

EXERCISES 1

1) Show that the kernel of a closed operator is closed. Is the same true for a closable operator.

2) Let Q be the multiplication operator on $L^2([0, 1])$. (Note that Q is bounded.)

$$Q : L^2([0, 1]) \rightarrow L^2([0, 1]) \quad f \mapsto Qf, \quad (Qf)(t) = tf(t)$$

Is the range of Q closed?

3) Define an unbounded operator T on $L^2([0, 1])$ with domain $D(T) = C([0, 1])$ (continuous functions) by

$$Tf = f(0)1,$$

where 1 is the constant function, $1(t) = 1$. Is T closable?