = \Psi_{2}(u, \dot{u}, ..., y, \dot{y}, ...)$$
(25)

without knowing the exact value of the road inclination α . The same result can also be found for road banking.

For the further analysis, the following system of equations is obtained from (3) and its first and second derivatives with respect to time

$$\mu = -\sigma_0 z, \qquad (26)$$

$$\dot{\mu} = -v_r \sigma_0 - \kappa r \omega \mu - \sigma_0 \theta f(v_r) \mu, \qquad (27)$$

$$\ddot{\mu} = -\dot{v}_r \sigma_0 - \kappa r (\dot{\omega} \mu + \omega \dot{\mu}) - \sigma_0 \theta (f'(v_r) \mu + f(v_r) \dot{\mu}) (28)$$

with $f(v_r) = \frac{|v_r|}{\varphi(v_r)}$ and $f'(v_r) = \frac{\partial f(v_r)}{\partial v_r} \dot{v}_r$. Solving (26)-(28), one gets

$$x_4 = \sigma_0 = \frac{\ddot{\mu} + \kappa r \dot{\omega} \mu + \kappa r \omega \dot{\mu} - \zeta(v_r, \mu, \dot{\mu}) \left(\dot{\mu} + \kappa r \omega \mu\right)}{-\dot{v}_r + \zeta(v_r, \mu, \dot{\mu}) v_r}$$
(29)

with $\zeta(v_r, \mu, \dot{\mu}) = \frac{f'(v_r)\mu + f(v_r)\dot{\mu}}{f(v_r)\mu}$. As μ , v_r , and ω are functions of the system input, the measured outputs, and their derivatives, σ_0 can be rewritten as $\psi_4(u, \dot{u}, ..., \mathbf{y}, \dot{\mathbf{y}}, ...)$.

In the same way, it can be easily found that

$$x_5 = z = \Psi_5(u, \dot{u}, ..., y, \dot{y}, ...),$$
 (30)

$$x_6 = \theta = \psi_6(u, \dot{u}, ..., y, \dot{y}, ...).$$
 (31)

The above analysis has shown that all the states of the system can be directly calculated from the system input, the measured outputs, and their derivatives, up to several singularities like $-\dot{v}_r + \zeta(v_r, \mu, \dot{\mu})v_r = 0$ in (29). Therefore, the system is proven to be observable with respect to the defined input and output up to the singularity points [16].

B. Unscented Kalman Filter

The unscented Kalman filter (UKF) is an extension of the linear Kalman filter for nonlinear systems. Using a method of nonlinear transformation, UKF was proven to be precise and robust for nonlinear state estimation in [17]. Because of the filter property, the observer using UKF is also robust against measurement noise. In addition, it is possible to take state constraints into account in the UKF algorithm [18], which is impossible or very difficult to realize using other observer design methods.

Consider the following discrete-time nonlinear system

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k, \quad \mathbf{w}_k \sim (0, \mathbf{Q}), \\ \mathbf{y}_{k+1} &= \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k, \quad \mathbf{v}_k \sim (0, \mathbf{R}), \end{aligned}$$
(32)

where w_k is the process noise vector with zero mean and covariance defined as Q, v_k is the measurement noise vector with zero mean and covariance defined as R. Both process and measurement noise are assumed to be in the additive form. The UKF-algorithm in additive form is defined as follows [17]:

 Initialization at k = 0: Initialize the state variables

$$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0],\tag{33}$$

where $E[\cdot]$ is the symbol for the expected value. Initialize the state covariance matrix

$$\boldsymbol{P}_{\boldsymbol{x},0} = E[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^T].$$
(34)

• For k = 1, 2, ...:

1) Calculate 2N+1 sigma-points based on the current state covariance

$$\chi_{i,k|k} = \begin{cases} \hat{\mathbf{x}}_{k}, & i = 0\\ \hat{\mathbf{x}}_{k} + \eta(\sqrt{\mathbf{P}_{x,k}})_{i}, & i = 1, ..., N\\ \hat{\mathbf{x}}_{k} - \eta(\sqrt{\mathbf{P}_{x,k}})_{i}, & i = N+1, ..., 2N \end{cases}$$
(35)

where N is the system dimension.

In (35)
$$\eta$$
 is a scaling parameter

$$\eta = \sqrt{N + \beta} \tag{36}$$

with the tuning parameter β . The coefficient $\beta \ge 0$ must be chosen to guarantee the semi-positive definiteness of the covarivance matrix, a good default choice is $\beta = 0$ [19].

Project the sigma points which are outside the feasible region on the boundary in order to obtain the constrained sigma points [18]

$$\boldsymbol{\chi}_{i,k|k}^{c} = \Phi\left(\boldsymbol{\chi}_{i,k|k}\right), \quad i = 0, ..., 2N,$$
(37)

where Φ refers to the projections.

2) Time update equations

Transform the constrained sigma points through the nonlinear state function

$$\begin{array}{ll} \boldsymbol{\chi}_{i,k+1|k}^{-} &= \boldsymbol{f} \left(\boldsymbol{\chi}_{i,k|k}^{c}, \boldsymbol{u}_{k} \right). \\ i &= 0, \dots, 2N \end{array}$$
(38)

Again apply the constraints on the transformed sigma points to obtain the constrained transformed sigma points

$$\chi_{i,k+1|k}^{-,c} = \Phi\left(\chi_{i,k+1|k}^{-}\right), \quad i = 0,...,2N.$$
 (39)

Calculate the a priori state estimation and a priori covariance

$$\hat{\mathbf{x}}_{k+1}^{-} = \sum_{i=0}^{2N} \left(w_i \boldsymbol{\chi}_{i,k+1|k}^{-,c} \right), \tag{40}$$

$$\boldsymbol{P}_{x,k+1}^{-} = \sum_{i=0}^{2N} w_i \left(\chi_{i,k+1|k}^{-,c} - \hat{\boldsymbol{x}}_{k+1}^{-} \right) \left(\chi_{i,k+1|k}^{-,c} - \hat{\boldsymbol{x}}_{k+1}^{-} \right)^T + \boldsymbol{Q},$$
(41)

where the covariance matrices of the process noise Q is summed to the covariance matrix calculated from the sigma points to consider the process noise. The weight factors w_i are defined as

$$\begin{array}{ll} w_i &= \frac{\beta}{N+\beta}, & i=0 \\ w_i &= \frac{1}{2(N+\beta)}, & i=1,...,2N. \end{array}$$
 (42)

3) Output prediction

Calculate 2N + 1 sigma points based on the a priori estimate

$$\chi_{i,k+1|k} = \begin{cases} \hat{\mathbf{x}}_{k+1}^{-}, & i = 0\\ \hat{\mathbf{x}}_{k+1}^{-} + \eta(\sqrt{\mathbf{P}_{x,k+1}^{-}})_{i}, & i = 1, ..., N\\ \hat{\mathbf{x}}_{k+1}^{-} - \eta(\sqrt{\mathbf{P}_{x,k+1}^{-}})_{i}, & i = N+1, ..., 2N \end{cases}$$
(43)

The same projection technique is used to avoid violation of state constraints:

$$\chi_{i,k+1|k}^{c} = \Phi\left(\chi_{i,k+1|k}\right), \quad i = 0,...,2N.$$
 (44)

Transform the sigma points through the nonlinear output function and calculate the mean value of the output vector

$$\gamma_{i,k+1|k} = \boldsymbol{g}\left(\boldsymbol{\chi}_{i,k+1|k}^{c}, \boldsymbol{u}_{k+1}\right), \quad i = 0, \dots, 2N, \quad (45)$$

$$\hat{\mathbf{y}}_{k+1}^{-} = \sum_{i=0}^{2N} \left(w_i \gamma_{i,k+1|k} \right).$$
(46)

4) Measurement update equations

The covariance of the measurement vector and the cross covariance are calculated according to

$$\begin{aligned} \boldsymbol{P}_{y,k+1} &= \sum_{i=0}^{2N} w_i \left(\boldsymbol{\gamma}_{i,k+1|k} - \hat{\boldsymbol{y}}_{k+1}^{-} \right) \left(\boldsymbol{\gamma}_{i,k+1|k} - \hat{\boldsymbol{y}}_{k+1}^{-} \right)^T + \boldsymbol{R}, \end{aligned}$$
(47)
$$\boldsymbol{P}_{xy,k+1} &= \sum_{i=0}^{2N} w_i \left(\boldsymbol{\chi}_{i,k+1|k}^c - \hat{\boldsymbol{x}}_{k+1}^- \right) \left(\boldsymbol{\gamma}_{i,k+1|k} - \hat{\boldsymbol{y}}_{k+1}^- \right)^T, \end{aligned}$$
(48)

where in order to consider the measurement noise the covariance matrices of the measurement noise R is summed to the measurement covariance calculated from the sigmapoints.

The Kalman gain is given by

$$\boldsymbol{K}_{k+1} = \boldsymbol{P}_{xy,k+1} \boldsymbol{P}_{y,k+1}^{-1}$$
(49)

and the UKF estimation and its covariance are computed from the standard Kalman update equations

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1}^{-} + \boldsymbol{K}_{k+1} \left(\boldsymbol{y}_{k+1} - \hat{\boldsymbol{y}}_{k+1}^{-} \right), \quad (50)$$

$$\boldsymbol{P}_{x,k+1} = \boldsymbol{P}_{x,k+1}^{-} - \boldsymbol{K}_{k+1} \boldsymbol{P}_{y,k+1}^{-1} \boldsymbol{K}_{k+1}^{1}.$$
(51)

C. Application to the Quarter Car Model with LuGre Tyre

The quarter car model with LuGre tyre in the continuous state space form can be expressed as

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{f}(\mathbf{x}(t)) + u(t), \quad t > 0, \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) + \mathbf{v}(t), \\ \mathbf{x} &\in D_{\mathbf{y}} \subset \mathbb{R}^{6}, \ u \in D_{u} \subset \mathbb{R}, \ \mathbf{v} \in D_{v} \subset \mathbb{R}^{2}, \ \mathbf{y} \in \mathbb{R}^{2}, \end{aligned}$$
 (52)

where the input, state, and measurement vectors u(t), $\mathbf{x}(t)$, and $\mathbf{y}(t)$ are given by (18), and $\mathbf{v}(t)$ is the measurement noise vector.

To apply the above described UFK algorithm, discretization of the continuous time model (52) is necessary. For the discretization, the classical forward Euler integration is used in this paper due to its simplicity.

V. SIMULATION RESULTS

In this section, simulation results are presented for the nonlinear observer using the UKF algorithm designed in this paper together with the applied wheel slip controller. The previously described quarter car model and LuGre tyre model are used for the simulation of the plant. The controller and observer work with a sample time of 0.001 s. In the simulation, the model parameters given in Table I are used.

TABLE I Nominal Parameters of Lugre Model, Quarter Car Model, and Actuator Model

Parameter	Value	Unit
m	300	Kg
r	0.3	m
J	1	Kgm ²
g	9.8	m/s ²
C_w	0.0003675	1/m
σ_0	110	1/m
μ_c	0.7	-
μ_s	1.4	-
v_s	3.967	m/s
κ	5.45	1/m
$M_{h,\text{max}}^{\text{act}}$	3000	Nm
$\dot{M}_{b,inc}$	15000	Nm/s
$\dot{M}_{b, dec}$	15000	Nm/s

Several typical anti-lock braking maneuvers are defined. In all the anti-lock braking maneuvers, the vehicle will start from the unbraked driving condition with an initial longitudinal velocity $v_{x,0}$ of 27.78 m/s (\approx 100 km/h). The anti-lock control will be activated for a wheel slip ratio larger than 0.2 and deactivated at low vehicle velocity smaller than 1 m/s.

In the observer, the initial velocity $\hat{v}_{x,0}$ is set to $1.1v_{x,0}$ with 10% estimate error and the other estimated states are set to $\hat{\omega} = 1.1\omega_{m,0}$, $\hat{v}_{x,0} = 1.1\dot{v}_{x,m0}$, $\hat{z} = 0$, $\hat{\theta} = 2$, and $\hat{\sigma}_0 = 100$ 1/m, where $\omega_{m,0}$, and $\dot{v}_{x,m0}$ are the noisy measurement signals of the wheel angular velocity and the vehicle longitudinal acceleration at the beginning of the simulation.

A. Anti-lock Braking on High-µ

Fig. 3 shows the simulation results for an anti-lock braking on high- μ . From Fig. 3 (left), it can be seen that through the wheel slip control using anti-windup PI strategy the magnitude and rate limits of the ABS actuator were not violated and the reference slip ratio was reached shortly after the anti-lock activation. Despite the presence of measurement noise in both ω_m and $\dot{v}_{x,m}$ the estimated velocity \hat{v}_x converged very quickly. The bottom plot in Fig. 3 (left) shows that the road coefficient $\hat{\theta}$ was already properly estimated before the activation of the anti-lock control. Good estimation results were also achieved for other estimated states, as shown in Fig. 3 (right).

B. Anti-lock Braking on Low-µ

Fig. 4 depicts the simulation results for an anti-lock braking on low- μ . In the estimation, the different road friciton was identified in less than 0.2 s, which allows the proper choice of the reference slip ratio for the low- μ road conditon, and thereby an optimal anti-lock braking.

C. Robustness Analysis during Anti-lock Braking

In the following the robustness of the observer is tested with several critical changes in the tyre parameter and road condition.

1) Anti-lock Braking on High-µ with Road Inclination

A road inclination α of 0.2 rad is assumed in the plant model, which is almost the maximal possible road inclination of normal streets. The simulation results are shown in Fig. 5. Due to the presence of road inclination there was a small offset in the σ_0 -estimation. Although that, good results were still achieved for other estimated states.

2) Anti-lock Braking on High-µ with Unknown Tyre Parameter

The tyre rubber stiffness σ_0 depends on tyre types. From tyre to tyre it can be quite different. A change of 20% from σ_0 compared to the nominal parameter is assumed here. From Fig. 6 it can be seen that the large change of σ_0 was well identified by the observer. The state estimation as well as the anti-lock control still work very well.

3) Anti-lock Braking with Negative µ-jump

The most critical situation during the anti-lock braking is a sudden change of the road friction coefficient θ , the so-called μ -jump. Fig. 7 depicts an anti-lock braking with negative μ -jump from high- μ to low- μ . The μ -jump happened at t = 1.5 s and it was very well handled by the UKF. The changed road friction coefficient was quickly taken into account in the wheel slip control. The overall anti-lock control keeps working stably in spite of the sudden change of the road friction coefficient θ .

4) Anti-lock Braking with Positive µ-jump

As shown in Fig. 8, the UKF also yields reliable and precise estimates during an anti-lock braking with positive μ -jump from low- μ to high- μ .

VI. CONCLUSION

A novel nonlinear observer based on an UKF algorithm is designed for state estimation during anti-lock braking. An observability analysis is carried out for the plant to prove the observability of defined state variables with respect to the measured signals. It is shown in simulations that the designed filter can estimate the state variables precisely, quickly, and robustly despite the presence of measurement noise, uncertainty in the tyre rubber stiffness, and unknown road inclination. Even the situation of a sudden change in the road friction coefficient can be handled by the filter very well. Using the information from the nonlinear filter, promising performance of anti-lock control is ensured.

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Fig. 3. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on high- μ with nominal model parameters.



Fig. 4. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on *low-µ* with *nominal* model parameters.



Fig. 5. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on *high-µ* with $\alpha = 0.2$ rad.



Fig. 6. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on high- μ with $\sigma_0 = 120\% \sigma_{0,n}$.



Fig. 7. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on *high-µ* with *negative* μ -jump at t = 1.5 s.



Fig. 8. Brake torque, slip ratio, vehicle velocity, wheel speed, and road friction coefficient (left); Vehicle longitudinal acceleration, internal friction state, and tyre rubber stiffness (right) during anti-lock braking on high- μ with positive μ -jump at t = 1.5 s.