Noncommutative C^k Functions, Multiple Operator Integrals, and Derivatives of Operator Functions

Abstract: Let $\mathcal{A}$ be a unital C^*-algebra, $f\colon \mathbb{R} \to \mathbb{C}$ be a continuous function, and $f_{\mathcal{A}}\colon \mathcal{A}_{\text{sa}} \to \mathcal{A}$ be the functional calculus map $\mathcal{A}_{\text{sa}} \ni a \mapsto f(a) \in \mathcal{A}$. It is elementary to show that $f_{\mathcal{A}}$ is continuous, so it is natural to wonder how the differentiability properties of f transfer to those of $f_{\mathcal{A}}$. This turns out to be a delicate problem. In this talk, I introduce a rich class of "noncommutative C^k functions" f such that $f_{\mathcal{A}}$ is k-times differentiable. I shall also discuss the interesting objects, called multiple operator integrals, used to express the derivatives of $f_{\mathcal{A}}$.