

Title: “Noises, Spatial Product Systems, and Free Product Systems”

Abstract: “This talk is actually about the last third of what I planned for Roland’s birthday conference last June, but what I did not manage to talk about in June. But, please, don’t worry, I will not assume any knowledge nor memory of what I managed to say in June. ☺ It just means that, instead of starting with those things that were related to Roland’s work more or less directly as in June, this time I will start with the part concentrating on my own inventions, repeating whatever will be needed from June when it is needed.

Even when limited to mathematics, the term ‘noise’ can stand for different things that, at least at first sight, have nothing much in common. (Brownian motion and other Lévy processes, but also Schürmann’s quantum Lévy processes; Tsirelson’s noises made by Liebscher into the theory of stationarily factoring measure types; quantum dynamics with invariant states or invariant conditional expectations.) I will not tell more about this in the talk. I just take as a starting point the basic insight

noises are – always! – intimately related to spatial tensor product systems
and start my talk with the latter and I will explain the connection with noises
as those occurring in quantum dynamics.

Every noise comes along with a monotone filtration (related to monotone independence). But when does such a noise (always monotone) also admit a free filtration (turning it into a free noise)? We are not (yet?) able to give a concrete answer to this question. What we can do, is:

- Construct a noise from a free product system. This parallels the construction being part of the intimate relation between noises and spatial tensor product systems, and works, roughly, by replacing tensor products (of Hilbert spaces or correspondences) with free products.
- Construct from every spatial tensor product system (that is, from a noise) a free product system (generated by it) and further, by the preceding item, a free noise. This parallels the construction of a product system from a subproduct system.

Both things will be visualized in the example of the full Fock space or module.”