# On invariant control design for systems admitting classical symmetries

The notion of symmetry of a dynamical system has been a subject of long standing interest in the treatment of dynamical systems. Roughly speaking, a transformation (or a family of transformations) is a symmetry (group) of a dynamical system if the transformation maps solutions to solutions. The knowledge of admitted symmetries of a dynamical system can help to obtain a qualitative understanding of the dynamics and the underlying problem that has been modeled.

Numerous control problems naturally admit symmetries (e.g. rotation & translation, unit scaling). Therefore, the problem of designing *invariant feedback laws*, i.e. control laws that preserve an existing symmetry of a nonlinear control system, arises [RR99]. Further, the availability of input signals can be used to address the question of designing a feedback such that a desired symmetry group is admitted by the controlled system leading to *controlled symmetries* [SB05]. In this talk both aspects are addressed by means of examples.

To this end, the design of invariant feedback laws is motivated by presenting a tracking control scheme for a kinematic car which is invariant under rotation and translation. After a short excursion to more general results taken from [MRR04], two controlled symmetries are designed in order to render the feedback for a predator-prey bioreactor set-point invariant and growth kinetic invariant, respectively.

## References


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