Faculty of Mathematics
Moritz Weber
Marcel Scherer
Jonas Metzinger

UNIVERSITÄT DES SAARLANDES

Fakultät für Mathematik und Informatik

Operator Algebras Problem Set 10

To be submitted by Wednesday, **July 2**, 4 pm.

Exercise 1

Suppose H is separable and let $M \subset \mathcal{B}(H)$ be an abelian von Neumann algebra. Show that there exists a single selfadjoint operator $a \in M$ which generates M, i.e. $\{a\}'' = M$.

Hint: Choose a sequence of projections $(p_n)_{n\in\mathbb{N}}$ in M which is strongly dense in the set of all projections in M (why does such a sequence exist?) and consider the operator $\sum_{n=0}^{\infty} 3^{-n} p_n$. You may use without proof the fact that the unit ball of $\mathcal{B}(H)$ is separable and metrizable in the strong operator topology.

Exercise 2

Let $M \subset \mathcal{B}(H)$ be a von Neumann algebra and let $(p_i)_{i\in I}$ be an arbitrary family of projections in M. The *supremum* of the family $(p_i)_{i\in I}$ is the projection onto the subspace

$$\overline{\operatorname{span}}\left(\bigcup_{i\in I} p_i(H)\right) \subset H$$

and is denoted by $\bigvee_{i\in I} p_i$. Show that $\bigvee_{i\in I} p_i \in M$.

Talk

This talk covers the *Cuntz algebra* and should give a sketch of the proof of Theorem 7.20.

(a) Define the *Cuntz algebra* $\mathcal{O}_n$ and briefly argue why it exists.

(b) Define *multi-indices* and *words* in $\mathcal{O}_n$.

(c) Prove Lemma 7.16 a), b) and c).

(d) Define $\mathcal{F}_n^k, \mathcal{F}_n$ and $\mathcal{S}$.

(e) Prove Lemma 7.15.

(f) Define ρ_ζ and φ, and note (without proof) some facts about φ.

(g) Define *purely infinite* C^*-algebras.

(h) Sketch the proof of Theorem 7.20, that is the Cuntz algebra $\mathcal{O}_n$ is purely infinite. (You may use the lemmas in section 7.6 without proving them.)