ULTRAPRODUCTS AND NON-COMMUTATIVE PROBABILITY

DAVID JEKEL

Abstract: Tracial von Neumann algebras serve as an analog of non-commutative probability spaces. Ultraproducts are a construction that allows us to obtain "limits" of these non-commutative probability spaces and the corresponding non-commuting random variables, including for instance limiting objects for random matrix models. It is natural to ask how to relate the random matrix ultraproduct $\prod_{n\to\mathcal{U}} L^\infty[0,1] \otimes \mathbb{M}_n$ with the deterministic ultraproduct $\prod_{n\to\mathcal{U}} \mathbb{M}_n$, or whether the operations of randomization and ultraproduct can be interchanged in some sense.

I will present joint work with David Gao which shows that indeed there is a way to "derandomize" an ultraproduct of von Neumann algebras with nontrivial center by fixing a character on the center. This also addresses another natural question in the model of von Neumann algebras concerning classification of tracial von Neumann algebras up to elementary equivalence (or equivalently, up to isomorphism of their ultrapowers). We can show that if two direct integrals are elementarily equivalent, then the fibers must be elementarily equivalent under some mild hypotheses; this is converse to a result of Farah and Ghasemi, and both directions have now been generalized by Ben Yaacov, Ibarlucia, and Tsankov. Thus, the classification of tracial von Neumann algebras up to elementary equivalence can be reduced to that of tracial factors.