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K-Theory of C^*-algebras

Summer term 2023

Exercise Sheet 3

The aim of this exercise sheet is to understand AF algebras and their K-theory. The main reference is the book 'C^*-algebras by example' by Davidson [2]. You may also take a look at [1].

Exercise 1. Bratteli-diagrams (Ch. III.1, III.2): finite dimensional C^*-algebras as direct sums of matrix algebras and the role of their matrix units $E_{ij}^{(k)}$, in particular Cor. III.2.2; Def. Bratteli-diagrams; examples; Prop. III.2.7 with (sketched) proof.

Exercise 2. Ideals and quotients in AF algebras: Ch. III.4 and III.5 (sketched proof of Thm. III.4.2.).

Exercise 3. Local characterization of AF algebras and short exact sequences (Ch. III.3, III.6): Thm. III.3.4, Thm. III.6.3, vague idea of the proofs.

Exercise 4. K_0 of stably finite C^*-algebras: Ch. IV.1 (mainly IV.1.6) and IV.2.

Exercise 5. Dimension groups and a bit of Riesz groups: Ch. IV.3 and a bit of Ch. IV.6 + IV.7 (mainly IV.3.3; summarize a bit of IV.6.1 + IV.6.4; briefly mention IV.7.2 + IV.7.3).

Exercise 6. Elliott's classification theorem of AF algebras (Thm. IV.4.3); mention a few things from Ch. IV.5 (summarize IV.5.1 + IV.5.2 + IV.5.3).

References

[1] Bruce Blackadar. *K-Theory for Operator Algebras*. Mathematical Sciences Research Institute publications. Cambridge University Press, 1998.

[2] Kenneth R. Davidson. *C*-algebras by example*. Fields Institute Monographs. 1996.