MULTIVARIABLE DE BRANGES-ROVNYAK SPACES

MAXIMILIAN LEIST

Abstract. Let H^∞ be the Banach algebra of bounded holomorphic functions on the unit disc $\mathbb{D}$ in $\mathbb{C}$, equipped with the supremum norm. Given $b \in H^\infty$ with $\|b\|_\infty \leq 1$, one defines the de Branges-Rovnyak space $\mathcal{H}(b)$ associated to b as the reproducing kernel Hilbert space on $\mathbb{D}$ with kernel

$$k^b(z,w) = \frac{1 - b(z)\overline{b(w)}}{1 - z\overline{w}}.$$

Although these spaces were originally introduced in the context of operator model theory, they are nowadays studied on their own right. In recent years, Jury and Martin extended much of the classical theory of these spaces to several variables, where H^∞ is replaced by the multiplier algebra of the Drury-Arveson space on the open unit ball of $\mathbb{C}^d$.

In this talk, we will first review parts of the classical theory of de Branges-Rovnyak spaces, in particular the theory of Aleksandrov-Clark measures. After that, we will discuss how the classical theory can be generalized to the multivariable setting. It will turn out that this generalization is not straightforward and requires the use of non-commutative methods.