

# $G$ -invariant Toeplitz algebras on the Fock space

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## Abstract

In this talk, we study commutative  $C^*$ -algebras generated by Toeplitz operators acting on the Fock space  $F^2(\mathbb{C}^n)$  whose symbols are invariant under translations by a Lagrangian subgroup  $G$  of  $\mathbb{C}^n$ .

The Fock space  $F^2(\mathbb{C}^n)$  is defined as the space of holomorphic functions in  $L^2(\mathbb{C}^n, \mu)$ , where  $\mu$  denotes the Gaussian measure on  $\mathbb{C}^n$ . A Toeplitz operator is defined as the compression of a multiplication operator on  $F^2(\mathbb{C}^n)$ .

As it turns out, there is an interesting connection between the behavior of algebras generated by Toeplitz operators and the geometry of the underlying domain  $\mathbb{C}^n$ . In this talk, we explore this connection by studying the commutativity of  $C^*$ -algebras generated by Toeplitz operators.

We begin by deducing the existence and commutativity of these algebras using some ideas from Quantum Harmonic Analysis in the sense of R. Werner [3]. Afterwards, we diagonalize the generating Toeplitz operators by means of a Bargmann-type transform and describe the Gelfand theory of the resulting algebras, extending some previous results by Esmeral and Vasilevski [2].

This talk is based on joint work with Robert Fulsche [1].

## References

- [1] R. Fulsche and M. A. Rodriguez Rodriguez. Commutative  $G$ -invariant Toeplitz  $C^*$ -algebras on the Fock space and their Gelfand theory through Quantum Harmonic Analysis. *Accepted in J. Operator Theory. Preprint available in arXiv:2307.15632*, 2023.
- [2] N. Vasilevski and K. Esmeral.  $C^*$ -algebra Generated by Horizontal Toeplitz Operators on the Fock Space. *Boletín de la Sociedad Matemática Mexicana*, 3, 2016.
- [3] R. Werner. Quantum Harmonic Analysis on Phase Space. *J. Math. Phys.*, 25:1404–1411, 1984.