



K-Theory of C^* -algebras

Summer term 2023

Exercise Sheet 2

The aim of this exercise sheet is to understand the Fredholm picture of K_0 . The main reference is the book ‘K-Theory and C^* -algebras’ by Wegge-Olsen [2]. For the basics about Hilbert C^* -modules you may also consult [1].

Exercise 1. Read Chapter 15, Sections 1-2 in [2]. What are Hilbert C^* -modules and their operators? What is the standard module \mathcal{H}_A over a C^* -algebra A , how does A relate to the operators on \mathcal{H}_A ?

Exercise 2. Read Chapter 15, Sections 3-4 in [2]. Summarize the main facts about complementable submodules and polar decompositions. How to see finitely generated projective A -modules as Hilbert- A -modules (mention Theorem 15.4.2)? Mention Kasparov’s Stabilization Theorem and its consequences.

Exercise 3. Read Chapter 17, Sections 1-2 in [2]. What are generalized Fredholm operators? State the Atkinson-Mingo Theorem. Define the generalized Fredholm index.

Exercise 4. Read Chapter 17, Section 3 in [2]. What are the main properties of the generalized Fredholm index? State Theorem 17.3.11.

References

- [1] E. Christopher Lance. *Hilbert C^* -modules: A toolkit for operator algebraists*. Cambridge University Press, 2011.
- [2] N. E. Wegge-Olsen. *K-theory and C^* -algebras: A friendly approach*. Oxford University Press, 1993.