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K-Theory of C^*-algebras
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Exercise Sheet 1

The aim of this exercise sheet is to understand $K^0(X) \cong K_0(C(X))$ for any compact space X. The main source for this are the lecture notes of Christian Voigt (in german; distributed via email). Additionally, you may read Chapter 1, Section 1 of [1].

Exercise 1. Read Chapter 1, Sections 1 and 2 of [2]. What are vector bundles and their morphisms? Definition of K^0 (Def. 1.5).

Exercise 2. Read Chapter 1, Section 3 of [2]. What are projective modules? How to see them as direct summands of free modules? What is the link to idempotents?

Exercise 3. Read Chapter 1, Section 4 of [2]. What is the theorem of Serre-Swan? What are the main ideas behind the proof? Briefly mention also Theorem 1.18.

Exercise 4. Read Chapter 2, Section 2 of [2]. What is the link between idempotents and projections? Summarize some properties.

Exercise 5. Prove Proposition 2.17 in [2]: $K^0(X) \cong K_0(C(X))$.

References

[1]   Bruce Blackadar. *K-Theory for Operator Algebras*. Mathematical Sciences Research Institute publications. Cambridge University Press, 1998.

[2]   Christian Voigt. *K-Theorie für Operatoralgebren*. Lecture Notes.