Abstract: We prove results that we found on our way to a deeper understanding of Schäffer's conjecture about inverse operators. In 1970 J.J. Schäffer proved that for any invertible $n \times n$ matrix T and for any operator norm $\|\cdot\|$, the inequality

$$|\det\{T\}| \left\|T^{-1}\right\| \leq \mathcal{S} \left\|T\right\|^{n-1}$$

holds with $\mathcal{S} = \mathcal{S}(n) \leq \sqrt{en}$. He conjectured that in fact this inequality holds with an $\mathcal{S}$ independent of n. This conjecture was refuted in the early 90's by E. Gluskin, M. Meyer and A. Pajor who have shown that for certain $T = T(n)$ the inequality can only hold when $\mathcal{S}$ is growing with n. Subsequent contributions of J. Bourgain and H. Queffélec provided increasing lower estimates on Schäffer's. Those results rely on probabilistic and number theoretic arguments for the existence of sequences $T(n)$ with growing $\mathcal{S}$. Constructive counterexamples to Schäffer's conjecture were not available since 1995. In this talk we propose a new and entirely constructive approach to Schäffer's conjecture. As a result, we present an explicit sequence of Toeplitz matrices T_λ with singleton spectrum $\{\lambda\} \subseteq \mathbb{D} \setminus \{0\}$ such that $\mathcal{S} \geq c(\lambda)\sqrt{n}$. A key ingredient in our approach will be to investigate l_p-norms of Fourier coefficients of powers of a Blaschke factor, which is an interesting and well-studied topic in its own right, initiated by J-P. Kahane in 1956. Finally, on our way, we prove new estimates for the asymptotic behavior of Jacobi polynomials with varying parameters and we highlight some flaws in the established literature on this topic.

This is based on a joint work with Oleg Szehr.